The background of the book cover is a scenic landscape photograph. It features a large, rugged mountain with patches of snow or light-colored rock near its peak. In the foreground, there is a dense forest of trees with vibrant autumn foliage in shades of red, orange, and yellow. A calm body of water in the lower half of the image reflects the mountains and the colorful trees. A bright sun is visible in the upper left portion of the sky, creating a lens flare effect. A thin, light blue wavy line arches over the mountain range. At the bottom of the cover, there are several vertical black lines of varying lengths, each ending in a small black dot, resembling a stylized signal or a digital waveform.

# Essentials of Digital Signal Processing

B. P. Lathi  
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Essentials of  
**Digital Signal Processing**

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## Essentials of Digital Signal Processing

This textbook offers a fresh approach to digital signal processing (DSP) that combines heuristic reasoning and physical appreciation with sound mathematical methods to illuminate DSP concepts and practices. It uses metaphors, analogies, and creative explanations along with carefully selected examples and exercises to provide deep and intuitive insights into DSP concepts.

Practical DSP requires hybrid systems including both discrete- and continuous-time components. This book follows a holistic approach and presents discrete-time processing as a seamless continuation of continuous-time signals and systems, beginning with a review of continuous-time signals and systems, frequency response, and filtering. The synergistic combination of continuous-time and discrete-time perspectives leads to a deeper appreciation and understanding of DSP concepts and practices.

### Notable Features:

- Written for upper-level undergraduates
- Provides an intuitive understanding and physical appreciation of essential DSP concepts without sacrificing mathematical rigor
- Illustrates concepts with 500 high-quality figures, more than 170 fully worked examples, and hundreds of end-of-chapter problems
- Encourages student learning with more than 150 drill exercises, including complete and detailed solutions
- Maintains strong ties to continuous-time signals and systems concepts, with immediate access to background material with a notationally consistent format, helping readers build on their previous knowledge
- Seamlessly integrates MATLAB throughout the text to enhance learning
- Develops MATLAB code from a basic level to reinforce connections to underlying theory and sound DSP practice

B. P. Lathi holds a PhD in Electrical Engineering from Stanford University and was previously a Professor of Electrical Engineering at California State University, Sacramento. He is the author of eight books, including *Signal Processing and Linear Systems* (second ed., 2005) and, with Zhi Ding, *Modern Digital and Analog Communications Systems* (fourth ed., 2009).

Roger A. Green is an Associate Professor of Electrical and Computer Engineering at North Dakota State University. He holds a PhD from the University of Wyoming. He is a contributor to the second edition of *Signal Processing and Linear Systems* by B. P. Lathi.



# Preface

Since its emergence as a field of importance in the 1970s, digital signal processing (DSP) has grown in exponential lockstep with advances in digital hardware. Today's digital age requires that undergraduate students master material that was, until recently, taught primarily at the graduate level. Many DSP textbooks remain rooted in this graduate-level foundation and cover an exhaustive (and exhausting!) number of topics. This book provides an alternative. Rather than cover the broadest range of topics possible, we instead emphasize a narrower set of core digital signal processing concepts. Rather than rely solely on mathematics, derivations, and proofs, we instead balance necessary mathematics with a physical appreciation of subjects through heuristic reasoning, careful examples, metaphors, analogies, and creative explanations. Throughout, our underlying goal is to make digital signal processing as accessible as possible and to foster an intuitive understanding of the material.

Practical DSP requires hybrid systems that include both discrete-time and continuous-time components. Thus, it is somewhat curious that most DSP textbooks focus almost exclusively on discrete-time signals and systems. This book takes a more holistic approach and begins with a review of continuous-time signals and systems, frequency response, and filtering. This material, while likely familiar to most readers, sets the stage for sampling and reconstruction, digital filtering, and other aspects of complete digital signal processing systems. The synergistic combination of continuous-time and discrete-time perspectives leads to a deeper and more complete understanding of digital signal processing than is possible with a purely discrete-time viewpoint. A strong foundation of continuous-time concepts naturally leads to a stronger understanding of discrete-time concepts.

## Notable Features

Some notable features of this book include the following:

1. This text is written for an upper-level undergraduate audience, and topic treatment is appropriately geared to the junior and senior levels. This allows a sufficiently detailed mathematical treatment to obtain a solid foundation and competence in DSP without losing sight of the basics.
2. An underlying philosophy of this textbook is to provide a simple and intuitive understanding of essential DSP concepts without sacrificing mathematical rigor. Much attention has been paid to provide clear, friendly, and enjoyable writing. A physical appreciation of the topics is attained through a balance of intuitive explanations and necessary mathematics. Concepts are illustrated using nearly 500 high-quality figures and more than 170 fully worked examples. Further reinforcement is provided through more than 150 drill exercises, complete detailed solutions of which are provided as an appendix to the book. Hundreds of end-of-chapter problems provide students with additional opportunities to learn and practice.
3. Unlike most DSP textbooks, this book maintains strong ties to continuous-time signals and systems concepts, which helps readers to better understand complete DSP systems. Further, by leveraging off a solid background of continuous-time concepts, discrete-time concepts are more easily and completely understood. Since the continuous-time background material is



included, readers have immediate access to as much or as little background material as necessary, all in a notationally consistent format.

4. MATLAB is effectively utilized throughout the text to enhance learning. This MATLAB material is tightly and seamlessly integrated into the text so as to seem a natural part of the material and problem solutions rather than an added afterthought. Unlike many DSP texts, this book does not have specific “MATLAB Examples” or “MATLAB Problems” any more than it has “Calculator Examples” or “Calculator Problems.” Modern DSP has evolved to the point that sophisticated computer packages (such as MATLAB) should be used every bit as naturally as calculus and calculators, and it is this philosophy that guides the manner with which MATLAB is incorporated into the book.

Many DSP books rely on canned MATLAB functions to solve various digital signal processing problems. While this produces results quickly and with little effort, students often miss how problem solutions are coded or how theory is translated into practice. This book specifically avoids high-level canned functions and develops code from a more basic level; this approach reinforces connections to the underlying theory and develops sound skills in the practice of DSP. Every piece of MATLAB code precisely conforms with book concepts, equations, and notations.

## Book Organization and Use

Roughly speaking, this book is organized into five parts.

1. Review of continuous-time signals and systems (Ch. 1) and continuous-time (analog) filtering (Ch. 2)
2. Sampling and reconstruction (Ch. 3)
3. Introduction to discrete-time signals and systems (Ch. 4) and the time-domain analysis of discrete-time systems (Ch. 5)
4. Frequency-domain analysis of discrete-time systems using the discrete-time Fourier transform (Ch. 6) and the  $z$ -transform (Ch. 7)
5. Discrete-time (digital) filtering (Ch. 8) and the discrete-Fourier transform (Ch. 9)

The first quarter of this book (Chs. 1 and 2, about 150 pages) focuses on continuous-time concepts, and this material can be scanned or skipped by readers who possess a solid background in these areas. The last three quarters of the book (Chs. 3 through 9, about 450 pages) cover traditional discrete-time concepts that form the backbone of digital signal processing. The majority of the book can be covered over a semester in a typical 3- or 4-credit-hour undergraduate-level course, which corresponds to around 45 to 60 lecture-hours of contact.

As with most textbooks, this book can be adapted to accommodate a range of courses and student backgrounds. Students with solid backgrounds in continuous-time signals and systems can scan or perhaps altogether skip the first two chapters. Students with knowledge in the time-domain analysis of discrete-time signals and systems can scan or skip Chs. 4 and 5. Courses that do not wish to emphasize filtering operations can eliminate coverage of Chs. 2 and 8. Many other options exist as well. For example, students enter the 3-credit Applied Digital Signal Processing and Filtering course at North Dakota State University having completed a 4-credit Signals and Systems course that covers both continuous-time and discrete-time concepts, including Laplace and  $z$ -transforms but not including discrete-time Fourier analysis. Given this student background, the NDSU DSP course covers Chs. 2, 3, 6, 8, and 9, which leaves enough extra time to introduce (and use) digital signal processing hardware from Texas Instruments; Chs. 1, 4, 5, and 7 are recommended for reading, but not required.

## Acknowledgments

We would like to offer our sincere gratitude to the many people who have generously given their time and talents to the creation, improvement, and refinement of this book. Books, particularly sizable ones such as this, involve a seemingly infinite number of details, and it takes the combined efforts of a good number of good people to successfully focus these details into a quality result. During the six years spent preparing this book, we have been fortunate to receive valuable feedback and recommendations from numerous reviewers, colleagues, and students. We are grateful for the reviews provided by Profs. Zekeriya Aliyazicioglu of California State Polytechnic University-Pomona, Mehmet Celenk of Ohio University, Liang Dong of Western Michigan University, Jake Gunther of Utah State University, Joseph P. Hoffbeck of the University of Portland, Jianhua Liu of Embry-Riddle Aeronautical University, Peter Mathys of the University of Colorado, Phillip A. Mlsna of Northern Arizona University, S. Hossein Mousavinezhad of Idaho State University, Kalyan Mondal of Fairleigh Dickinson University, Anant Sahai of UC Berkeley, Jose Sanchez of Bradley University, and Xiaomu Song of Widener University. We also offer our heartfelt thanks for the thoughtful comments and suggestions provided by the many anonymous reviewers, who outnumbered the other reviewers more than two-to-one. We wish that we could offer a more direct form of recognition to these reviewers. Some of the most thoughtful and useful comments came from students taking the Applied Digital Signal Processing and Filtering course at North Dakota State University. Two students in particular – Kyle Kraning and Michael Boyko – went above the call of duty, providing more than one hundred corrections and comments. For their creative contributions of cartoon ideas, we also give thanks to NDSU students Stephanie Rosen (Chs. 1, 4, and 5) and Tanner Voss (Ch. 2). Book writing is a time-consuming activity, and one that inevitably causes hardship to those who are close to an author. Thus, we offer our final thanks to our families for their sacrifice, support, and love.

*B. P. Lathi*  
*Roger A. Green*



DSP is always on the future's horizon!





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# Chapter 1

## Review of Continuous-Time Signals and Systems

This chapter reviews the basics of continuous-time (CT) signals and systems. Although the reader is expected to have studied this background as a prerequisite for this course, a thorough yet abbreviated review is both justified and wise since a solid understanding of continuous-time concepts is crucial to the study of digital signal processing.

### Why Review Continuous-Time Concepts?

It is natural to question how continuous-time signals and systems concepts are relevant to digital signal processing. To answer this question, it is helpful to first consider elementary signals and systems structures.

In the most simplistic sense, the study of signals and systems is described by the block diagram shown in Fig. 1.1a. An input signal is fed into a system to produce an output signal. Understanding this block diagram in a completely general sense is quite difficult, if not impossible. A few well-chosen and reasonable restrictions, however, allow us to fully understand and mathematically quantify the character and behavior of the input, the system, and the output.

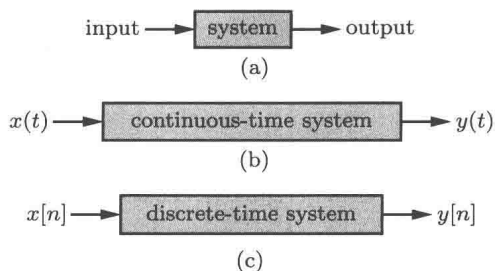


Figure 1.1: Elementary block diagrams of (a) general, (b) continuous-time, and (c) discrete-time signals and systems.

Introductory textbooks on signals and systems often begin by restricting the input, the system, and the output to be continuous-time quantities, as shown in Fig. 1.1b. This diagram captures the basic structure of continuous-time signals and systems, the details of which are reviewed later in this chapter and the next. Restricting the input, the system, and the output to be discrete-time (DT) quantities, as shown in Fig. 1.1c, leads to the topic of discrete-time signals and systems.

Typical digital signal processing (DSP) systems are hybrids of continuous-time and discrete-time systems. Ordinarily, DSP systems begin and end with continuous-time signals, but they process

signals using a digital signal processor of some sort. Specialized hardware is required to bridge the continuous-time and discrete-time worlds. As the block diagram of Fig. 1.2 shows, general DSP systems are more complex than either Figs. 1.1b or 1.1c allow; both CT and DT concepts are needed to understand complete DSP systems.

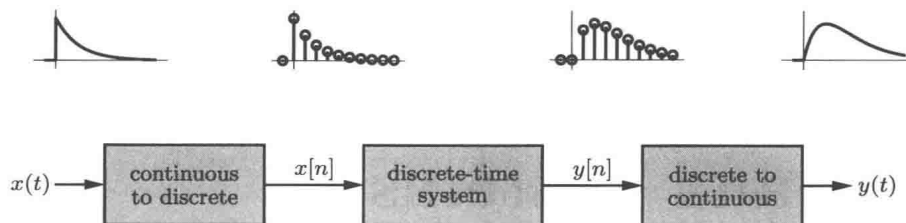


Figure 1.2: Block diagram of a typical digital signal processing system.

A more detailed explanation of Fig. 1.2 helps further justify why it is important for us to review continuous-time concepts. The continuous-to-discrete block converts a continuous-time input signal into a discrete-time signal, which is then processed by a digital processor. The discrete-time output of the processor is then converted back to a continuous-time signal.<sup>†</sup> Only with knowledge of continuous-time signals and systems is it possible to understand these components of a DSP system. Sampling theory, which guides our understanding of the CT-to-DT and DT-to-CT converters, can be readily mastered with a thorough grasp of continuous-time signals and systems. Additionally, the discrete-time algorithms implemented on the digital signal processor are often synthesized from continuous-time system models. All in all, continuous-time signals and systems concepts are useful and necessary to understand the elements of a DSP system.

Nearly all basic concepts in the study of continuous-time signals and systems apply to the discrete-time world, with some modifications. Hence, it is economical and very effective to build on the previous foundations of continuous-time concepts. Although discrete-time math is inherently simpler than continuous-time math (summation rather than integration, subtraction instead of differentiation), students find it difficult, at first, to grasp basic discrete-time concepts. The reasons are not hard to find. We are all brought up on a steady diet of continuous-time physics and math since high school, and we find it easier to identify with the continuous-time world. It is much easier to grasp many concepts in continuous-time than in discrete-time. Rather than fight this reality, we might use it to our advantage.

## 1.1 Signals and Signal Categorizations

A *signal* is a set of data or information. Examples include telephone and television signals, monthly sales of a corporation, and the daily closing prices of a stock market (e.g., the Dow Jones averages). In all of these examples, the signals are functions of the independent variable *time*. This is not always the case, however. When an electrical charge is distributed over a body, for instance, the signal is the charge density, a function of *space* rather than time. In this book we deal almost exclusively with signals that are functions of time. The discussion, however, applies equally well to other independent variables.

Signals are categorized as either continuous-time or discrete-time and as either analog or digital. These fundamental signal categories, to be described next, facilitate the systematic and efficient analysis and design of signals and systems.

<sup>†</sup>As we shall later see, the continuous-to-discrete block is typically comprised of a signal conditioning circuit followed by a CT-to-DT converter and an analog-to-digital converter (ADC). Similarly, the discrete-to-continuous block is typically comprised of a digital-to-analog converter (DAC) followed by a DT-to-CT converter and finally another conditioning circuit.