

Development of Research in Microscale and Nanoscale Thermal and Fluid Sciences

MECHANICAL
ENGINEERING
THEORY AND
APPLICATIONS

Lixin Cheng
Editor

NOVA

MECHANICAL ENGINEERING THEORY AND APPLICATIONS

**DEVELOPMENT OF RESEARCH IN
MICROSCALE AND NANOSCALE
THERMAL AND FLUID SCIENCES**

LIXIN CHENG
EDITOR



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PREFACE

Applications of microscale and nanoscale thermal and fluid transport phenomena are involved in traditional industries and highly specialized fields such as bioengineering, chemical and biochemical engineering, micro-fabricated fluidic systems, microelectronics, aerospace technology, micro heat pipes, chips cooling etc. The research in the relevant subjects has been becoming especially important since the late 20th century. However, microscale and nanoscale thermal and fluid transport phenomena are quite different from those at conventional scale or macroscale. Research on the thermal and fluid transport phenomena at microscale and nanoscale has extensively been conducted to understand the very complex phenomena in the past decades. New instrumental methods have been applied to measure the basic physical parameters at microscale and are continuously under development. New test data have been obtained through state-of-the-art experimental facilities. New prediction methods and mathematical models have also been developed to cover both macroscale and microscale channels and are being continuously under investigation. However, there are quite contradictory results in the available research. Furthermore, new theories and mechanisms are also urgently needed for the fluid flow and heat transfer phenomena at microscale and nanoscale. There are many issues to be clarified from both theoretical and applied aspects. In recent years, interdisciplinary research areas are also rapidly under development. For example, as a new research frontier of nanotechnology, the research of nanofluid two-phase flow and thermal physics is rapidly growing. However, it has also posed new challenges as there are quite contradictory results in the available research. There are still a number of issues needed to be solved in the practical applications.

To foster the research development of numerous evolving research topics, technologies and applications based on microscale and nanoscale thermal and fluid transport phenomena, I formed a new journal-*International Journal of Microscale and Nanoscale Thermal Fluid and Transport Phenomena (IJMNTFTP)* in 2010, which provides a high-quality forum specially for a wide range of papers dealing with original research results, technical notes and state-of-the-art reviews pertaining to thermal and fluid transport phenomena at microscale and nanoscale. It is aimed at meeting such urgent needs and to bring these important frontier research works together worldwide. It covers a wide range of topics on fundamentals and applications of micro-scale and nano-scale transfer processes of mass, momentum and energy such as micro-scale and nanoscale heat transfer and fluid flow, nanofluid heat transfer and flow, microfluidics, nanofluidics and technologies based on these transport processes such as various micro-scale and nano-scale thermal and fluid devices, micro and nano energy

systems, micro-cooling technology in the computer and electronics industries and information technologies, MEMS, NEMS and the interdisciplinary research related to micro-scale and nano-scale thermal and fluid transport phenomena in bio-engineering, medical engineering and life engineering etc. Over the past three years, the new journal is going well and has provided an excellent platform for researchers and readers to exchange their research results.

It is my great pleasure to present this preface to this new edited book which includes selected papers from volume 5, 2014 of the *IJMNTFTP*. It is my greatest wish that the book can advance the knowledge in microscale and nanoscale thermal and fluid sciences and thus further promote the research in the microscale and nanoscale thermal and fluid transport phenomena in our community.

Prof. Lixin Cheng

A handwritten signature in black ink, appearing to read 'Lixin Cheng', with a stylized, flowing script.

24 03 2016

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Chapter 1

MIXED CONVECTIVE BOUNDARY LAYER FLOW OVER A VERTICAL WEDGE EMBEDDED IN A POROUS MEDIUM SATURATED WITH A NANOFLUID

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ABSTRACT

A boundary layer analysis is presented for the mixed convection past a vertical wedge in a porous medium saturated with a nano fluid. The prescribed heat and mass flux boundary conditions are considered. The entire regime of the mixed convection is included, as the mixed convection parameter ξ varies from 0 (pure free convection) to 1 (pure forced convection). The transformed nonlinear system of equations is solved by using an implicit infinite difference method. Numerical results for friction factor, surface heat transfer rate and mass transfer rate have been presented for parametric variations of the buoyancy ratio parameter N_r , Brownian motion parameter N_b , thermophoresis parameter N_t and Lewis number Le . The dependency of the friction factor, surface heat transfer rate (Nusselt number) and mass transfer rate (Sherwood number) on these parameters has been discussed.

Keywords: mixed convection, porous medium, nanofluid

NOMENCLATURE

D_B	Brownian diffusion coefficient [$m^2 s^{-1}$]
D_T	thermophoretic diffusion coefficient [$m^2 s^{-1}$]

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f	dimensionless stream function
h	rescaled nano-particle volume fraction
g	gravitational acceleration vector [m s^{-2}]
k_m	effective thermal conductivity of the porous medium [$\text{W m}^{-1} \text{K}^{-1}$]
K	permeability of porous medium
Le	Lewis number
m	wedge angle parameter [$\gamma/(\pi - \gamma)$]
N_r	Buoyancy Ratio
N_b	Brownian motion parameter
N_t	thermophoresis parameter
Nu	Nusselt number
P	pressure [Pa]
q_w	wall heat flux [W m^{-2}]
Ra_x	local Rayleigh number
T	temperature [K]
T_w	wall temperature at vertical wedge [K]
T_∞	ambient temperature attained as y tends to infinity [K]
U_∞	ambient velocity [m s^{-1}]
u, v	velocity components [m s^{-1}]
(x, y)	Cartesian coordinates [m]

Greek Symbols

α_m	thermal diffusivity of porous medium [$\text{m}^2 \text{s}^{-1}$]
β	volumetric expansion coefficient of fluid [K^{-1}]
γ	wedge half angle [rad]
ε	porosity
η	dimensionless distance
ξ	mixed convection parameter
θ	dimensionless temperature
μ	viscosity of fluid [$\text{kg m}^{-1} \text{s}^{-1}$]
ρ_f	fluid density [kg m^{-3}]
ρ_p	nano-particle mass density [kg m^{-3}]
$(\rho c)_f$	heat capacity of the fluid [$\text{J m}^{-3} \text{K}^{-1}$]
$(\rho c)_m$	effective heat capacity of porous medium [$\text{J m}^{-3} \text{K}^{-1}$]
$(\rho c)_p$	effective heat capacity of nano-particle material [$\text{J m}^{-3} \text{K}^{-1}$]
τ	parameter defined by equation (5)
ϕ	nano-particle volume fraction
ϕ_w	nano-particle volume fraction at vertical wedge
ϕ_∞	ambient nano-particle volume fraction attained
ψ	stream function
λ	power law constant

INTRODUCTION

Several researchers are interested in the study of convective heat transfer in nanofluids. Nanofluids are solid-liquid composite materials consisting of solid nanoparticles or nanofibers with sizes typically of 1-100 nm suspended in liquid. It is reported that a small amount (<1% volume fraction) of Cu nanoparticles or carbon nanotubes dispersed in ethylene glycol or oil will increase the inherently poor thermal conductivity of the liquid by 40% and 150%, respectively [1,2]. Conventional particle-liquid suspensions require high concentrations (>10%) of particles to achieve such enhancement. However, problems of rheology and stability are amplified at high concentrations, precluding the widespread use of conventional slurries as heat transfer fluids. In some cases, the observed enhancement in thermal conductivity of nanofluids is orders of magnitude larger than predicted by well-established theories. Nanofluids display a strong temperature dependence of the thermal conductivity [3] and a three-fold higher critical heat flux compared with the base fluids [4, 5]. If confirmed and found consistent, they would make nanofluids promising for applications in thermal management. Furthermore, suspensions of metal nanoparticles are also being developed for other purposes, such as medical applications including cancer therapy. The interdisciplinary nature of nanofluid research presents a great opportunity for exploration and discovery at the frontiers of nanotechnology.

Porous media heat transfer problems have several engineering applications such as geothermal energy recovery, crude oil extraction, ground water pollution, thermal energy storage and flow through filtering media. Cheng and Minkowycz [6] presented similarity solutions for free convective heat transfer from a vertical plate in a fluid-saturated porous medium. Gorla and co-workers [7, 8] solved the nonsimilar problem of free convective heat transfer from a vertical plate embedded in a saturated porous medium with an arbitrarily varying surface temperature or heat flux. The problem of combined convection from vertical plates in porous media was studied by Minkowycz et al. [9] and Ranganathan and Viskanta [10]. Gorla and Kumari [11] presented an analysis for the combined convection along a non-isothermal wedge in a porous medium. All these studies were concerned with Newtonian fluid flows. The boundary layer flows in nano fluids have been analyzed recently by Nield and Kuznetsov [12] and Nield and Kuznetsov [13]. Kuznetsov and Nield [14] considered the thermal instability in porous medium saturated by a nanofluid. For high Lewis numbers, the primary contribution of the nanoparticles was via a buoyancy effect. Kuznetsov and Nield [15] investigated the onset of convection in a horizontal layer of a porous medium saturated by a nanofluid by taking account of local thermal non-equilibrium between the particles and fluid phases as well as between solid matrix and fluid phases. For Large Lewis numbers, the effect of local thermal non-equilibrium is small. Kuznetsov and Nield [16] analyzed a double-diffusive nanofluid convection in porous media. Nield and Kuznetsov [17] studied the double-diffusive nanofluid natural convection in a porous medium when the base fluid was a binary fluid such as salty water. Ahmad and Pop [18] and Nazar et al. [19] analyzed the mixed convection from a vertical plate and circular cylinder, respectively.

Convective heat transfer over a stationary wedge is important for the thermal design of various types of industrial equipments such as heat exchangers, canisters for nuclear waste disposal, nuclear reactor cooling systems and geothermal reservoirs etc.

The present work has been undertaken in order to analyze the mixed convection past a vertical wedge embedded in a porous medium saturated by a nanofluid. The effects of Brownian motion and thermophoresis are included for the nanofluid. Numerical solutions of the boundary layer equations are obtained and discussion is provided for several values of the nanofluid parameters governing the problem. In this paper, the case of opposing boundary layer flow when the boundary layer can separate from the surface of the wedge is not treated. This will be the subject matter for a future investigation.

ANALYSIS

Let us consider here the problem of mixed convection boundary-layer flow of a nanofluid over a non-isothermal vertical wedge with half angle γ embedded in a saturated porous medium. The wall heat and mass fluxes are prescribed. Figure 1 shows the flow model and physical coordinate system. The origin of the coordinate system is placed at the vertex of the wedge, where x is the coordinate along the surface of the wedge is measured from the origin and y is the coordinate normal to the surface, respectively. u and v are the components of the Darcy velocity in the x and y directions, respectively. Assuming low velocity and porosity, the Darcy model is employed in the analysis. Fluid properties are assumed to be constant except the density variation in the buoyancy force term. The viscous dissipation effect is neglected for the low velocity. It is assumed that nanoparticles are suspended in the nanofluid using either a surfactant or surface charge technology. This prevents the particles from agglomeration and deposition on the porous matrix.

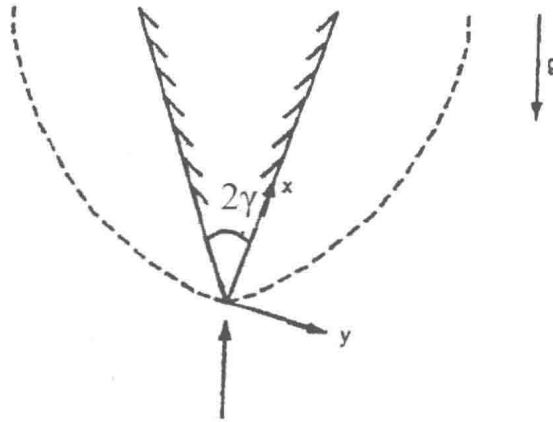


Figure 1. Flow Model and Coordinate System.

We consider the two-dimensional problem. We consider at $y=0$, the heat and mass fluxes take values q_w and m_w , respectively. The ambient values of T and ϕ are denoted by T_∞ and ϕ_∞ , respectively. The Oberbeck-Boussinesq approximation is employed. Homogeneity and local thermal equilibrium in the porous medium is assumed. We consider the porous medium whose porosity is denoted by ϵ and permeability by K .

We now make the standard boundary layer approximation based on a scale analysis and write the governing equations. The model used for the nanofluid incorporates the effects of Brownian motion and thermophoresis. The standard boundary layer approximation is as given below.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\frac{\partial u}{\partial y} = \frac{(1-\phi_\infty)\rho_{f\infty}\beta g_x K}{\mu} \frac{\partial T}{\partial y} - \frac{(\rho_p - \rho_{f\infty})g_x K}{\mu} \frac{\partial \phi}{\partial y} \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_m \frac{\partial^2 T}{\partial y^2} + \tau \left[D_B \frac{\partial \phi}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right] \quad (3)$$

$$\frac{1}{\varepsilon} \left(u \frac{\partial \phi}{\partial x} + v \frac{\partial \phi}{\partial y} \right) = D_B \frac{\partial^2 \phi}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} \quad (4)$$

where

$$\alpha_m = \frac{k_m}{(\rho c)_f}, \quad \tau = \frac{\varepsilon(\rho c)_p}{(\rho c)_f} \quad (5)$$

Here ρ_f , μ and β are the density, viscosity and volumetric volume expansion coefficient of the fluid while ρ_p is the density of the particles. The gravitational acceleration is denoted by g . We have introduced the effective heat capacity $(\rho c)_m$ and effective thermal conductivity k_m of the porous medium. The coefficients that appear in equations (3) and (4) are the Brownian diffusion coefficient D_B and the thermophoretic diffusion coefficient D_T .

The boundary conditions are taken to be

$$v = 0, \quad -k_m \left(\frac{\partial T}{\partial y} \right)_{y=0} = q_w, \quad -D \left(\frac{\partial \phi}{\partial y} \right)_{y=0} = m_w, \quad \text{at } y = 0, \quad (6)$$

$$u \rightarrow U_\infty, T \rightarrow T_\infty, \phi \rightarrow \phi_\infty \text{ as } y \rightarrow \infty \quad (7)$$

Here, $U_\infty = C x^m$ is the velocity of the potential flow outside the boundary layer; B is a constant and m is the wedge angle parameter. We assume that

$$q_w = Ax^\lambda, m_w = Bx^\lambda$$

We introduce a stream line function ψ defined by

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \quad (8)$$

so that Equation (1) is satisfied identically.

Proceeding with the analysis, we define the following transformations:

$$\begin{aligned}
\eta &= \frac{y}{x} \cdot Pe_x^{1/2} \cdot \xi^{-1} \\
\xi &= \frac{Pe_x^{1/2}}{Pe_x^{1/2} + Ra_x^{1/3}} \\
s(\xi, \eta) &= \frac{\psi}{\alpha_m} \cdot \xi \cdot Pe_x^{-1/2} \\
\theta &= \frac{(T - T_\infty) \xi^{-1} \sqrt{Pe_x}}{(q_w x / k_m)} \\
f &= \frac{(\phi - \phi_\infty) \xi^{-1} \sqrt{Pe_x}}{(m_w x / D)} \\
Pe_x &= \frac{U_\infty x}{\alpha_m} = \frac{C \cdot x^{m+1}}{\alpha_m} \\
Ra_x &= \frac{(1 - \phi_\infty) \rho_{f\infty} \cdot \beta g_x \cdot K \cdot x^2 q_w}{\mu \cdot \alpha_m \cdot k_m} \\
g_x &= g \cos \gamma = x \text{ component of "g"}
\end{aligned} \tag{9}$$

In the above equations, the primes denote the differentiation with respect to η only. ξ is the mixed convection parameter. It is noted that $\xi=0$ and 1 correspond to pure free and pure forced convection cases, respectively. The entire regime of mixed convection corresponds to the values of ξ between 0 and 1. Substituting the above expressions into the governing equations (1)-(4), we have the following transformed equations

$$S'' - (1 - \xi)^3 [\theta' - N_r f'] = 0 \tag{10}$$

$$\begin{aligned}
\theta'' - \left\{ (\lambda + 1) + \frac{1}{6} (\xi - 1) (2\lambda + 1 - 3m) \right\} S' \\
+ \left\{ \frac{1}{6} (1 - \xi) (2\lambda + 1 - 3m) + \frac{1}{2} (m + 1) \right\} S \theta' \\
+ N_b \xi f' \theta' + N_t \xi \theta'^2 = \frac{1}{6} \xi (\xi - 1) (2\lambda + 1 - 3m) \left\{ S' \frac{\partial \theta}{\partial \xi} - \theta' \frac{\partial S}{\partial \xi} \right\}
\end{aligned} \tag{11}$$

$$\begin{aligned}
f'' - \left\{ (\lambda + 1) Le + \frac{1}{6} (\xi - 1) (2\lambda + 1 - 3m) Le \right\} f S' + \left\{ \frac{1}{6} (1 - \xi) (2\lambda + 1 - 3m) Le \right. \\
\left. + \frac{1}{2} (m + 1) Le \right\} S f' + \frac{N_t}{N_b} \theta'' = \frac{1}{6} \xi (\xi - 1) (2\lambda + 1 - 3m) Le \left\{ S' \frac{\partial f}{\partial \xi} - f' \frac{\partial S}{\partial \xi} \right\}
\end{aligned} \tag{12}$$

where the four parameters are defined as

$$N_r = \frac{(\rho_P - \rho_{f\infty}) k_m m_w}{\rho_{f\infty} \beta q_w D_B (1 - \phi_\infty)}$$

$$N_b = \frac{\varepsilon (\rho c)_P D_B m_w x}{Pe_x^{1/2} k_m D}$$

$$N_t = \frac{\varepsilon(\rho c)_p D_T q_w x}{T_\infty Pe_x^{1/2} k_m k_m}$$

$$Le = \frac{\alpha_m}{\varepsilon D_B} \quad (13)$$

The transformed boundary conditions are:

$$\begin{aligned} S(\xi, 0) &= 0, \theta'(\xi, 0) = -1, f'(\xi, 0) = -1 \\ S'(\xi, \infty) &= \xi^2, \theta(\xi, \infty) = 0, f(\xi, \infty) = 0 \end{aligned} \quad (14)$$

The local friction factor may be written as

$$C_{fx} = \frac{2\mu \left(\frac{\partial u}{\partial y} \right)_{y=0}}{\rho_f \infty U_\infty^2} = \frac{2 \left[Pe_x^{\frac{1}{2}} + Ra_x^{\frac{1}{3}} \right]^3}{Re_x Pe_x} S''(\xi, 0) \quad (15)$$

The heat transfer rate q_w at the surface of the wedge is

$$q_w = -k_f \left(\frac{\partial T}{\partial y} \right)_{y=0} \quad (16)$$

The heat transfer coefficient is given by

$$h = \frac{q_w}{(T_w - T_\infty)} \quad (17)$$

The local Nusselt number is given by

$$Nu_x = \frac{h \cdot x}{k_m} = Pe_x^{\frac{1}{2}} \cdot \xi^{-1} [\theta(\xi, 0)]^{-1} \quad (18)$$

where h denotes the local heat transfer coefficient and k represents the thermal conductivity.

The Mass Transfer rate is given by:

$$N_w = -D \left(\frac{\partial \phi}{\partial y} \right)_{y=0} = h_m (\phi_w - \phi_\infty)$$

where h_m = mass transfer coefficient,

The local Sherwood number is given by

$$Sh_x = \frac{h_m \cdot x}{D} = Pe_x^{\frac{1}{2}} \cdot \xi^{-1} [f(\xi, 0)]^{-1} \quad (19)$$

Numerical Method and Validation

Equations (10) through (12) represent an initial-value problem with ξ playing the role of mixed convection parameter. This general non-linear problem cannot be solved in closed form and, therefore, a numerical solution is necessary to describe the physics of the problem. The implicit, tridiagonal finite-difference method similar to that discussed by Blottner [20] has proven to be adequate and sufficiently accurate for the solution of this kind of problems. Therefore, it is adopted in the present work. All first-order derivatives with respect to ξ are replaced by two-point backward-difference formulae when marching in the positive ξ direction. Then, all second-order differential equations in η are discretized using three-point central difference quotients. This discretization process produces a tri-diagonal set of algebraic equations at each line of constant ξ which is readily solved by the well known Thomas algorithm (see Blottner [20]). During the solution, iteration is employed to deal with the nonlinearity aspect of the governing differential equations. The problem is solved line by line starting with line $\xi = 0$ where similarity equations are solved to obtain the initial profiles of velocity, temperature and concentration and marching forward (or backward) in ξ until the desired line of constant ξ is reached. Variable step sizes in the η direction with $\Delta\eta_1 = 0.001$ and a growth factor $G = 1.035$ such that $\Delta\eta_n = G\Delta\eta_{n-1}$ and constant step sizes in the ξ direction with $\Delta\xi = 0.01$ are employed. These step sizes are arrived at after many numerical experimentations performed to assess grid independence. The convergence criterion employed in the present work is based on the difference between the current and the previous iterations. When this difference reached 10^{-5} for all points in the η directions, the solution was assumed converged and the iteration process was terminated.

RESULTS AND DISCUSSION

Equations (10-12) were solved numerically to satisfy the boundary conditions (14) for parametric values of Le , N_r (buoyancy ratio number), N_b (Brownian motion parameter), N_t (thermophoresis parameter) and m (wedge angle parameter) using finite difference method.

In order to assess the accuracy of numerical results, we presented results for Nusselt number for $N_r = N_t = N_b = 0$ in Figure 2. A comparison of our results with literature data from Hsieh et al. [22] and Kumari and Gorla [11] suggests that the present results are highly accurate.

Figure 3 shows that as N_r increases, the velocity decreases and the temperature and concentration increase. Similar effects are observed from Figures 4 and 5 as N_t and N_b vary. The parameter N_r appears only in the momentum boundary layer equation (10). Buoyancy is principally a macroscale effect. The buoyancy influences the velocity and temperature fields, however, has a minor effect on nano particle diffusion. This explains the minor influence of buoyancy on concentration profiles.

Thermophoresis parameter, N_t appears in the thermal and concentration boundary layer equations. As we note, it is coupled with temperature function and plays a strong role in determining the diffusion of heat and nanoparticle concentration in the boundary layer. From Figure 4, we note that the temperature and nanoparticle concentration are elevated as N_t increases where as the velocity decreases with increasing values of N_t .