



PRINCIPLES OF  
**NEURODYNAMICS**

PERCEPTRONS AND THE THEORY  
OF  
BRAIN MECHANISMS

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## PREFACE

It is only after much hesitation that the writer has reconciled himself to the addition of the term "neurodynamics" to the list of such recent linguistic artifacts as "cybernetics", "bionics", "autonomics", "biomimesis", "synnoetics", "intelelectronics", and "robotics". It is hoped that by selecting a term which more clearly delimits our realm of interest and indicates its relationship to traditional academic disciplines, the underlying motivation of the perceptron program may be more successfully communicated. The term "perceptron", originally intended as a generic name for a variety of theoretical nerve nets, has an unfortunate tendency to suggest a specific piece of hardware, and it is only with difficulty that its well-meaning popularizers can be persuaded to suppress their natural urge to capitalize the initial "P". On being asked, "How is Perceptron performing today?" I am often tempted to respond, "Very well, thank you, and how are Neutron and Electron behaving?"

That the aims and methods of perceptron research are in need of clarification is apparent from the extent of the controversy within the scientific community since 1957, concerning the value of the perceptron concept. There seem to have been at least three main reasons for negative reactions to the program. First, was the admitted lack of mathematical rigor in preliminary reports. Second, was the handling of the first public announcement of the program in 1958 by the popular press, which fell to the task with all of the exuberance and sense of discretion of a pack of happy bloodhounds. Such headlines as "Frankenstein Monster Designed by Navy Robot That Thinks" (Tulsa, Oklahoma Times) were hardly designed to inspire scientific confidence. Third, and perhaps most significant, there has been a failure to comprehend the difference in motivation between the perceptron program and the various engineering projects concerned with automatic pattern recognition, "artificial intelligence", and advanced computers. For this writer, the perceptron program is not primarily concerned with the inven-

tion of devices for "artificial intelligence", but rather with investigating the physical structures and neurodynamic principles which underlie "natural intelligence". A perceptron is first and foremost a brain model, not an invention for pattern recognition. As a brain model, its utility is in enabling us to determine the physical conditions for the emergence of various psychological properties. It is by no means a "complete" model, and we are fully aware of the simplifications which have been made from biological systems; but it is, at least, an analyzable model. The results of this approach have already been substantial; a number of fundamental principles have been established, which are presented in this report, and these principles may be freely applied, wherever they prove useful, by inventors of pattern recognition machines and artificial intelligence systems.

The purpose of this report is to set forth the principles, motivation, and accomplishments of perceptron theory in their entirety, and to provide a self-sufficient text for those who are interested in a serious study of neurodynamics. The writer is convinced that this is as definitive a treatment as can reasonably be accomplished in a volume of manageable size. Since this volume attempts to present a consistent theoretical position, however, the student would be well advised to round out his reading with several of the alternative approaches referenced in Part I. Within the last year, a number of comprehensive reviews of the literature have appeared, which provide convenient jumping-off points for such a study.\*

The work reported here has been performed jointly at the Cornell Aeronautical Laboratory in Buffalo and at Cornell University in Ithaca. Both programs have been under the support of the Information Systems Branch of the Office of Naval Research -- the Buffalo program since July, 1957, and the Ithaca

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\* See, for example, Minsky's article, "Steps Toward Artificial Intelligence", Proc. I.R.E., 49, January, 1961, for an entertaining statement of the views of the loyal opposition, which includes an excellent bibliography.

program since September, 1959. A number of other agencies have contributed to particular aspects of the program. The Rome Air Development Center has assisted in the development of the Mark I perceptron, and we are indebted to the Atomic Energy Commission for making the facilities of the NYU computing center available to us.

A great many individuals have participated in this work. R. D. Joseph and H. D. Block, in particular, have contributed ideas, suggestions, and criticisms to an extent which should entitle them to co-authorship of several chapters of this volume. I am especially indebted to both of them for their heroic performance in proofreading the mathematical exposition presented here, a task which has occupied many weeks of their time, and which has saved me from committing many a mathematical felony. Carl Kesler, Trevor Barker, David Feign, and Louise Hay have rendered invaluable assistance in programming the various digital computers employed on the project, while the engineering work on the Mark I was carried out primarily by Charles Wightman and Francis Martin at C.A.L. The experimental program with the Mark I was carried out by John Hay. In addition to all of those who have contributed directly to the research activities, the writer is indebted to Professors Mark Kac, Barkley Rosser, and other members of the Cornell faculty for their administrative support and encouragement, and to Alexander Stieber, W. S. Holmes, and the administrative staffs of the Cornell Aeronautical Laboratory and the Office of Naval Research whose confidence and support have carried the program successfully through its infancy.

Frank Rosenblatt  
15 March 1961

## ERRATA

Page 9, line 10: Change Kohler to Köhler

Page 111, footnote (line 4): Change ; to ,

Page 121, line 12: Change "remaining" to "following"

Page 124, line 14: Change  $b_i X^\#$  to  $b_i (X^\#)$

Page 125, line 18: Change  $b_i X^\#$  to  $b_i (X^\#)$

Page 126, line 4: Change  $b_i X^\#$  to  $b_i (X^\#)$

Page 127, line 7: Change  $b_i X^\#$  to  $b_i (X^\#)$

Page 139, fourth label under "Typical A-units" should read (Probability  $Q_j - Q_{ij}$ ) instead of (Probability  $Q_j - Q_i$ ).

Page 142, Figure 10c: Change  $R = 1.0$  to  $R = .10$

Page 147, Equation 6.10: Include  $E_{ijA}$  in the argument of  $\rho_x$ .

Page 149, footnote (line 4); Change "displayed" to "displaced"

Page 155, line 8: Change  $\alpha_i^*$  to  $a_i^*$

Page 159, first two equations: Change  $e_{ir}^*$  to  $c_{ir}^*$

Page 160, line 5: Change first word to "close"

Page 165, heading of first table: Correct spelling of "excitatory"

Page 166, line 14: Change page reference from 75 to 102

Page 326, line 1: continuous

Page 356, line 8: Change first symbol from  $M_i^{(t)}$  to  $M^{(i)}(t)$

Page 356, line 19: Change  $(t_0)$  to  $(0)$

Page 362, line 11: fifth word should be "region"

Page 391, line 19: Insert the sentence: "Signal transmission times are again taken to be instantaneous."

Page 391, line 22: Change  $t+1$  to  $t+\tau$

Page 395, label of last matrix, change  $<$  to  $>$

Page 399: In label of last matrix, change 302 to 303

Page 443: In subheading of table, change  $x = 2$  to  $x = 3$

Page 622: Change name in eleventh entry to "Chiu"

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## **PART I**

### **DEVELOPMENT OF BASIC CONCEPTS**



