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# Wave Propagation in Solid and Porous Half-Space Media



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# Wave Propagation in Solid and Porous Half-Space Media

*Dedicated to our wives Azar, Xinming, and  
Mojgan*

*Trying is not doing; doing is trying.*

# Preface

The main scope of this book is to present the established analytical and experimental techniques to address the dynamic responses of elastic as well as porous half-space media when they are subjected to dynamic loads and the related topics in a concise and suitable manner. The book introduces the reader to the dynamic response of the surface of an elastic half-space excited by concentrated vertical or tangential force. Based on the presented analyses, it also addresses the dynamic response of a rigid massless footing of arbitrary shape resting on the surface of an elastic half-space medium for three modes of vertical, horizontal, and rocking vibrations. The book also presents solutions to the three pure modes of vibration for massive rectangular foundations by employing the impedance matching technique and provides design charts for these modes of vibrations. The solution for these modes is extended to develop a solution to the dynamics of simultaneous horizontal and rocking motions of a rectangular foundations resting on the surface of elastic half-space medium. Moreover, the book presents the required theoretical background needed for analysis of interaction of two rectangular foundations founded on the surface of an elastic half-space. In addition to the theoretical topics, the book describes a finite model to simulate an elastic half-space and introduces experimental techniques to verify the presented solution. Furthermore, experimental methods are presented to determine the two important elastic properties of shear modulus and Poisson's ratio for the medium. In order to verify the present theoretical results some experiments, procedures, and results are also provided.

This book presents the required theoretical background needed to develop mathematical models and their solutions for the above topics. Furthermore, it offers the engineering information and quantitative data needed for design analysis and applications of the presented analytical procedures for different disciplines such as: mechanical, civil, and bioengineering. The book in its entirety constitutes as an extensive guidance for its reader. It also provides a systematic solution for the dynamic analysis of elastic, porous, and layered half-space media. It also extends the provided analytical solution to address a variety of practical problems in engineering and to determine the essential elastic properties of the medium. The book is intended to lay the foundation for understanding mathematical modeling, vibration analysis,

and the design of engineering systems which can be modeled by a half-space medium in a complete and succinct manner. Throughout the book, an attempt has been made to provide a conceptual framework that includes exposure to the required background in mathematics and the fundamentals of the theory of elasticity. The knowledge of the presented topics will enable the reader to pursue further advances in the field.

## **Level of the Book**

The primary audience of this book is the graduate students in mechanical engineering, engineering mechanics, civil engineering, bioengineering, ocean engineering, mathematics, and science disciplines. In particular, it is geared toward the students interested in enhancing their knowledge by taking the second graduate course in the areas of vibration of continuous systems, application of wave propagation, and soil dynamics. The presented topics have been prepared to serve as an aid to engineering designers. It can also be utilized as a guide for professional engineers in research and industry who are seeking to expand their expertise and are expected to extend their knowledge for setting design specifications and ensuring their fulfillment.

## **Organization of the Book**

The book is presented in ten chapters: introduction, fundamentals of elasticity, vibration analysis for single-layer cylinders, modal analysis for single-layer cylinders, vibration of multilayer thick cylinders, constrained-layer damping for cylindrical structures, and vibration of thick cylindrical panels. Furthermore, it offers helpful and significant tabulated results, which can be used as design guidelines for these structures.

To make effective use of the presented topics, the following procedure is suggested. The realization of the topics may require a review of certain theoretical concepts and methods which can be achieved through references in Chaps. 1 through 5. To become acquainted with the state of the art in this particular field and learn about the historical background on this topic, the reader should begin with Chap. 1, which lists an extensive number of key references with brief discussions on their methodology, required assumptions, and their achievements. Chapter 2 reviews the succinct fundamental theoretical background and concepts needed from the theory of elasto-dynamics, which will enable the reader to follow the derivation of the required governing equations and their solutions in Chaps. 3 and 4. Chapter 5 is intended to present numerical results for the non-dimensional frequency responses of rigid rectangular foundations resting on an elastic half-space for three modes of vertical, horizontal, and rocking vibrations, as well as coupled horizontal and rocking vibrations. Chapter 6 presents a finite size experimental model for a



semi-infinite elastic half-space model and the experimental procedures for verifying the theoretical results. It also presents available techniques for determining the dynamic properties of the medium needed for analytical analysis. Chapters 7 and 8 provide analytical method to determine dynamic response of a rigid foundation subjected to a distance blast and to identify position of a vertical exciting force on the surface of an elastic half-space medium using sensor fusion, respectively. Chapter 9 presents an overview of techniques established for analyzing Surface Vibration of a multilayered elastic medium due to harmonic concentrated force. Chapter 10 will cover the three-dimensional wave propagation in porous media.

## **Method of Presentation**

The scope of each chapter is clearly outlined and the governing equations are derived with an adequate explanation of the procedures. The covered topics are logically and completely presented without unnecessary overemphasis. The topics are presented in a book form rather than in the style of a handbook. Tables, charts, equations, and references are used in abundance. Proofs and derivations are often emphasized and the physical model and final results are accompanied with illustrations and interpretations. Certain specific information that is required in carrying out the design analysis in detail has been stressed.

## **Prerequisites**

The book is written for graduate students, so the assumption is that the readers are familiar with the fundamentals of differential equations, as well as a basic knowledge of linear algebra, Fourier transform, and numerical methods. The presented topics are aimed to establish a conceptual framework that enables the reader to pursue further advances in the field. Although the governing equations will be derived with adequate explanations of the procedures, it is assumed that the readers have a working knowledge of theory of elasticity, fluid-structure interaction, and vibration engineering.

## **Unit System**

Through the chapters, for the sake of generality, computed results and the required parameters are provided in non-dimensional forms. Nevertheless, the system of units adopted for case studies is, unless otherwise stated, the British Gravitational system of units (BG). The units of degree (deg) or radian (rad) are utilized for variables representing angular quantities.

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