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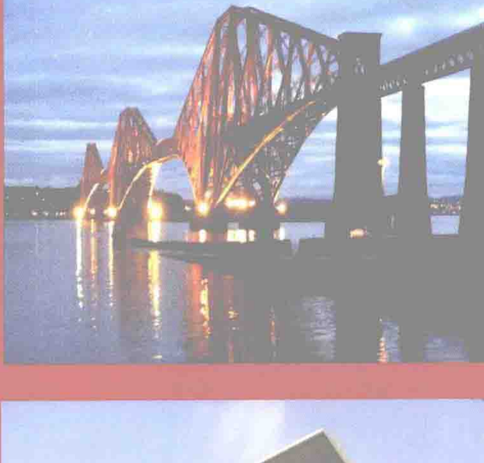
Examples in Structural Analysis

Second Edition



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Preface

Prior to the development of quantitative structural theories in the mid-18th century and since, builders relied on an intuitive and highly developed sense of structural behaviour. The advent of modern mathematical modelling and numerical methods has to a large extent replaced this skill with a reliance on computer generated solutions to structural problems. Professor Hardy Cross[1] aptly expressed his concern regarding this in the following quote:

> *'There is sometimes cause to fear that the scientific technique, the proud servant of the engineering arts, is trying to swallow its master.'*

It is inevitable and unavoidable that designers will utilize continually improving computer software for analyses. However, it is essential that the use of such software should only be undertaken by those with the appropriate knowledge and understanding of the mathematical modelling, assumptions and limitations inherent in the programs they use.

Students adopt a variety of strategies to develop their knowledge and understanding of structural behaviour, e.g. the use of:

- computers to carry out sensitivity analyses,
- physical models to demonstrate physical effects such as buckling, bending, the development of tension and compression and deformation characteristics,
- the study of worked examples and carrying out analyses using 'hand' methods.

This textbook focuses on the provision of numerous fully detailed and comprehensive worked examples for a wide variety of structural problems. In each chapter a résumé of the concepts and principles involved in the method being considered is given and illustrated by several examples. A selection of problems is then presented which students should undertake on their own prior to studying the given solutions.

Students are strongly encouraged to attempt to visualise/sketch the deflected shape of a loaded structure and predict the type of force in the members prior to carrying out the analysis; i.e.

(i) in the case of pin-jointed frames identify the location of the tension and compression members,

(ii) in the case of beams/rigid-jointed frames, sketch the shape of the bending moment diagram and locate points of contraflexure indicating areas of tension and compression.

A knowledge of the location of tension zones is vital when placing reinforcement in reinforced concrete design and similarly with compression zones when assessing the effective buckling lengths of steel members.

When developing their understanding and confirming their own answers by studying the solutions provided, students should also analyse the structures using a computer analysis, and identify any differences and the reasons for them.

The methods of analysis adopted in this text represent the most commonly used 'hand' techniques with the exception of the direct stiffness method in Chapter 7. This matrix based method is included to develop an understanding of the concepts and procedures adopted in most computer software analysis programs. A method for inverting matrices is given in Appendix 3 and used in the solutions for this chapter – it is *not* necessary for students to undertake this procedure. It is included to demonstrate the process involved when solving the simultaneous equations as generated in the direct stiffness method.

Whichever analysis method is adopted during design, it must always be controlled by the designer, i.e. not a computer! This can only be the case if a designer has a highly developed knowledge and understanding of the concepts and principles involved in structural behaviour. The use of worked examples is one of a number of strategies adopted by students to achieve this.

In this 2nd Edition the opportunity has been taken to modify the x-y-z co-ordinate system/ symbols and Chapter 6 on buckling instability, to reflect the conventions adopted in the structural Eurocode EN 1993-1-1 for steel structures, i.e.

- x-x along the member,
- y-y the major principal axis of the cross-section (e.g. parallel to the flange in a steel beam) and
- z-z the minor principal axes of the cross-section (e.g. perpendicular to the flange in a steel beam).

Local and flexural buckling equations as given in the EN 1993-1-1 are also considered.

Chapter 4 for the analysis of beams has been expanded to include moment redistribution and moment envelopes. Chapter 5 has been expanded to include the analysis of singly-redundant, rigid-jointed frames using the unit load method.

In addition, two new chapters have been added: Chapter 9 relating to the construction and use of influence lines for beams and Chapter 10, the use of approximate methods of analysis for pin-jointed frames, multi-span beams and rigid-jointed frames.

1 Cross, H. *Engineers and Ivory Towers*. New York: McGraw Hill, 1952

William M.C. McKenzie

To Karen, Gordon, Claire and Eilidh

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I wish to thank Caroline for her endless support and encouragement.

William M. C. McKenzie is a lecturer in structural engineering at Edinburgh Napier University on undergraduate and postgraduate courses, including the MSc course in Advanced Structural Engineering. He graduated with a 1st Class Honours degree and a Ph.D. from Heriot-Watt University, Edinburgh and has been involved in consultancy, research and teaching for more than 35 years.

His publications include research papers relating to stress analysis using holographic interferometry. He is also the author of six design textbooks relating to both the British Standards and the Eurocodes for structural design and one structural analysis textbook.

As a member of the Institute of Physics he is both a Chartered Engineer and a Chartered Physicist. He has presented numerous CPD courses/seminars and guest lectures to industry throughout the UK, in China, Singapore and Malaysia, in relation to the use of the Structural Eurocodes.

Contents

1. Structural Analysis and Design

1.1 *Introduction*

The design of structures, of which analysis is an integral part, is frequently undertaken using computer software. This can only be done safely and effectively if those undertaking the design fully understand the concepts, principles and assumptions on which the computer software is based. It is vitally important therefore that design engineers develop this knowledge and understanding by studying and using hand-methods of analysis based on the same concepts and principles, e.g. equilibrium, energy theorems, elastic, elasto-plastic and plastic behaviour and mathematical modelling.

In addition to providing a mechanism for developing knowledge and understanding, hand-methods also provide a useful tool for readily obtaining approximate solutions during preliminary design and an independent check on the answers obtained from computer analyses.

The methods explained and illustrated in this text, whilst not exhaustive, include those most widely used in typical design offices, e.g. method-of-sections/joint resolution/unit load/McCaulay's method/moment distribution/plastic analysis etc.

In Chapter 7 a résumé is given of the direct stiffness method; the technique used in developing most computer software analysis packages. The examples and problems in this case have been restricted and used to illustrate the processes undertaken when using matrix analysis; this is **not** regarded as a hand-method of analysis.

1.2 *Equilibrium*

All structural analyses are based on satisfying one of the fundamental laws of physics, i.e.

$$F = ma \qquad \text{Equation (1)}$$

where

F is the force system acting on a body
m is the mass of the body
a is the acceleration of the body

Structural analyses carried out on the basis of a force system inducing a **dynamic** response, for example structural vibration induced by wind loading, earthquake loading, moving machinery, vehicular traffic etc., have a non-zero value for 'a' the acceleration. In the case of analyses carried out on the basis of a **static** response, for example stresses/deflections induced by the self-weights of materials, imposed loads which do **not** induce vibration etc., the acceleration 'a' is equal to **zero**.

Static analysis can be regarded as a special case of the more general dynamic analysis in which:

$$F = ma = 0 \qquad \text{Equation (2)}$$

F can represent the applied force system in any direction; for convenience this is normally considered in either two or three mutually perpendicular directions as shown in Figure 1.1.

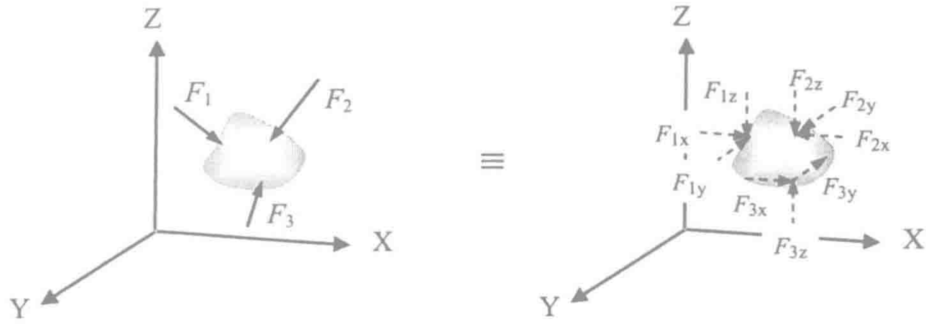

Figure 1.1

The application of Equation (2) to the force system indicated in Figure 1.1 is:

Sum of the forces in the direction of the X-axis	$\Sigma F_x = 0$	Equation (3)
Sum of the forces in the direction of the Y-axis	$\Sigma F_y = 0$	Equation (4)
Sum of the forces in the direction of the Z-axis	$\Sigma F_z = 0$	Equation (5)

Since the structure is neither moving in a linear direction, nor in a rotational direction a further three equations can be written down to satisfy Equation (2):

Sum of the moments of the forces about the X-axis	$\Sigma M_x = 0$	Equation (6)
Sum of the moments of the forces about the Y-axis	$\Sigma M_y = 0$	Equation (7)
Sum of the moments of the forces about the Z-axis	$\Sigma M_z = 0$	Equation (8)

Equations (3) to (8) represent the static equilibrium of a body (structure) subject to a three-dimensional force system. Many analyses are carried out for design purposes assuming two-dimensional force systems and hence only two linear equations (e.g. equation (3) and equation (5) representing the x and z axes) and one rotational equation (e.g. equation (7) representing the y-axis) are required. The x, y and z axes must be mutually perpendicular and can be in any orientation, however for convenience two of the axes are usually regarded as horizontal and vertical, (e.g. gravity loads are vertical and wind loads frequently regarded as horizontal). It is usual practice, when considering equilibrium, to assume that clockwise rotation is positive and anti-clockwise rotation is negative. The following conventions have been adopted in this text:

x-direction: horizontal direction - positive is left-to right → +ve

z- direction: vertical direction - positive is upwards ↑ +ve

y- direction: rotation about the y-axis - positive is clockwise ↷ +ve

plane (XZ) of the structure

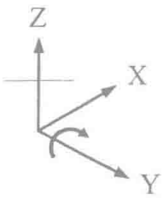

Figure 1.2