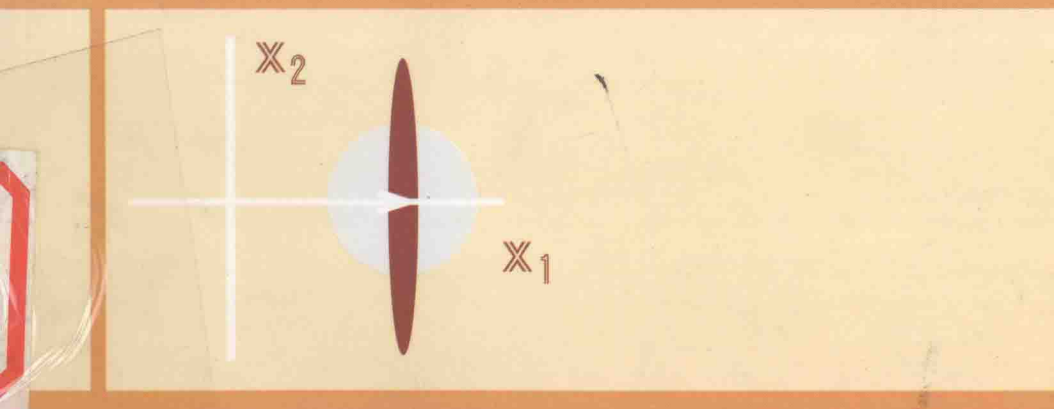


D.F.Walls G.J.Milburn

Quantum Optics

量子光学



Springer-Verlag

世界图书出版公司

D.F. Walls G.J. Milburn

Quantum Optics

With 98 Figures

Springer-Verlag

世界图书出版公司

北京·广州·上海·西安

书 名: Quantum Optics
作 者: D. F. Walls G.J.Milburn
中译名: 量子光学
出版者: 世界图书出版公司北京公司
印刷者: 北京世图印刷厂
发 行: 世界图书出版公司北京公司 (北京朝内大街 137 号 100010)
联系电话: 010-64015659, 64038347
电子信箱: kjsk@vip.sina.com
开 本: 24 印 张: 15.5
出版年代: 1999 年 6 月第 1 版 2004 年 1 月第 2 次印刷
书 号: 7-5062-3627-3
版权登记: 图字: 01-97-1786
定 价: 55.00 元

世界图书出版公司北京公司已获得 Springer-Verlag 授权在中国大陆
独家重印发行。

Quantum Optics

SPRINGER
STUDY
EDITION

Professor D.F. WALLS, F.R.S.

University of Auckland
Private Bag 92019
Auckland
New Zealand

Dr. G.J. MILBURN
Physics Department
University of Queensland
St. Lucia QLD 4067
Australia

First Edition 1994
Second Printing 1995

ISBN 3-540-58831-0 Springer-Verlag Berlin Heidelberg New York

CIP data applied for

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilm or in any other way, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer-Verlag. Violations are liable for prosecution under the German Copyright Law.

© Springer-Verlag Berlin Heidelberg 1994
Printed in the United States of America

The use of general descriptive names, registered names, trademarks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

This reprint has been authorized by Springer-Verlag (Berlin/Heidelberg/New York) for sale in the People's Republic of China only and not for export therefrom.
Reprinted in China by Beijing World Publishing Corporation, 1998

Preface

This text book originated out of a graduate course of lectures in Quantum Optics given at the University of Waikato and the University of Auckland. A broad range of material is covered in this book ranging from introductory concepts to current research topics. A pedagogic description of the techniques of quantum optics and their applications to physical systems is presented. Particular emphasis is given to systems where the theoretical predictions have been confirmed by experimental observation.

The material presented in this text could be covered in a two semester course. Alternatively the introductory material in Chaps. 1–6 and selected topics from the later chapters would be suitable for a one semester course. For example, for material involving the interaction of light with atoms Chaps. 10–13 would be appropriate, whereas for material on squeezed light Chaps. 7 and 8 are required. Chaps. 14–16 describe the interrelation of fundamental topics in quantum mechanics with quantum optics. The final chapter on atomic optics gives an introduction to this new and rapidly developing field.

One of us (D.F. Walls) would like to thank Roy Glauber and Hermann Haken for the wonderful introduction they gave me to this exciting field. We would also like to thank our students and colleagues at the Universities of Waikato, Auckland and Queensland who have contributed so much to the material in this book. In particular, Crispin Gardiner, Ken McNeil, Howard Carmichael, Peter Drummond, Margaret Reid, Shoukry Hassan, Matthew Collett, Sze Tan, Alistair Lane, Brian Kennedy, Craig Savage, Monika Marte, Murray Holland and Pippa Storey. Finally, we would like to thank all our friends and colleagues in Quantum Optics too numerous to name with whom we have shared in the excitement of the development of this field.

The completion of this book would not have been possible without the excellent work of Susanna van der Meer who performed the word processing through many iterations.

Auckland, New Zealand
St. Lucia, Australia
January 1994

D.F. WALLS
G.J. MILBURN

Contents

1. Introduction	1
2. Quantisation of the Electromagnetic Field	7
2.1 Field Quantisation	7
2.2 Fock or Number States	10
2.3 Coherent States	12
2.4 Squeezed States	15
2.5 Two-Photon Coherent States	18
2.6 Variance in the Electric Field	20
2.7 Multimode Squeezed States	22
2.8 Phase Properties of the Field	23
Exercises	26
3. Coherence Properties of the Electromagnetic Field	29
3.1 Field-Correlation Functions	29
3.2 Properties of the Correlation Functions	31
3.3 Correlation Functions and Optical Coherence	32
3.4 First-Order Optical Coherence	34
3.5 Coherent Field	38
3.6 Photon Correlation Measurements	39
3.7 Quantum Mechanical Fields	41
3.7.1 Squeezed States	42
3.7.2 Squeezed Vacuum	44
3.8 Phase-Dependent Correlation Functions	44
3.9 Photon Counting Measurements	46
3.9.1 Classical Theory	46
3.9.2 Constant Intensity	48
3.9.3 Fluctuating Intensity – Short-Time Limit	48
3.10 Quantum Mechanical Photon Count Distribution	51
3.10.1 Coherent Light	52
3.10.2 Chaotic Light	52
3.10.3 Photo-Electron Current Fluctuations	53
Exercises	55

4. Representations of the Electromagnetic Field	57
4.1 Expansion in Number States	57
4.2 Expansion in Coherent States	58
4.2.1 P Representation	58
a) Correlation Functions	60
b) Covariance Matrix	61
c) Characteristic Function	62
4.2.2 Wigner's Phase-Space Density	63
a) Coherent State	64
b) Squeezed State	64
c) Number State	64
4.2.3 Q Function	65
4.2.4 R Representation	66
4.2.5 Generalized P Representations	68
a) Number State	71
b) Squeezed State	71
4.2.6 Positive P Representation	71
Exercises	72
5. Quantum Phenomena in Simple Systems in Nonlinear Optics	73
5.1 Single-Mode Quantum Statistics	73
5.1.1 Degenerate Parametric Amplifier	73
5.1.2 Photon Statistics	75
5.1.3 Wigner Function	75
5.2 Two-Mode Quantum Correlations	77
5.2.1 Non-degenerate Parametric Amplifier	77
5.2.2 Squeezing	80
5.2.3 Quadrature Correlations and the Einstein–Podolsky–Rosen Paradox	81
5.2.4 Wigner Function	82
5.2.5 Reduced Density Operator	83
5.3 Quantum Limits to Amplification	85
5.4 Amplitude Squeezed State with Poisson Photon Number Statistics	86
Problems	89
6. Stochastic Methods	91
6.1 Master Equation	91
6.2 Equivalent c -Number Equations	98
6.2.1 Photon Number Representation	98
6.2.2 P Representation	99
6.2.3 Properties of Fokker–Planck Equations	101

6.2.4	Steady State Solutions – Potential Conditions . .	101
6.2.5	Time Dependent Solution	103
6.2.6	Q Representation	104
6.2.7	Wigner Function	106
6.2.8	Generalized P Representation	108
	a) Complex P Representation	109
	b) Positive P Representation	110
6.3	Stochastic Differential Equations	111
6.3.1	Use of the Positive P Representation	115
6.4	Linear Processes with Constant Diffusion	115
6.5	Two Time Correlation Functions in Quantum Markov Processes	117
6.5.1	Quantum Regression Theorem	118
6.6	Application to Systems with a P Representation	118
	Exercises	119
7.	Input–Output Formulation of Optical Cavities	121
7.1	Cavity Modes	121
7.2	Linear Systems	124
7.3	Two-Sided Cavity	126
7.4	Two Time Correlation Functions	127
7.5	Spectrum of Squeezing	129
7.6	Parametric Oscillator	129
7.7	Squeezing in the Total Field	132
7.8	Fokker–Planck Equation	132
	Exercises	135
8.	Generation and Applications of Squeezed Light	137
8.1	Parametric Oscillation and Second Harmonic Generation	137
8.1.1	Semi-classical Steady States and Stability Analysis	139
8.1.2	Parametric Oscillation	139
8.1.3	Second Harmonic Generation	140
8.1.4	Squeezing Spectrum	141
8.1.5	Parametric Oscillation	142
8.1.6	Experiments	143
8.2	Twin Beam Generation and Intensity Correlations . .	146
8.2.1	Second Harmonic Generation	150
8.2.2	Experiments	152
8.2.3	Dispersive Optical Bistability	153
8.3	Applications of Squeezed Light	158
8.3.1	Interferometric Detection of Gravitational Radiation	158

8.3.2 Sub-Shot-Noise Phase Measurements	172
Exercises	174
9. Nonlinear Quantum Dissipative Systems	177
9.1 Optical Parametric Oscillator: Complex P Function . .	177
9.2 Optical Parametric Oscillator: Positive P Function . .	182
9.3 Quantum Tunnelling Time	186
9.4 Dispersive Optical Bistability	191
9.5 Comment on the Use of the Q and Wigner Representations	193
Exercises	193
9.A Appendix	194
9.A.1 Evaluation of Moments for the Complex P function for Parametric Oscillation (9.17)	194
9.A.2 Evaluation of the Moments for the Complex P Function for Optical Bistability (9.48)	195
10. Interaction of Radiation with Atoms	197
10.1 Quantization of the Electron Wave Field	197
10.2 Interaction Between the Radiation Field and the Electron Wave Field	199
10.3 Interaction of a Two-Level Atom with a Single Mode Field	204
10.4 Quantum Collapses and Revivals	205
10.5 Spontaneous Decay of a Two-Level Atom	206
10.6 Decay of a Two-Level Atom in a Squeezed Vacuum	208
10.7 Phase Decay in a Two-Level System	210
Exercises	211
11. Resonance Fluorescence	213
11.1 Master Equation	213
11.2 Spectrum of the Fluorescent Light	217
11.3 Photon Correlations	221
11.4 Squeezing Spectrum	225
Exercises	228
12. Quantum Theory of the Laser	229
12.1 Master Equation	229
12.2 Photon Statistics	232
12.2.1 Spectrum of Intensity Fluctuations	233
12.3 Laser Linewidth	235
12.4 Regularly Pumped Laser	236

12.A Appendix: Derivation of the Single-Atom Increment	240
Exercises	244
13. Intracavity Atomic Systems	245
13.1 Optical Bistability	245
13.2 Nondegenerate Four Wave Mixing	252
13.3 Experimental Results	258
Exercises	259
14. Bells Inequalities in Quantum Optics	261
14.1 The Einstein–Podolsky–Rosen (EPR) Argument	261
14.2 Bell Inequalities and the Aspect Experiment	262
14.3 Violations of Bell’s Inequalities Using a Parametric Amplifier Source	268
14.4 One-Photon Interference	273
Exercises	279
15. Quantum Nondemolition Measurements	281
15.1 Concept of a QND measurement	282
15.2 Back Action Evasion	284
15.3 Criteria for a QND Measurement	284
15.4 The Beam Splitter	287
15.5 Ideal Quadrature QND Measurements	290
15.6 Experimental Realisation	291
15.7 A Photon Number QND Scheme	294
Exercises	296
16. Quantum Coherence and Measurement Theory	297
16.1 Quantum Coherence	297
16.2 The Effect of Fluctuations	303
16.3 Quantum Measurement Theory	306
16.4 Examples of Pointer Observables	310
16.5 Model of a Measurement	311
Exercises	313
17. Atomic Optics	315
17.1 Young’s Interference with Path Detectors	316
17.1.1 The Feynman Light Microscope	319
17.2 Atomic Diffraction by a Standing Light Wave	321
17.3 Optical Stern–Gerlach Effect	325
17.4 Quantum Non-Demolition Measurement of the Photon Number by Atomic Beam Deflection	330

XII Contents

17.5 Measurement of Atomic Position	334
17.5.1 Atomic Focussing and Contractive States...	337
Exercises	339
17.A Appendix.....	339
References	341
Subject Index	347

1. Introduction

The first indication of the quantum nature of light came in 1900 when M. Planck discovered he could account for the spectral distribution of thermal light by postulating that the energy of a harmonic oscillator is quantized. Further evidence was added by A. Einstein who showed in 1905 that the photoelectric effect could be explained by the hypothesis that the energy of a light beam was distributed in discrete bundles later known as photons.

Einstein also contributed to the understanding of the absorption and emission of light from atoms with his development of a phenomenological theory in 1917. This theory was later shown to be a natural consequence of the quantum theory of electromagnetic radiation.

Despite this early connection with quantum theory physical optics has developed more or less independently of quantum theory. The vast majority of physical-optics experiments can adequately be explained using classical theory of electromagnetic radiation based on Maxwell's equations. An early attempt to find quantum effects in an optical interference experiment by G.I. Taylor in 1909 gave a negative result. Taylor's experiment was an attempt to repeat T. Young's famous two slit experiment with one photon incident on the slits. The classical explanation based on the interference of electric field amplitudes and the quantum explanation based on the interference of the probability amplitudes for the photon to pass through either slit coincide in this experiment. Interference experiments of Young's type do not distinguish between the predictions of classical theory and quantum theory. It is only in higher-order interference experiments involving the interference of intensities that differences between the predictions of classical and quantum theory appear. In such an experiment two electric fields are detected on a photomultiplier and their intensities are allowed to interfere. Whereas classical theory treats the interference of intensities, in quantum theory the interference is still at the level of probability amplitudes. This is one of the most important differences between quantum theory and classical theory.

The first experiment in intensity interferometry was the famous experiment of R. Hanbury Brown and R.Q. Twiss. This experiment studied the correlation in the photo-current fluctuations from two detectors. Later experiments were photon counting experiments, and the correlations between photon numbers were studied.

The Hanbury-Brown-Twiss experiment observed an enhancement in the two-time intensity correlation function of short time delays for a thermal light

source known as photon bunching. This was a consequence of the large intensity fluctuations in the thermal source. Such photon bunching phenomena may be adequately explained using a classical theory with a fluctuating electric field amplitude. For a perfectly amplitude stabilized light field such as an ideal laser operating well above threshold there is no photon bunching. A photon counting experiment where the number of photons arriving in an interval T are counted, shows that there is still a randomness in the photon arrivals. The photon-number distribution for an ideal laser is Poissonian. For thermal light a super-Poissonian photocount distribution results.

While the above results may be derived from both classical and quantum theory, the quantum theory makes additional unique predictions. This was first elucidated by R.J. Glauber in his quantum formulation of optical coherence theory in 1963. One such prediction is photon antibunching where the initial slope of the two-time correlation function is positive. This corresponds to greater than average separations between the photon arrivals or photon antibunching. The photocount statistics may also be sub-Poissonian. A classical theory of fluctuating field amplitudes would require negative probabilities in order to give photon antibunching. In the quantum picture it is easy to visualize photon arrivals more regular than Poissonian.

It was not, however, until 1975 when H.J. Carmichael and D.F. Walls predicted that light generated in resonance fluorescence from a two-level atom would exhibit photon antibunching that a physically accessible system exhibiting nonclassical behaviour was identified. Photon antibunching was observed during the next year in this system in an experiment by H.J. Kimble, M. Dagenais and L. Mandel. This was the first nonclassical effect observed in optics and ushered in a new era in quantum optics.

The experiments of Kimble et al. used an atomic beam and hence the photon antibunching was convolved with the atomic number fluctuations in the beam. With developments in ion-trap technology it is now possible to trap a single ion for several minutes. H. Walther and coworkers in Munich have studied resonance fluorescence from a single atom in a trap. They have observed both photon antibunching and sub-Poissonian statistics in this system.

In the 1960's improvements in photon counting techniques proceeded in tandem with the development of new laser light sources. Light from incoherent (thermal) and coherent (laser) sources could now be distinguished by their photon counting properties. The groups of F.T. Arecchi in Milan, L. Mandel in Rochester and R.E. Pike in Malvern measured the photocount statistics of the laser. They showed that the photocount statistics went from super-Poissonian below threshold to Poissonian far above threshold. Concurrently, the quantum theory of the laser was being developed by H. Haken in Stuttgart, M.O. Scully and W. Lamb at Yale, and M. Lax and W.H. Louisell in New Jersey. In these theories both the atomic variables and the electromagnetic field were quantized. The result of these calculations were that the laser functioned as an essentially classical device. In fact H. Risken showed that it could be modelled by a van der Pol oscillator.

It is only quite recently that the role the noise in the pumping process plays in obscuring the quantum aspects of the laser has been understood. If the noise in the pumping process can be suppressed the output of the laser may exhibit sub-Poissonian statistics. In other words, the intensity fluctuations may be reduced below the shot-noise level characteristic of normal lasers. Y. Yamamoto in Tokyo has pioneered experimental developments in the area of semiconductor lasers with suppressed pump noise. In a high impedance constant current driven semiconductor laser the fluctuations in the pumping electrons are reduced below Poissonian. This results in the photon statistics of the emitted photons being sub-Poissonian.

It took another nine years after the observation of photon antibunching for another prediction of the quantum theory of light to be observed – squeezing of quantum fluctuations. The electric field for a nearly monochromatic plane wave may be decomposed into two quadrature components with the time dependence $\cos \omega t$ and $\sin \omega t$, respectively. In a coherent state, the closest quantum counterpart to a classical field, the fluctuations in the two quadratures are equal and minimize the uncertainty product given by Heisenberg's uncertainty relation. The quantum fluctuations in a coherent state are equal to the zero-point vacuum fluctuations and are randomly distributed in phase. In a squeezed state the quantum fluctuations are no longer independent of phase. One quadrature phase may have reduced quantum fluctuations at the expense of increased quantum fluctuations in the other quadrature phase such that the product of the fluctuations still obeys Heisenberg's uncertainty relation.

Squeezed states offer the possibility of beating the quantum limit in optical measurements by making phase-sensitive measurements which utilize only the quadrature with reduced quantum fluctuations. The generation of squeezed states requires a nonlinear phase-dependent interaction. The first observation of squeezed states was achieved by R.E. Slusher in 1985 at the AT&T Bell Laboratories in four-wave mixing in atomic sodium. This was soon followed by demonstrations of squeezing in an optical parametric oscillator by H.J. Kimble and by four-wave mixing in optical fibres by M.D. Levenson.

Squeezing-like photon antibunching is a consequence of the quantization of the light field. The usefulness of squeezed light was demonstrated in experiments in optical interferometry by Kimble and Slusher. Following the original suggestion of C.M. Caves at Caltech they injected squeezed light into the empty port of an interferometer. By choosing the phase of the squeezed light so that the quantum fluctuations entering the empty port were reduced below the vacuum level they observed an enhanced visibility of the interference fringes.

In the nonlinear process of parametric down conversion a high frequency photon splits into two photons with frequencies such that their sum equals that of the high-energy photon. The two photons (photon twins) produced in this process possess quantum correlations and have identical intensity fluctuations. This may be exploited in experiments where the intensity fluctuations in the difference photocurrent for the two beams is measured. The intensity difference fluctuations in the twin beams have been shown to be considerably below the

shot-noise level in experiments by E. Giacobino in Paris and P. Kumar in Evanston.

The twin beams may also be used in absorption measurements where the sample is placed in one of the beams and the other beam is used as a reference. The driving laser is tuned so that the frequency of the twin beams matches the frequency at which the sample absorbs. When the twin beams are detected and the photocurrents are subtracted, the presence of even very weak absorption can be seen because of the small quantum noise in the difference current.

The photon pairs generated in parametric down conversion also carry quantum correlations of the Einstein-Podolsky-Rosen type. Intensity correlation experiments to test Bell inequalities were designed using a correlated pair of photons. The initial experiments by A. Aspect in Paris utilized a two photon cascade to generate the correlated photons, however, recent experiments have used parametric down conversion. These experiments have consistently given results in agreement with the predictions of quantum theory and in violation of classical predictions. At the basis of the difference between the two theories is the interference of probability amplitudes which is characteristic of quantum mechanics. In these intensity interference experiments as opposed to interference experiments of the Young's type the two theories yield different predictions. This was strikingly demonstrated in an intensity interference experiment which has only one incident photon but has phase-sensitive detection. In this experiment proposed by S.M. Tan, D.F. Walls and M.J. Collett a single photon may take either path to two homodyne detectors. Nonlocal quantum correlations between the two detectors occur, which are a consequence of the interference of the probability amplitudes for the photon to take either path.

The major advances made in quantum optics, in particular the ability to generate and detect light with less quantum fluctuations than the vacuum, makes optics a fertile testing ground for quantum measurement theory. The idea of quantum non-demolition measurements arose in the context of how to detect the change in position of a free mass acted on by a force such as a gravitational wave. However, the concept is general. Basically one wishes to measure the value of an observable without disturbing it so that subsequent measurements can be made with equal accuracy as the first. Demonstrations of quantum non-demolition measurements have been achieved in optics. In experiments by M.D. Levenson and P. Grangier two electromagnetic-field modes have been coupled via a nonlinear interaction. A measurement of the amplitude quadrature of one mode (the probe) allows one to infer the value of the amplitude quadrature of the other mode (the signal) without disturbing it. This quantum non-demolition measurement allows one to evade the back action noise of the measurement by shunting the noise into the phase quadrature which is undetected.

The techniques developed in quantum optics include quantum treatments of dissipation. Dissipation has been shown to play a crucial role in the destruction of quantum coherence, which has profound implications for quantum measurement theory. The difficulties in generating a macroscopic superposition of

quantum states (Schrödinger's cat) is due to the fragility of such states to the presence of even small dissipation. Several schemes to generate these superposition states in optics have been proposed but to date there has been no experimental manifestation.

Matter-wave interferometry is a well established field, for example, electron and neutron interferometry. More recently, however, such effects have been demonstrated with atoms. Interferometry with atoms offers the advantage of greater mass and therefore greater sensitivity for measurements of changes of gravitational potentials. Using techniques of laser cooling the de Broglie wavelength of atoms may be increased. With slow atoms the passage time in the interferometer is increased thus leading to an increase in sensitivity. Atoms also have internal degrees of freedom which may be used to tag which path an atom took. Thus demonstrations of the principle of complementarity using a double-slit interference experiment with which path detectors may be realized with atoms.

Atoms may be diffracted from the periodic potential structure of a standing light wave. A new field of atomic optics is rapidly emerging. In atomic optics the role of the light and atoms are reversed. Optical elements such as mirrors and beam splitters consist of light fields which reflect and split atomic beams. The transmission of an atom by a standing light wave may be state selective (the optical Stern-Gerlach effect) and this property may be used as a beam splitter. The scattering of an atom by a standing light wave may depend on the photon statistics of the light. Hence, measuring the final momentum distribution of the atoms may give information on the photon statistics of the light field. Thus atomic optics may extend the range of quantum measurements possible with quantum optical techniques. For example, the position an atom passes through a standing light wave may be determined by measuring the phase shift it imparts to the light.

The field of quantum optics now occupies a central position involving the interaction of atoms with the electromagnetic field. It covers a wide range of topics ranging from fundamental tests of quantum theory to the development of new laser-light sources. In this text we introduce the analytic techniques of quantum optics. These techniques are applied to a number of illustrative examples. While the main emphasis of the book is theoretical, descriptions of the experiments which have played a central role in the development of quantum optics are included.

A summary of the topics included in this text book is given as follows:

A familiarity with non-relativistic quantum mechanics is assumed. As we will be concerned with the quantum properties of light and its interaction with atoms, the electromagnetic field is quantised in the second chapter. Commonly used basis states for the field, the number states, the coherent states, and the squeezed state are introduced and their properties discussed. A definition of optical coherence is given via a set of field correlation functions in Chap. 3. Various representations for the electromagnetic field are introduced in Chap. 4 using the number states and the coherent states as a basis.