Dynamics of Structures

*Theory and
Applications
to Earthquake
Engineering*

FOURTH EDITION

Anil K. Chopra

DYNAMICS OF STRUCTURES
Theory and Applications to Earthquake Engineering

International Edition

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DYNAMICS OF STRUCTURES

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Dedicated to Hamida and Nasreen with gratitude for suggesting the idea of working on a book and with appreciation for patiently enduring and sharing these years of preparation with me. Their presence and encouragement made this idea a reality.

Overview

Foreword

The need for a textbook on earthquake engineering was first pointed out by the eminent consulting engineer, John R. Freeman (1855–1932). Following the destructive Santa Barbara, California earthquake of 1925, he became interested in the subject and searched the Boston Public Library for relevant books. He found that not only was there no textbook on earthquake engineering, but the subject itself was not mentioned in any of the books on structural engineering. Looking back, we can see that in 1925 engineering education was in an undeveloped state, with computing done by slide rule and curricula that did not prepare the student for understanding structural dynamics. In fact, no instruments had been developed for recording strong ground motions, and society appeared to be unconcerned about earthquake hazards.

In recent years books on earthquake engineering and structural dynamics have been published, but the present book by Professor Anil K. Chopra fills a niche that exists between more elementary books and books for advanced graduate studies. The author is a well-known expert in earthquake engineering and structural dynamics, and his book will be valuable to students not only in earthquake-prone regions but also in other parts of the world, for a knowledge of structural dynamics is essential for modern engineering. The book presents material on vibrations and the dynamics of structures and demonstrates the application to structural motions caused by earthquake ground shaking. The material in the book is presented very clearly with numerous worked-out illustrative examples, so that even a student at a university where such a course is not given should be able to study the book on his or her own time. Readers who are now practicing engineering should have no difficulty in studying the subject by means of this book. An especially interesting feature of the book is the application of structural dynamics theory to important issues in the seismic response and design of multistory buildings. The information presented in this book

will be of special value to those engineers who are engaged in actual seismic design and want to improve their understanding of the subject.

Although the material in the book leads to earthquake engineering, the information presented is also relevant to wind-induced vibrations of structures, as well as man-made motions such as those produced by drophammers or by heavy vehicular traffic. As a textbook on vibrations and structural dynamics, this book has no competitors and can be recommended to the serious student. I believe that this is the book for which John R. Freeman was searching.

George W. Housner
California Institute of Technology

Preface

PHILOSOPHY AND OBJECTIVES

This book on dynamics of structures is conceived as a textbook for courses in civil engineering. It includes many topics in the theory of structural dynamics, and applications of this theory to earthquake analysis, response, design, and evaluation of structures. No prior knowledge of structural dynamics is assumed in order to make this book suitable for the reader learning the subject for the first time. The presentation is sufficiently detailed and carefully integrated by cross-referencing to make the book suitable for self-study. This feature of the book, combined with a practically motivated selection of topics, should interest professional engineers, especially those concerned with analysis and design of structures in earthquake country.

In developing this book, much emphasis has been placed on making structural dynamics easier to learn by students and professional engineers because many find this subject to be difficult. To achieve this goal, the presentation has been structured around several features: The mathematics is kept as simple as each topic will permit. Analytical procedures are summarized to emphasize the key steps and to facilitate their implementation by the reader. These procedures are illustrated by over 120 worked-out examples, including many comprehensive and realistic examples where the physical interpretation of results is stressed. Some 500 figures have been carefully designed and executed to be pedagogically effective; many of them involve extensive computer simulations of dynamic response of structures. Photographs of structures and structural motions recorded during earthquakes are included to relate the presentation to the real world.

The preparation of this book has been inspired by several objectives:

- Relate the structural idealizations studied to the properties of real structures.
- Present the theory of dynamic response of structures in a manner that emphasizes physical insight into the analytical procedures.
- Illustrate applications of the theory to solutions of problems motivated by practical applications.
- Interpret the theoretical results to understand the response of structures to various dynamic excitations, with emphasis on earthquake excitation.
- Apply structural dynamics theory to conduct parametric studies that bring out several fundamental issues in the earthquake response, design, and evaluation of multistory buildings.

This mode of presentation should help the reader to achieve a deeper understanding of the subject and to apply with confidence structural dynamics theory in tackling practical problems, especially in earthquake analysis, design, and evaluation of structures, thus narrowing the gap between theory and practice.

EVOLUTION OF THE BOOK

Since the book first appeared in 1995, it has been revised and expanded in several ways, resulting in the second edition (2001) and third edition (2007). Prompted by an increasing number of recordings of ground motions in the proximity of the causative fault, Chapter 6 was expanded to identify special features of near-fault ground motions and compare them with the usual far-fault ground motions. Because of the increasing interest in seismic performance of bridges, examples on dynamics of bridges and their earthquake response were added in several chapters. In response to the growing need for simplified dynamic analysis procedures suitable for performance-based earthquake engineering, Chapter 7 was expanded to provide a fuller discussion relating the earthquake-induced deformations of inelastic and elastic systems, and to demonstrate applications of the inelastic design spectrum to structural design for allowable ductility, displacement-based design, and seismic evaluation of existing structures. Chapter 19 (now Chapter 20) was rewritten completely to incorporate post-1990 advances in earthquake analysis and response of inelastic buildings. Originally limited to three building codes—United States, Canada, and Mexico—Chapter 21 (now Chapter 22) was expanded to include the Eurocode. The addition of Chapter 22 (now Chapter 23) was motivated by the adoption of performance-based guidelines for evaluating existing buildings by the structural engineering profession.

In response to reader requests, the frequency-domain method of dynamic analysis was included, but presented as an appendix instead of weaving it throughout the book. This decision was motivated by my goal to keep the mathematics as simple as each topic permits, thus making structural dynamics easily accessible to students and professional engineers.

WHAT'S NEW IN THIS EDITION

Dynamics of Structures has been well received in the 19 years since it was first published. It continues to be used as a textbook at universities in the United States and many other countries, and enjoys a wide professional readership as well. Translations in Japanese, Korean, Chinese, Greek, Spanish, and Persian have been published. Preparation of the fourth edition provided me with an opportunity to improve, expand, and update the book.

Chapter 14 has been added, requiring renumbering of Chapters 14 to 22 as 15 to 23 (the new numbering is reflected in the rest of the Preface); Chapters 5 and 16 underwent extensive revision; Chapters 12 and 13 have been expanded; and Chapters 22 and 23 have been updated. Specific changes include:

- Chapter 14 on nonclassically damped systems has been added. This addition has been motivated by growing interest in such systems that arise in several practical situations: for example, structures with supplemental energy-dissipating systems or on a base isolation system, soil–structure systems, and fluid-structure systems.
- Chapters 5 and 16 on numerical evaluation of dynamic response have been rewritten to conform with the ways these numerical methods are usually implemented in computer software, and to offer an integrated presentation of nonlinear static analysis—also known as pushover analysis—and nonlinear dynamic analysis.
- A section has been added at the end of Chapter 12 to present a general version of the mode acceleration superposition method for more complex excitations, such as wave forces on offshore drilling platforms.
- Chapter 13 has been extended to include two topics that so far have been confined to the research literature, but are of practical interest: (1) combining peak responses of a structure to individual translational components of ground motion to estimate its peak response to multicomponent excitation; and (2) response-spectrum-based equations to determine an envelope that bounds the joint response trajectory of all simultaneously acting forces that control the seismic design of a structural element.
- Chapters 22 and 23 have been updated to reflect the current editions of building codes for designing new buildings, and of performance-based guidelines and standards for evaluating existing buildings.
- The addition of Chapter 14 prompted minor revision of Chapters 2, 4, 6, 10, and 12.
- Several new figures, photographs, worked-out examples, and end-of-chapter problems have been added.

Using the book in my teaching and reflecting on it over the years suggested improvements. The text has been clarified and polished throughout, and a few sections have been reorganized to enhance the effectiveness of the presentation.

SUBJECTS COVERED

This book is organized into three parts: I. Single-Degree-of-Freedom Systems; II. Multi-Degree-of-Freedom Systems; and III. Earthquake Response, Design, and Evaluation of Multistory Buildings.

Part I includes eight chapters. In the opening chapter the structural dynamics problem is formulated for simple elastic and inelastic structures, which can be idealized as single-degree-of-freedom (SDF) systems, and four methods for solving the differential equation governing the motion of the structure are reviewed briefly. We then study the dynamic response of linearly elastic systems (1) in free vibration (Chapter 2), (2) to harmonic and periodic excitations (Chapter 3), and (3) to step and pulse excitations (Chapter 4). Included in Chapters 2 and 3 is the dynamics of SDF systems with Coulomb damping, a topic that is normally not included in civil engineering texts, but one that has become relevant to earthquake engineering, because energy-dissipating devices based on friction are being used in earthquake-resistant construction. After presenting numerical time-stepping methods for calculating the dynamic response of SDF systems (Chapter 5), the earthquake response of linearly elastic systems and of inelastic systems is studied in Chapters 6 and 7, respectively. Coverage of these topics is more comprehensive than in texts presently available; included are details on the construction of response and design spectra, effects of damping and yielding, and the distinction between response and design spectra. The analysis of complex systems treated as generalized SDF systems is the subject of Chapter 8.

Part II includes Chapters 9 through 18 on the dynamic analysis of multi-degree-of-freedom (MDF) systems. In the opening chapter of Part II the structural dynamics problem is formulated for structures idealized as systems with a finite number of degrees of freedom and illustrated by numerous examples; also included is an overview of methods for solving the differential equations governing the motion of the structure. Chapter 10 is concerned with free vibration of systems with classical damping and with the numerical calculation of natural vibration frequencies and modes of the structure. Chapter 11 addresses several issues that arise in defining the damping properties of structures, including experimental data—from forced vibration tests on structures and recorded motions of structures during earthquakes—that provide a basis for estimating modal damping ratios, and analytical procedures to construct the damping matrix, if necessary. Chapter 12 is concerned with the dynamics of linear systems, where the classical modal analysis procedure is emphasized. Part C of this chapter represents a "new" way of looking at modal analysis that facilitates understanding of how modal response contributions are influenced by the spatial distribution and the time variation of applied forces, leading to practical criteria on the number of modes to include in response calculation. In Chapter 13, modal analysis procedures for earthquake analysis of classically damped systems are developed; both response history analysis and response spectrum analysis procedures are presented in a form that provides physical interpretation; the latter procedure estimates the peak response of MDF systems directly from the earthquake response or design spectrum. The procedures are illustrated by numerous examples, including coupled lateral-torsional response of unsymmetric-plan buildings and torsional response of nominally symmetric buildings. The chapter ends

with response spectrum-based procedures to consider all simultaneously acting forces that control the design of a structural element, and to estimate the peak response of a structure to multicomponent earthquake excitation. The modal analysis procedure is extended in Chapter 14 to response history analysis; of nonclassically damped systems subjected to earthquake excitation. For this purpose, we first revisit classically damped systems and re-cast the analysis procedures of Chapters 10 and 13 in a form that facilitates their extension to the more general case.

Chapter 15 is devoted to the practical computational issue of reducing the number of degrees of freedom in the structural idealization required for static analysis in order to recognize that the dynamic response of many structures can be well represented by their first few natural vibration modes. In Chapter 16 numerical time-stepping methods are presented for MDF systems not amenable to classical modal analysis: systems with nonclassical damping or systems responding into the range of nonlinear behavior. Chapter 17 is concerned with classical problems in the dynamics of distributed-mass systems; only one-dimensional systems are included. In Chapter 18 two methods are presented for discretizing one-dimensional distributed-mass systems: the Rayleigh–Ritz method and the finite element method. The consistent mass matrix concept is introduced, and the accuracy and convergence of the approximate natural frequencies of a cantilever beam, determined by the finite element method, are demonstrated.

Part III of the book contains five chapters concerned with earthquake response design, and evaluation of multistory buildings, a subject not normally included in structural dynamics texts. Several important and practical issues are addressed using analytical procedures developed in the preceding chapters. In Chapter 19 the earthquake response of linearly elastic multistory buildings is presented for a wide range of two key parameters: fundamental natural vibration period and beam-to-column stiffness ratio. Based on these results, we develop an understanding of how these parameters affect the earthquake response of buildings and, in particular, the relative response contributions of the various natural modes, leading to practical information on the number of higher modes to include in earthquake response calculations. Chapter 20 is concerned with the important subject of earthquake response of multistory buildings deforming into their inelastic range. Part A of the chapter presents rigorous nonlinear response history analysis; identifies the important influence of modeling assumptions, key structural parameters, and ground motion details on seismic demands; and determines the strength necessary to limit the story ductility demands in a multistory building. Recognizing that rigorous nonlinear response history analysis remains an onerous task, the modal pushover analysis (MPA) procedure—an approximate analysis procedure—is developed in Part B of the chapter. In this procedure, seismic demands are estimated by nonlinear static analyses of the structure subjected to modal inertia force distributions. Base isolation is the subject of Chapter 21. Our goal is to study the dynamic behavior of buildings supported on base isolation systems with the limited objective of understanding why and under what conditions isolation is effective in reducing the earthquake-induced forces in a structure. In Chapter 22 we present the seismic force provisions in four building codes—*International Building Code* (United States), *National Building Code of Canada*, *Eurocode*, and *Mexico Federal District Code*—together with their relationship to the theory of structural dynamics developed in Chapters 6, 7, 8,

and 13. Subsequently, the code provisions are evaluated in light of the results of dynamic analysis of buildings presented in Chapters 19 and 20. Performance-based guidelines and standards for evaluating existing buildings consider inelastic behavior explicitly in estimating seismic demands at low performance levels, such as life safety and collapse prevention. In Chapter 23, selected aspects of the nonlinear dynamic procedure and of the nonlinear static procedure in these documents—ATC-40, FEMA 356, and ASCE 41-06— are presented and discussed in light of structural dynamics theory developed in Chapters 7 and 20.

A NOTE FOR INSTRUCTORS

This book is suitable for courses at the graduate level and at the senior undergraduate level. No previous knowledge of structural dynamics is assumed. The necessary background is available through the usual courses required of civil engineering undergraduates. These include:

- Static analysis of structures, including statically indeterminate structures and matrix formulation of analysis procedures (background needed primarily for Part II)
- Structural design
- Rigid-body dynamics
- Mathematics: ordinary differential equations (for Part I), linear algebra (for Part II), and partial differential equations (for Chapter 17 only)

By providing an elementary but thorough treatment of a large number of topics, this book permits unusual flexibility in selection of the course content at the discretion of the instructor. Several courses can be developed based on the material in this book. Here are a few examples.

Almost the entire book can be covered in a one-year course:

- *Title:* Dynamics of Structures I (1 semester)

  Syllabus: Chapter 1; Sections 1 and 2 of Chapter 2; Parts A and B of Chapter 3; Chapter 4; selected topics from Chapter 5; Sections 1 to 7 of Chapter 6; Sections 1 to 7 of Chapter 7; selected topics from Chapter 8; Sections 1 to 4 and 9 to 11 of Chapter 9; Parts A and B of Chapter 10; Sections 1 and 2 of Chapter 11; Parts A and B of Chapter 12; Sections 1, 2, 7, and 8 (excluding the CQC method) of Chapter 13; and selected topics from Part A of Chapter 22

- *Title:* Dynamics of Structures II (1 semester)

  Syllabus: Sections 5 to 7 of Chapter 9; Sections 3 to 5 of Chapter 11; Parts C and D of Chapter 12; Sections 3 to 11 of Chapter 13; selected parts of Chapters 14, 15, 17, 19 to 21, and 23; and Appendix A.

The selection of topics for the first course has been dictated in part by the need to provide comprehensive coverage, including dynamic and earthquake analysis of MDF systems, for students taking only one course.

Abbreviated versions of the outline above can be organized for two quarter courses. One possibility is as follows:

- *Title:* Dynamics of Structures I (1 quarter)

  Syllabus: Chapter 1; Sections 1 and 2 of Chapter 2; Sections 1 to 4 of Chapter 3; Sections 1 and 2 of Chapter 4; selected topics from Chapter 5; Sections 1 to 7 of Chapter 6; Sections 1 to 7 of Chapter 7; selected topics from Chapter 8; Sections 1 to 4 and 9 to 11 of Chapter 9; Parts A and B of Chapter 10; Part B of Chapter 12; Sections 1, 2, 7, and 8 (excluding the CQC method) of Chapter 13.

- *Title:* Dynamics of Structures II (1 quarter)

  Syllabus: Sections 5 to 7 of Chapter 9; Sections 3 to 9 of Chapter 13; and selected topics from Chapters 19 to 23

A one-semester course emphasizing earthquake engineering can be organized as follows:

- *Title:* Earthquake Dynamics of Structures

  Syllabus: Chapter 1; Sections 1 and 2 of Chapter 2; Sections 1 and 2 of Chapter 4; Chapters 6 and 7; selected topics from Chapter 8; Sections 1 to 4 and 9 to 11 of Chapter 9; Parts A and B of Chapter 10; Part A of Chapter 11; Sections 1 to 3 and 7 to 11 of Chapter 13; and selected topics from Chapters 19 to 23.

Solving problems is essential for students to learn structural dynamics. For this purpose the first 18 chapters include 373 problems. Chapters 19 through 23 do not include problems, for two reasons: (1) no new dynamic analysis procedures are introduced in these chapters; (2) this material does not lend itself to short, meaningful problems. However, the reader will find it instructive to work through the examples presented in Chapters 19 to 23 and to reproduce the results. The computer is essential for solving some of the problems, and these have been identified. In solving these problems, it is assumed that the student will have access to computer programs such as MATLAB or MATHCAD. Solutions to these problems are available to instructors as a download from the publisher.

In my lectures at Berkeley, I develop the theory on the blackboard and illustrate it by PowerPoint slides of the more complex figures in the book; enlarged versions of many of the figures, which are suitable for making slides for use in the classroom, are available to instructors as a download from the publisher. Despite requests for a *complete* set of PowerPoint slides, they have not been developed because I do not think this approach is the most effective strategy for teaching dynamics of structures. Resources for the International Edition are accessible to instructors at www.pearsoninternationaleditions.com/chopra.