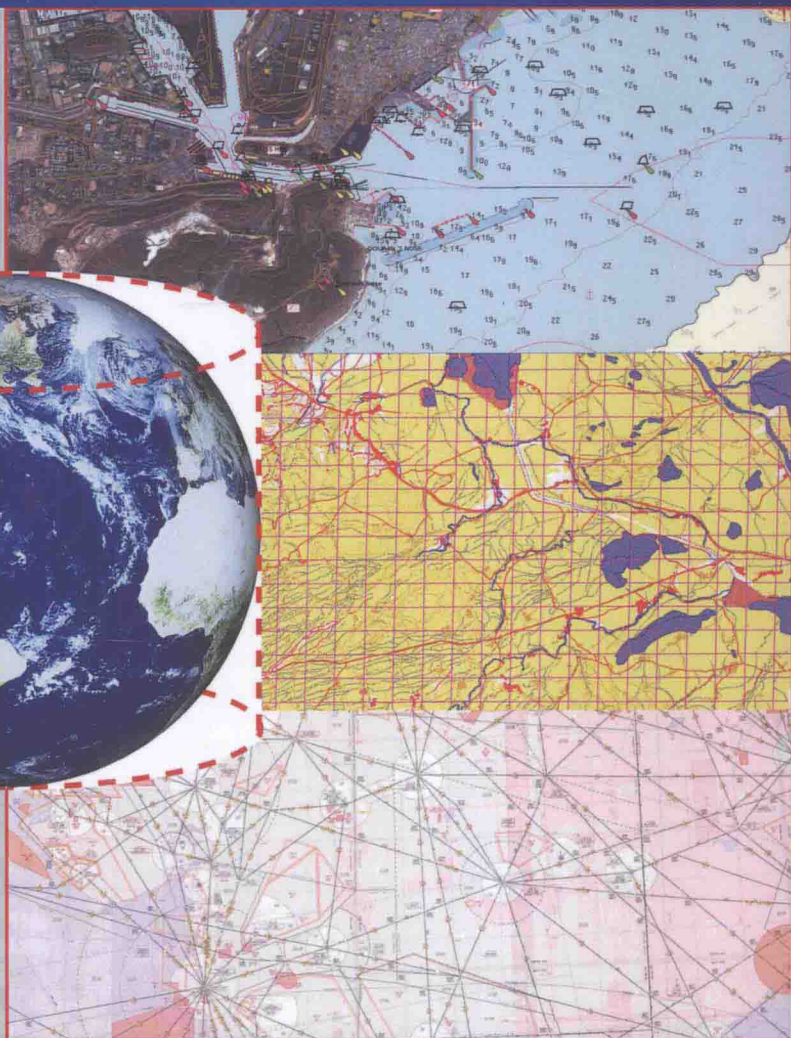


# Computing in Geographic Information Systems

$$(x, y) = f(\phi, \lambda)$$



$$(\phi, \lambda) = f^{-1}(x, y)$$



Narayan Panigrahi



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Narayan Panigrahi



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*This book is dedicated to the loving memory of my  
parents  
Shri Raghu Nath Panigrahi and Smt Yasoda Panigrahi  
Village Pallipadnapur, District Ganjam, State Odisha  
of India  
who brought me up with dedication and placed  
education second to none despite their modest means*

# List of Figures

1.1	Block diagram depicting the macro GIS functions . . . . .	6
1.2	Multi-tier architecture in GIS . . . . .	9
1.3	Collaborative diagram depicting various contributing branches of science and technology; GIS as a platform for scientific computing . . . . .	12
1.4	Organization of chapters . . . . .	17
2.1	Separation of geoid and ellipsoid undulation . . . . .	21
2.2	Auxiliary circle, the 2D projected ellipsoid . . . . .	24
2.3	Geodetic and geocentric latitude . . . . .	29
2.4	Reduced latitude . . . . .	30
3.1	Spherical coordinate system . . . . .	41
3.2	Cylindrical coordinate system . . . . .	42
3.3	Polar coordinate system . . . . .	44
3.4	Celestial coordinate system . . . . .	47
3.5	Celestial coordinate of constellation Sirius defined by RA and declination . . . . .	48
3.6	Universal transverse Mercator grid system . . . . .	49
3.7	Transformation of the datum surface . . . . .	56
4.1	Map projection, the mapping of Earth coordinates to map coordinates . . . . .	63
4.2	Process flow of map projection . . . . .	64
4.3	Schematic of azimuthal map projection . . . . .	65
4.4	Schematic of cylindrical map projection . . . . .	69
4.5	Schematic of conical map projection . . . . .	72
4.6	Flattened cone after cutting along a central meridian . . . . .	72
4.7	Map projections based on the position of the viewer . . . . .	77
4.8	Geometry of map developable surfaces: (A) planar, (B) cylindrical, (C) conical placed tangent to the datum surface . . . . .	78
4.9	Geometry of map developable surfaces: (A) planar, (B) cylindrical, (C) conical placed secant to the datum surface . . . . .	79
4.10	Geometry of the map projections depending upon the orientation of the map surface with the datum surface: (A) normal, (B) transverse, (C) oblique . . . . .	79

5.1	Steps of computing key points from satellite image using scale invariant feature transform (SIFT), detection of key points from image using DOG and maximization rule. . . . .	96
5.2	Gaussian blurred image pyramid, depicting the scale space of an image . . . . .	97
5.3	Detection of keypoint from image using DoG and maximization rule . . . . .	98
5.4	Example of registration of satellite image pair using Log-Polar transformation: (a) base image, (b) image with geometric error, (c) image (b) registered and resampled with respect to image (a) . . . . .	99
5.5	Satellite images: (a) base image, (b) image with geometric distortion, (c) image, (b) registered with respect to image (a), (d) final registered image (b) . . . . .	100
6.1	Edge surface with Gaussian curvature $K = 0, \lambda_1 = 0$ and $\lambda_2 < 0$ . The principal eigenvalues are directed in orthogonal directions. . . . .	112
6.2	Saddle surface with Gaussian curvature $K < 0, \lambda_1 < 0$ and $\lambda_2 > 0$ The, principal eigenvalues directed in orthogonal directions of the dominant curvatures . . . . .	113
6.3	Blob-like surface with Gaussian curvature $K > 0, \lambda_1 < 0$ and $\lambda_2 < 0$ , a convex surface . . . . .	113
7.1	Polygonal curves . . . . .	124
7.2	Existence of a diagonal . . . . .	125
7.3	Dual graph triangulation . . . . .	126
7.4	Types of line segment intersections . . . . .	130
7.5	Diagonal test in a polygon . . . . .	132
7.6	Graham's scan . . . . .	133
7.7	Push and pop operation . . . . .	135
7.8	Computing the lower tangent . . . . .	137
7.9	QuickHulls initial quadrilateral . . . . .	138
7.10	QuickHull elimination procedure . . . . .	139
7.11	Voronoi diagram . . . . .	140
7.12	Delaunay triangulation . . . . .	142
7.13	Basic triangulation changes . . . . .	145
7.14	Point insertion . . . . .	145
7.15	Delaunay triangulations and convex hulls . . . . .	148
7.16	Planes and circles . . . . .	150
8.1	Variogram with range, nugget and sill . . . . .	162
8.2	Commonly used variogram models: (a) spherical; (b) exponential; (c) linear; and (d) Gaussian . . . . .	163

10.1	(a) Ray diagram of working sonar; (b) multi-beam sonar working principle . . . . .	186
10.2	New York Harbor nautical chart . . . . .	192
10.3	Chart colours and representation . . . . .	193
10.4	Topobathymetry production of bENC . . . . .	195
10.5	Example of a map . . . . .	198
10.6	Example of a chart . . . . .	199
12.1	(a) Geodesic distance; (b) Manhattan distance . . . . .	213
12.2	Planimetric area of a triangle . . . . .	220
12.3	Computation of volume using contour data . . . . .	221
12.4	Slope computed as the ratio of rise over run in terrain surface . . . . .	222
12.5	DEM grid with cardinal designator for the height . . . . .	223
12.6	Line of sight between the observer and various points of the terrain . . . . .	225
12.7	Line of sight between the observer and ship at sea . . . . .	227
13.1	Satellite image of Chilka Lake in the state of Odisha in India depicting a land, sea and lake with its vector map draped on it . . . . .	235
13.2	A contour map covering a portion of land and sea . . . . .	235
13.3	Topobathymetry surface with vector data of topography and S-57 bathymetry data of sea . . . . .	236
13.4	Topobathymetry surface depicting the sea contours and sounding measures of the sea depth in fathoms . . . . .	236
13.5	An instance of a flythrough visualization of a DEM draped with raster map . . . . .	237
13.6	3D perspective visualization of an undulated terrain with sun shaded relief map draped on it . . . . .	237
13.7	Colour-coded satellite image of an undulated terrain surface depicting relief . . . . .	238
13.8	Computation of communication line of sight between transmitter and receiver with the corresponding terrain profile along the LOS . . . . .	239
13.9	Computation of line-of-sight fan 360 degrees around the observer . . . . .	240
13.10	Line of sight between observer and the target the visible portion is depicted in green and invisible in red . . . . .	240

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## *List of Tables*

---

1.1	Input Domain of a GIS . . . . .	4
1.2	Computing Algorithms and Their Usage in GIS . . . . .	15
4.1	Criteria of Projecting Earth Surface and Classes of Map Projections . . . . .	75
4.2	Applications of Map Projections . . . . .	84
5.1	Applications of Image Registration Algorithms . . . . .	104
8.1	The Spatial Interpolation Methods Considered in This Chapter . . . . .	157
9.1	Comparison of Univariate and Bivariate Data . . . . .	172
10.1	Differences between a Chart and a Map . . . . .	197
12.1	Spatial Location Measures and Their Applications . . . . .	210
13.1	Important Reference Parameters of Ellipsoids in Use . . . . .	233

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## *Introduction*

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The progress of GIS (Geographic Information System) over the past two decades has been phenomenal. The quantity and quality of research literature contributed, new applications developed and systems engineered using GIS are indicators of its growing popularity among researchers, industry and the user community. Though GIS derives its acronym from Geographic Information System, it has emerged as a platform for computing spatio-temporal data obtained through a heterogeneous array of sensors from Land-Air-Sea in a continuous time frame. Therefore, GIS can easily be connoted as Spatio-Temporal Information (STI) system.

The capability of continuous acquisition of high spatial and high spectral data has resulted in the availability of a large volume of spatial data. This has led to the design, analysis, development and optimization of new algorithms for extraction of spatio-temporal patterns from the data. The trend analysis in spatial data repository has led to the development of data analytics. The progress in the design of new computing techniques to analyze, visualize, quantify and measure spatial objects using high volume spatial data has led to research in the development of robust and optimized algorithms in GIS.

The collaborative nature of GIS has borrowed modeling techniques, scientific principles and algorithms from different fields of science and technology. Principles of geodesy, geography, geomatics, geometry, cartography, statistics, remote sensing, and digital image processing (DIP) have immensely contributed to its growth. In this book I have attempted to compile the essential computing principles required for the development of GIS. The modeling, mathematical transformations, algorithms and computation techniques which form the basis of GIS are discussed. Each chapter gives the underlying computing principle in the form of CDF (Concept-Definition-Formula). The overall arrangement of the chapters follows the principle of IPO (Input-Processing-Output) of spatial data by GIS.

This book is intended to encourage the scientific thoughts of students, researchers and users by explaining the mathematical principles of GIS.

---

## *Preface*

---

Each time I wanted to experiment and analyze the spatial data presented to me, I was confronted with many queries such as: Which GIS function will be suitable to read the spatial data format? Which set of functions will be suitable for the analysis? How to visualize and analyze the resulted outputs? Which COTS GIS has all the related functions to meaningfully read, analyze, visualize and measure the spatio-temporal event in the data?

Even if I were to select a COTS GIS system which is most suitable to answer all these queries, the cumbersome process of fetching the COTS GIS along with its high cost and strict licensing policy discourages me from procuring it. That made me a very poor user of COTS GIS and associated tools.

But the quest to analyze, visualize, estimate and measure spatial information has led me to search for the mathematical methods, formulae, algorithms that can accomplish the task. To visualize terrain as it is through modeling of spatial data has always challenged the computing skills that I acquired during my academic and professional career.

The alternatives left are to experiment with the growing list of open source GIS tools available or to design and develop a GIS software. Compelled by all these circumstances I developed a set of GIS tools for visualization and analysis *ab initio*.

The design and development of GIS functions need deeper understanding of the algorithms and mathematical methods inherent in the process. The first principle approach of development has its own merit and challenges. This has led me to delve into the mathematical aspects of geodesy, cartography, map projection, spatial interpolation, spatial statistics, coordinate transformation etc. This book is the outcome of the associated scientific computations along with the applications of computational geometry, differential geometry and affine geometry in GIS.

Putting all these scientific principles together I came up with a new definition. GIS is a collaborative platform for visualization and analysis of spatio-temporal data using computing methods of geodesy, photogrammetry, cartography, computer science, computational geometry, affine geometry, differential geometry, spatial statistics, spatial interpolation, remote sensing, and digital image processing.

This book is intended for students, researchers and professionals engaged in analysis, visualization and estimation of spatio-temporal data, objects and events.

---

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My wife Smita is my perennial source of strength and support. She has been a constant guiding factor throughout the compilation of this book. My son Sabitra Sankalp and daughter Mahashweta motivated me throughout and made the long hours of thinking and consolidation a pleasure. Sabitra has contributed enough to understand the scientific principles of GIS and helped in proofreading some of the mathematical equations presented. The kind blessings of Shri Sashi Bhusan Tripathy and Smt Kalyani Tripathy are a boon.

Thanks to all the reviewers of this manuscript whose suggestions and new ideas have improved its quality. The suggestions of Dr G. Athithan, Outstanding Scientist, and Prof. P. Venkatachalam of IIT, Bombay are gratefully acknowledged.

This book would not have been possible without the relentless efforts of a few individuals who have contributed in many aspects to enhance the quality, including Cyan Subhra Mishra, trainee, who added all the questions and meticulously worked out the answers for each chapter and enhanced the book's relevance to the student community. He has also carefully reviewed the mathematical aspects of map projections.

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---

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# Contents

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List of Figures	xv
List of Tables	xix
Introduction	xxi
Preface	xxiii
Acknowledgments	xxv
Author Bio	xxvii
<b>1 Introduction</b>	<b>1</b>
1.1 Definitions and Different Perspectives of GIS . . . . .	2
1.1.1 Input Domain of GIS . . . . .	2
1.1.2 Functional Profiling of GIS . . . . .	3
1.1.3 Output Profiling of GIS . . . . .	7
1.1.4 Information Architecture of GIS . . . . .	7
1.1.4.1 Different Architectural Views of GIS . . . . .	8
1.1.5 GIS as a Platform for Multi-Sensor Data Fusion . . . . .	11
1.1.6 GIS as a Platform for Scientific Visualization . . . . .	12
1.2 Computational Aspects of GIS . . . . .	13
1.3 Computing Algorithms in GIS . . . . .	14
1.4 Purpose of the Book . . . . .	14
1.5 Organization of the Book . . . . .	17
1.6 Summary . . . . .	18
<b>2 Computational Geodesy</b>	<b>19</b>
2.1 Definition of Geodesy . . . . .	19
2.2 Mathematical Models of Earth . . . . .	20
2.2.1 Physical Surface of Earth . . . . .	21
2.2.2 The Reference Geoid . . . . .	21
2.2.3 The Reference Ellipsoid . . . . .	22
2.3 Geometry of Ellipse and Ellipsoid . . . . .	22
2.3.1 Relation between 'e' and 'f' . . . . .	25
2.4 Computing Radius of Curvature . . . . .	25
2.4.1 Radius of Curvature at Prime Vertical Section . . . . .	27

2.5	Concept of Latitude . . . . .	28
2.5.1	Modified Definition of Latitude . . . . .	28
2.5.2	Geodetic Latitude . . . . .	28
2.5.3	Geocentric Latitude . . . . .	29
2.5.4	Spherical Latitude . . . . .	29
2.5.5	Reduced Latitude . . . . .	29
2.5.6	Rectifying Latitude . . . . .	30
2.5.7	Authalic Latitude . . . . .	31
2.5.8	Conformal Latitude . . . . .	31
2.5.9	Isometric Latitude . . . . .	32
2.5.10	Astronomical Latitude . . . . .	32
2.6	Applications of Geodesy . . . . .	33
2.7	The Indian Geodetic Reference System (IGRS) . . . . .	33
2.8	Summary . . . . .	34
<b>3</b>	<b>Reference Systems and Coordinate Transformations</b>	<b>35</b>
3.1	Definition of Reference System . . . . .	35
3.2	Classification of Reference Systems . . . . .	36
3.3	Datum and Coordinate System . . . . .	37
3.4	Attachment of Datum to the Real World . . . . .	37
3.5	Different Coordinate Systems Used in GIS . . . . .	38
3.5.1	The Rectangular Coordinate System . . . . .	39
3.5.2	The Spherical Coordinate System . . . . .	39
3.5.3	The Cylindrical Coordinate System . . . . .	40
3.5.4	The Polar and Log-Polar Coordinate System . . . . .	42
3.5.5	Earth-Centered Earth-Fixed (ECEF) Coordinate System . . . . .	43
3.5.6	Inertial Terrestrial Reference Frame (ITRF) . . . . .	45
3.5.7	Celestial Coordinate System . . . . .	46
3.5.8	Concept of GRID, UTM, Mercator's GRID and Military GRID . . . . .	48
3.6	Shape of Earth . . . . .	50
3.6.1	Latitude and Longitude . . . . .	50
3.6.2	Latitude . . . . .	51
3.6.3	Longitude . . . . .	51
3.7	Coordinate Transformations . . . . .	52
3.7.1	2D Coordinate Transformations . . . . .	53
3.7.2	3D Coordinate Transformations . . . . .	54
3.8	Datum Transformation . . . . .	55
3.8.1	Helmert Transformation . . . . .	57
3.8.2	Molodensky Transformation . . . . .	58
3.9	Usage of Coordinate Systems . . . . .	58
3.10	Summary . . . . .	59

<b>4</b>	<b>Basics of Map Projection</b>	<b>61</b>
4.1	What Is Map Projection? Why Is It Necessary? . . . . .	61
4.2	Mathematical Definition of Map Projection . . . . .	62
4.3	Process Flow of Map Projection . . . . .	63
4.4	Azimuthal Map Projection . . . . .	64
4.4.1	Special Cases of Azimuthal Projection . . . . .	66
4.4.2	Inverse Azimuthal Projection . . . . .	67
4.5	Cylindrical Map Projection . . . . .	68
4.5.1	Special Cases of Cylindrical Projection . . . . .	69
4.5.1.1	Gnomonic Projection . . . . .	70
4.5.1.2	Stereographic Projection . . . . .	70
4.5.1.3	Orthographic Projection . . . . .	70
4.5.2	Inverse Transformation . . . . .	70
4.6	Conical Map Projection . . . . .	71
4.7	Classification of Map Projections . . . . .	74
4.7.1	Classification Based on the Cartographic Quantity Preserved . . . . .	75
4.7.2	Classification Based on the Position of the Viewer . . . . .	76
4.7.3	Classification Based on Method of Construction . . . . .	77
4.7.4	Classification Based on Developable Map Surface . . . . .	78
4.7.5	Classification Based on the Point of Contact . . . . .	79
4.8	Application of Map Projections . . . . .	80
4.8.1	Cylindrical Projections . . . . .	80
4.8.1.1	Universal Transverse Mercator (UTM) . . . . .	80
4.8.1.2	Transverse Mercator projection . . . . .	81
4.8.1.3	Equidistant Cylindrical Projection . . . . .	81
4.8.1.4	Pseudo-Cylindrical Projection . . . . .	81
4.8.2	Conic Map Projection . . . . .	82
4.8.2.1	Lambert's Conformal Conic . . . . .	82
4.8.2.2	Simple Conic Projection . . . . .	82
4.8.2.3	Albers Equal Area Projection . . . . .	82
4.8.2.4	Polyconic Projection . . . . .	82
4.8.3	Azimuthal Projections . . . . .	83
4.9	Summary . . . . .	83
<b>5</b>	<b>Algorithms for Rectification of Geometric Distortions</b>	<b>87</b>
5.1	Sources of Geometric Distortion . . . . .	88
5.1.1	Definition and Terminologies . . . . .	89
5.1.2	Steps in Image Registration . . . . .	89
5.2	Algorithms for Satellite Image Registration . . . . .	91
5.2.1	Polynomial Affine Transformation (PAT) . . . . .	91
5.2.2	Similarity Transformation . . . . .	92
5.3	Scale Invariant Feature Transform (SIFT) . . . . .	93
5.3.1	Detection of Scale-Space Extrema . . . . .	94
5.3.2	Local Extrema Detection . . . . .	94

5.3.3	Accurate Key Point Localization . . . . .	95
5.3.4	Eliminating Edge Responses . . . . .	98
5.4	Fourier Mellin Transform . . . . .	100
5.4.1	The Log-Polar Transformation Algorithm . . . . .	101
5.5	Multiresolution Image Analysis . . . . .	102
5.6	Applications of Image Registration . . . . .	103
5.7	Summary . . . . .	105
<b>6</b>	<b>Differential Geometric Principles and Operators</b>	<b>107</b>
6.1	Gradient (First Derivative) . . . . .	107
6.2	Concept of Curvature . . . . .	108
6.3	Hessian: The Second Order Derivative . . . . .	110
6.4	Gaussian Curvature . . . . .	111
6.5	Mean Curvature . . . . .	112
6.6	The Laplacian . . . . .	114
6.7	Properties of Gaussian, Hessian and Difference of Gaussian	114
6.7.1	Gaussian Function . . . . .	115
6.7.2	Hessian Function . . . . .	115
6.7.3	Difference of Gaussian . . . . .	116
6.8	Summary . . . . .	117
<b>7</b>	<b>Computational Geometry and Its Application to GIS</b>	<b>119</b>
7.1	Introduction . . . . .	119
7.2	Definitions . . . . .	120
7.2.1	Triangulation and Partitioning . . . . .	120
7.2.2	Convex Hull . . . . .	121
7.2.3	Voronoi Diagram and Delaunay Triangulation . . .	121
7.3	Geometric Computational Techniques . . . . .	122
7.4	Triangulation of Simple Polygons . . . . .	123
7.4.1	Theory of Polygon Triangulation . . . . .	124
7.4.2	Dual Tree . . . . .	126
7.4.3	Polygon Triangulation . . . . .	127
7.4.3.1	Order Type . . . . .	127
7.4.4	Line Segment Intersection . . . . .	129
7.4.5	Finding Diagonals in a Polygon . . . . .	131
7.4.6	Naive Triangulation Algorithm . . . . .	132
7.5	Convex Hulls in Two Dimensions . . . . .	133
7.5.1	Graham's Scan: . . . . .	133
7.5.1.1	Steps of Graham's Scan . . . . .	134
7.6	Divide and Conquer Algorithm . . . . .	135
7.6.1	Divide and Conquer Convex Hull . . . . .	136
7.6.1.1	Lower Tangent . . . . .	136
7.6.2	Quick Hull . . . . .	137
7.7	Voronoi Diagrams . . . . .	139
7.7.1	Properties of Voronoi Diagrams . . . . .	140

7.8	Delaunay Triangulation . . . . .	141
7.8.1	Properties of Delaunay Triangulation . . . . .	141
7.9	Delaunay Triangulation: Randomized Incremental Algorithm . . . . .	143
7.9.1	Incremental Update . . . . .	143
7.10	Delaunay Triangulations and Convex Hulls . . . . .	147
7.11	Applications of Voronoi Diagram and Delaunay Triangulation . . . . .	151
7.11.1	Applications of Voronoi Diagrams . . . . .	152
7.12	Summary . . . . .	152
<b>8</b>	<b>Spatial Interpolation Techniques</b>	<b>155</b>
8.1	Non-Geostatistical Interpolators . . . . .	156
8.1.1	Nearest Neighbours . . . . .	156
8.1.2	Triangular Irregular Network . . . . .	156
8.1.3	Natural Neighbours . . . . .	156
8.1.4	Inverse Distance Weighting . . . . .	158
8.1.5	Regression Models . . . . .	159
8.1.6	Trend Surface Analysis . . . . .	159
8.1.7	Splines and Local Trend Surfaces . . . . .	159
8.1.8	Thin Plate Splines . . . . .	159
8.1.9	Classification Methods . . . . .	160
8.1.10	Regression Tree . . . . .	160
8.1.11	Fourier series . . . . .	160
8.1.12	Lapse Rate . . . . .	161
8.2	Geostatistics . . . . .	161
8.2.1	Introduction of Geostatistics . . . . .	161
8.2.2	Semivariance and Variogram . . . . .	162
8.2.3	Kriging Estimator . . . . .	163
8.2.4	Simple Kriging . . . . .	164
8.2.5	Ordinary Kriging . . . . .	165
8.2.6	Kriging with a Trend . . . . .	165
8.2.7	Block Kriging . . . . .	165
8.2.8	Factorial Kriging . . . . .	165
8.2.9	Dual Kriging . . . . .	166
8.2.10	Simple Kriging with Varying Local Means . . . . .	166
8.2.11	Kriging with an External Drift . . . . .	166
8.2.12	Cokriging . . . . .	166
8.3	Summary . . . . .	167
<b>9</b>	<b>Spatial Statistical Methods</b>	<b>169</b>
9.1	Definition of Statistics . . . . .	169
9.2	Spatial Statistics . . . . .	170
9.3	Classification of Statistical Methods . . . . .	171
9.3.1	Descriptive Statistics . . . . .	171
9.4	Role of Statistics in GIS . . . . .	173
9.5	Descriptive Statistical Methods . . . . .	174