



The background of the cover features several technical diagrams. At the top, there are vector fields labeled E , J_0 , J_1 , \hat{n} , S , and V . Below these, a graph plots milliamperes/m against an unlabeled horizontal axis. The graph contains several curves: a solid line, a dashed line, a dotted line with circular markers, and a wavy line. A legend in the top right of the graph area identifies the curves as $|J_x|$ top, $|J_z|$ top, $|J_x|$ bottom, $|J_z|$ bottom, and GO. The label (b) is positioned to the left of the main title.

APPLIED FREQUENCY-DOMAIN ELECTROMAGNETICS

ROBERT PAKNYS



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Robert Paknys

Concordia University, Montreal, Canada

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To Marilyn and Michael

About the Author

Robert Paknys was born in Montreal, Canada. He received the BEng degree from McGill University in 1979, and the MSc and PhD degrees from Ohio State University in 1982 and 1985, respectively, all in electrical engineering.

He was an assistant professor at Clarkson University during 1985-1987 and an engineer at MPB Technologies during 1987-1989. He joined Concordia University in 1989 as a faculty member in electrical and computer engineering, and is a professor. He has served as a consultant for the government and industry.

He was a visiting professor at the University of Auckland in 1996, the University of Houston in 2004 and the Ecole Polytechnique de Montreal in 2010.

Professor Paknys is a registered professional engineer, a member of CNC-URSI Commission B, a senior member of the IEEE, and a past associate editor for the *IEEE Transactions on Antennas and Propagation*.

Preface

The technologies related to electromagnetic waves go back to Hertz, Marconi and the radar systems of World War II. The knowledge gained during those eras propelled the subsequent development of microwave and satellite communications and the ubiquitous wireless technology of today. Understanding electromagnetic scattering is pivotal in the applications of radar target identification, underground geophysical probing as well as security applications such as airport scanners and seeing through walls. Computational electromagnetic modelling is a key element in the design of commercial and military aircraft, and navy ships, where the placement of dozens of collocated antennas must be carefully considered, so that intersystem interference can be mitigated.

Researchers behind these and other advances in technology need to understand both the classical theory of electromagnetics and modern techniques for solving Maxwell's equations. To this end, this book provides a graduate-level treatment of selected topics. Chapters 1 and 2 present background material on Maxwell's equations, plane waves and rigorous and approximate boundary conditions. Chapter 3 develops solutions for rectangular, cylindrical and dielectric waveguides and resonators. In Chapter 4, some crucial theorems, principles and potential theory are explained in detail. Chapter 5 presents the solutions to some canonical problems that have an exact solution, such as the cylinder, wedge and sphere. Chapter 6 describes the method of moments. Chapter 7 covers the finite element method. Chapter 8 is about the uniform geometrical theory of diffraction, and Chapter 9 covers physical optics and the physical theory of diffraction. Chapters 10–12 are about Green's functions and their applications.

Analytical methods provide physical insights that are valuable in the design process and the invention of new devices. The separation of variables method is applied to waveguides, cylinders, wedges and other canonical shapes. Asymptotic methods address the evaluation of integrals, as well as diffraction theory. Green's function concepts are presented in the two-dimensional (2D) scalar and three-dimensional (3D) dyadic forms, and their interpretation is given in relation to the surface equivalence principle.

Numerical methods are indispensable as they allow us to solve highly arbitrary and realistic problems that the purely analytical techniques cannot. The method of moments and the finite element method are described in dedicated chapters. The level of presentation allows the reader to immediately begin applying the methods to some problems of moderate complexity. It also provides an explanation of the underlying theory so that its capabilities and limitations can be understood. This has value as it helps one make informed decisions when using modern CAD tools.

Often, in the preliminary stages of research, it is very useful to investigate field behaviour by using 2D problems. This way, it is often possible to greatly simplify the problem while still retaining the essential characteristics of the fields. It is also a good way to learn the subject, as it minimizes the mathematical complexity and makes the field solutions easier to physically interpret. The book emphasizes a 2D approach, however, where appropriate, 3D is also used.

The book is aimed at graduate students and engineers in industry and R&D labs. The minimum assumed background is an undergraduate course in waves and transmission lines. The first three chapters aim to put all readers on an equal footing – thereby readers with diverse backgrounds and levels of

familiarity are accommodated. The coverage is intended to assist research students who are beginning to explore the current engineering literature, as well as more experienced researchers who need to learn about new topics.

The way people look for relevant literature has changed dramatically in the past 20 years. For this reason, no attempt has been made to compile a comprehensive list of references, which in any case would be prone to rapid obsolescence. Rather, each chapter contains a small list of references that should help readers proceed and find the key books and papers that address their specific interests. Many fine works have been omitted, and should any authors feel slighted, I offer my apologies in advance.

The topics are not necessarily arranged by the subject category, but in the order that they are most easily learned and applied. Some topics are revisited at a gradually increasing depth. For instance, waveguides are in Chapters 3 and 4, and the surface equivalence principle is in Chapters 4 and 10.

The homework problems have been developed with an intention to provide motivation and opportunities for practice, as well as revealing new concepts. There are problems for review purposes, for analytical development and for programming.

For both analytical and numerical techniques, it is a rewarding step to generate numerical results. The computer-oriented homework problems allow the reader to apply numerical techniques. Some of the problems involve minor modifications of existing programs instead of coding from scratch. Therefore, larger amounts of material and more ambitious problems can be covered in a given time. Many other problems involve little or no computer work, so instructors can choose to opt out of the computation-oriented format or else solve some of the problems with their own code.

The supporting code is written in Fortran 90, which is widely used in computational science and high-performance computing. Well-tested subroutines are provided for special functions, diffraction coefficients, root finding, numerical integration and matrix manipulations. The Netlib repository is extensively used. In this book, the object-oriented capability of Fortran 90 has been used to develop easy-to-use interfaces that hide the complexity of large subroutines.

Computing and plotting can be done with public-domain software that is available under Linux, Windows and Mac OS X. It is assumed that the reader has some prior experience with a programming or scripting language, but not necessarily Fortran 90. Appendix F summarizes the essentials, so that the reader can begin computational work with little difficulty.

R. Paknys
Montreal
September 2015

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