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# Barron's 巴朗

# AP 微积分（第13版）

## Barron's AP Calculus 13th Edition

[美] 博　克（David Bock）　多诺万（Dennis Donovan）
霍基特（Shirley O. Hockett）　著

- 考点透析 应试技巧
- 6套微积分 AB 模拟试题
- 6套微积分 BC 模拟试题

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## Barron's 巴朗五大 Essential 5 要点提示

As you review the content in this book to work toward earning that **5** on your AP CALCULUS AB exam, here are five things that you **MUST** know above everything else:

**1** **Learn the basic facts:**

- derivatives (p. 115) and antiderivatives (p. 217) of common functions;
- the product (p. 115), quotient (p. 115), and chain rules (p. 116) for finding derivatives;
- the midpoint, left and right rectangle, and trapezoid approximations for estimating definite integrals (p. 259);
- finding antiderivatives by substitution (p. 219);
- the important theorems: Rolle's theorem (p. 130), the Mean Value theorem (p. 130), and especially the Fundamental Theorem of Calculus (p. 251).

(Barron's *AP Calculus Flash Cards* are a great way to study these!)

**2** **Understand that a derivative is an instantaneous rate of change, and be able to apply that concept to:**

- find equations of tangent lines (p. 163);
- determine where a function is increasing/decreasing (p. 164), concave up/down (p. 165), or has maxima, minima, or points of inflection (pp. 165, 171);
- analyze the speed, velocity, and acceleration of an object in motion (p. 181);
- solve related rates problems (p. 189), using implicit differentiation (p. 126) when necessary.

**3** **Understand that integrals represent accumulation functions based on antiderivatives, and be able to apply those concepts to:**

- the average value of a function (p. 272);
- area (p. 293) and volume (p. 300);
- position of object in motion and distance traveled (p. 347);
- total amount when given the rate of accumulation (p. 352);
- differential equations, including solutions and slope fields (p. 369).

**4** **Be able to apply any of the above calculus concepts to functions defined algebraically, graphically, or in tables.**

**5** **Be able to maximize your score on the exam by:**

- answering *all* the multiple-choice questions;
- knowing how and when to use your calculator, and when not to;
- understanding what work you need to show;
- knowing how to explain, interpret, and justify answers when a question requires that. (The free-response solutions in this book model such answers.)

As you review the content in this book to work toward earning that **5** on your AP CALCULUS BC exam, here are five things that you **MUST** know above everything else:

巴朗五大要点提示 Barron's Essential 5

**1** **Master the Essential 5 listed for the AB Calculus Exam.** These form the core for questions that determine your AB subscore, and provide the essential knowledge base you'll need for questions related to the additional BC topics.

**2** **Understand how to extend AB Calculus concepts to more advanced situations, including:**

- using L'Hôpital's rule to find limits of indeterminate forms (p. 131);
- using limits to analyze improper integrals (p. 309);
- solving logistic differential equations (p. 387) and estimating solutions using Euler's method (p. 373);
- finding antiderivatives using integration by parts (p. 226) or partial fractions (p. 225);
- finding arc lengths (p. 307).

**3** **Be able to apply calculus concepts to parametrically defined functions (pp. 77, 125, 256, 297, 349) and polar functions (pp. 80, 191, 298).**

**4** **Know how to analyze the position, velocity, speed, acceleration, and distance traveled for an object in motion in two dimensions by applying calculus concepts to vectors (p. 183).**

**5** **Understand infinite series.** You must be able to:

- determine whether a series converges or diverges (p. 409);
- use Taylor's theorem to represent functions as power series (p. 426);
- determine the interval of convergence for a power series (p. 421);
- find bounds on the error for estimates based on series (pp. 420, 433).

# 目 录

# Contents

# Introduction 绪 论

This book is intended for students who are preparing to take either of the two Advanced Placement Examinations in Mathematics offered by the College Entrance Examination Board, and for their teachers. It is based on the May 2014 course description published by the College Board, and covers the topics listed there for both Calculus AB and Calculus BC.

Candidates who are planning to take the CLEP Examination on Calculus with Elementary Functions are referred to the section of this Introduction on that examination on page 10.

## THE COURSES 课程

Calculus AB and BC are both full-year courses in the calculus of functions of a single variable. Both courses emphasize:

(1) student understanding of concepts and applications of calculus over manipulation and memorization;

(2) developing the student's ability to express functions, concepts, problems, and conclusions analytically, graphically, numerically, and verbally, and to understand how these are related; and

(3) using a graphing calculator as a tool for mathematical investigations and for problem-solving.

Both courses are intended for those students who have already studied college-preparatory mathematics: algebra, geometry, trigonometry, analytic geometry, and elementary functions (linear, polynomial, rational, exponential, logarithmic, trigonometric, inverse trigonometric, and piecewise). The AB topical course outline that follows can be covered in a full high-school academic year even if some time is allotted to studying elementary functions. The BC course assumes that students already have a thorough knowledge of all the topics noted above.

## TOPICS THAT MAY BE TESTED ON THE CALCULUS AB EXAM 微积分AB考试中可能考查的知识点

1. **Functions and Graphs**
   Rational, trigonometric, inverse trigonometric, exponential, and logarithmic functions.

2. **Limits and Continuity**
   Intuitive definitions; one-sided limits; functions becoming infinite; asymptotes and graphs; indeterminate limits of the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$ using algebra; $\lim_{\theta \to 0} \frac{\sin \theta}{\theta}$; estimating limits using tables or graphs.

Definition of continuity (in terms of limits); kinds of discontinuities; theorems about continuous functions; Extreme Value and Intermediate Value Theorems.

3. **Differentiation**
Definition of derivative as the limit of a difference quotient and as instantaneous rate of change; derivatives of power, exponential, logarithmic, trig and inverse trig functions; product, quotient, and chain rules; differentiability and continuity; estimating a derivative numerically and graphically; implicit differentiation; derivative of the inverse of a function; the Mean Value Theorem; recognizing a given limit as a derivative.

4. **Applications of Derivatives**
Rates of change; slope; critical points; average velocity; tangent line to a curve at a point and local linear approximation; increasing and decreasing functions; using the first and second derivatives for the following: local (relative) max or min, concavity, inflection points, curve sketching, global (absolute) max or min and optimization problems; relating a function and its derivatives graphically; motion along a line; related rates; differential equations and slope fields.

5. **The Definite Integral**
Definite integral as the limit of a Riemann sum; area; definition of definite integral; properties of the definite integral; use of Riemann sums (left, right and midpoint evaluations) and trapezoidal sums to approximate a definite integral; estimating definite integrals from tables and graphs; comparing approximating sums; average value of a function; Fundamental Theorem of Calculus; graphing a function from its derivative; accumulated change as integral of rate of change.

6. **Integration**
Antiderivatives and basic formulas; antiderivatives by substitution; applications of antiderivatives; separable differential equations; motion problems.

7. **Applications of Integration to Geometry**
Area of a region, including between two curves; volume of a solid of known cross section, including a solid of revolution.

8. **Further Applications of Integration and Riemann Sums**
Velocity and distance problems involving motion along a line; other applications involving the use of integrals of rates as net change or the use of integrals as accumulation functions; average value of a function over an interval.

9. **Differential Equations**
Basic definitions; geometric interpretations using slope fields; solving first-order separable differential equations analytically; exponential growth and decay.

## TOPICS THAT MAY BE TESTED ON THE CALCULUS BC EXAM 微积分BC考试中可能考查的知识点

BC ONLY

Any of the topics listed above for the Calculus AB exam may be tested on the BC exam. The following additional topics are restricted to the BC exam.

1. **Functions and Graphs**
Parametrically defined functions; polar functions; vector functions.

BC ONLY

2. **Limits and Continuity**
   No additional topics.

3. **Differentiation**
   Derivatives of polar, vector, and parametrically defined functions; indeterminate forms; L'Hôpital's rule.

4. **Applications of Derivatives**
   Tangents to parametrically defined curves; slopes of polar curves; analysis of curves defined parametrically or in polar or vector form.

5. **The Definite Integral**
   Integrals involving parametrically defined functions.

6. **Integration**
   By parts; by partial fractions (involving nonrepeating linear factors only); improper integrals.

7. **Applications of Integration to Geometry**
   Area of a region bounded by parametrically defined or polar curves; arc length.

8. **Further Applications of Integration and Riemann Sums**
   Velocity and distance problems involving motion along a planar curve; velocity and acceleration vectors.

9. **Differential Equations**
   Euler's method; applications of differential equations, including logistic growth.

10. **Sequences and Series**
    Definition of series as a sequence of partial sums and of its convergence as the limit of that sequence; harmonic, geometric, and p-series; integral, ratio, and comparison tests for convergence; alternating series and error bound; power series, including interval and radius of convergence; Taylor polynomials and graphs; finding a power series for a function; Maclaurin and Taylor series; Lagrange error bound for Taylor polynomials; computations using series.

## THE EXAMINATIONS 考试

The Calculus AB and BC Examinations and the course descriptions are prepared by committees of teachers from colleges or universities and from secondary schools. The examinations are intended to determine the extent to which a student has mastered the subject matter of the course.

Each examination is 3 hours and 15 minutes long, as follows:

Section I has two parts. Part A has 28 multiple-choice questions for which 55 minutes are allowed. The use of calculators is *not* permitted in Part A.

Part B has 17 multiple-choice questions for which 50 minutes are allowed. Some of the questions in Part B require the use of a graphing calculator.

Section II, the free-response section, has a total of six questions in two parts:

Part A has two questions, of which some parts *require* the use of a graphing calculator. After 30 minutes, however, *you will no longer be permitted to use a calculator.* If you finish Part A early, you will not be permitted to start work on Part B.

Part B has four questions and you are allotted an additional 60 minutes, but *you are not allowed to use a calculator.* You may work further on the Part A questions (without your calculator).

The section that follows gives important information on the use (and misuse!) of the graphing calculator.

## THE GRAPHING CALCULATOR: USING YOUR GRAPHING CALCULATOR ON THE AP EXAM
## 图形计算器：在AP考试中使用您的图形计算器

### The Four Calculator Procedures 计算器使用四步

Each student is expected to bring a graphing calculator to the AP Exam. Different models of calculators vary in their features and capabilities; however, there are four procedures you must be able to perform on your calculator:

**C1.** Produce the graph of a function within an arbitrary viewing window.
**C2.** Solve an equation numerically.
**C3.** Compute the derivative of a function numerically.
**C4.** Compute definite integrals numerically.

### Guidelines for Calculator Use 计算器使用指南

**1.** On multiple-choice questions in Section I, Part B, *you may use any feature or program on your calculator.* Warning: Don't rely on it too much! Only a few of these questions require the calculator, and in some cases using it may be too time-consuming or otherwise disadvantageous.

**2.** On the free-response questions of Section II Part A:

(a) You may use the calculator to perform any of the four listed procedures. When you do, you need only write the equation, derivative, or definite integral (called the "setup") that will produce the solution, then write the calculator result to the required degree of accuracy (three places after the decimal point unless otherwise specified). Note especially that a setup must be presented in standard algebraic or calculus notation, not in calculator syntax. For example, you *must* include in your work the setup $\int_0^{\pi} \cos t \, dt$ even if you use your calculator to evaluate the integral.

(b) For a solution for which you use a calculator capability other than the four listed above, you must write down the mathematical steps that yield the answer. A correct answer alone will not earn full credit and will likely earn no credit.

(c) You must provide *mathematical reasoning* to support your answer. Calculator results alone will not be sufficient.

### The Procedures Explained 步骤说明

Here is more detailed guidance for the four allowed procedures.

**C1.** "Produce the graph of a function within an arbitrary viewing window." Be sure that you create the graph *in the window specified,* then copy it carefully onto your exam paper. If no window is prescribed in the question, clearly indicate the window dimensions you have used. When a graph is used to support justification in a free response question, it must be clearly labeled as to what is being graphed.

**C2.** "Solve an equation numerically" is equivalent to "Find the zeros of a function" or "Find the point of intersection of two curves." Remember: you must first show your

setup—write the equation out algebraically; then it is sufficient just to write down the calculator solution.

**C3.** "Compute the derivative of a function numerically." When you seek the value of the derivative of a function at a specific point, you may use your calculator. First, indicate what you are finding—for example, $f'(6)$—then write the numerical answer obtained from your calculator. Note that if you need to find the derivative of the function, rather than its value at a particular point, you must write the derivative symbolically. Note: some calculators are able to perform symbolic operations.

**C4.** "Compute definite integrals numerically." If, for example, you need to find the area under a curve, you must first show your setup. Write the complete integral, including the integrand in terms of a single variable, with the limits of integration. You may then simply write the calculator answer; you need not compute an antiderivative.

## Accuracy 精确度

*Calculator answers must be correct to three decimal places*. To achieve this required accuracy, never type in decimal numbers unless they came from the original question. Do *not* round off numbers at intermediate steps, as this is likely to produce error accumulations resulting in loss of credit. If necessary, store intermediate answers in the calculator's memory. Do *not* copy them down on paper; storing is faster and avoids transcription errors. Round off only after your calculator produces the final answer.

## Sample Solutions of Free-Response Questions 开放式题目的样例解析

The following set of examples illustrates proper use of your calculator on the examination. In all of these examples, the function is

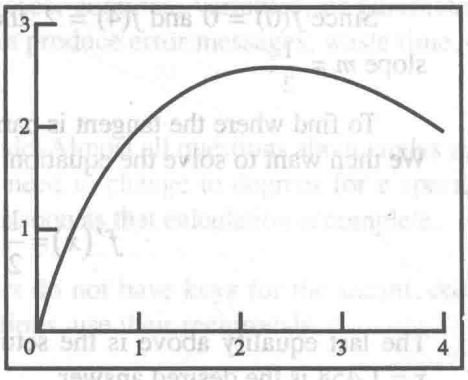
$$f(x)=\frac{10x}{x^2+4} \quad \text{for} \quad 0\leq x\leq 4.$$

1. Graph $f$ in $[0,4]\times[0,3]$.

   Set the calculator window to the dimensions printed in your exam paper.

   Graph $y=\dfrac{10x}{x^2+4}$.

   Copy your graph carefully into the window on the exam paper.




Viewing window $[0,4]\times[0,3]$

2. Write the tangent line for $f(x)$ at $x=1$.

   Note that $f(1)=2$. Then, using your calculator, evaluate the derivative:

$$f'(1)=1.2$$

   Then write the tangent-line (or local linear) approximation

$$\begin{aligned} f(x) &\simeq f(1)+f'(1)(x-1) \\ &\simeq 2+1.2(x-1)=1.2x+0.8 \end{aligned}$$

   You need not simplify, as we have, after the last equals sign just above.

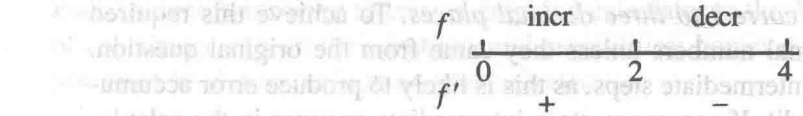
3. Find the coordinates of any maxima of $f$. Justify your answer.

Since finding a maximum is not one of the four allowed procedures, you must use calculus and show your work, writing the derivative algebraically and setting it equal to zero to find any critical numbers:

$$f'(x) = \frac{(x^2+4)10 - 10x(2x)}{(x^2+4)^2} = \frac{40-10x^2}{(x^2+4)^2}$$
$$= \frac{10(2-x)(2+x)}{(x^2+4)^2}$$

Then $f'(x) = 0$ at $x = 2$ and at $x = -2$; but $-2$ is not in the specified domain.

We analyze the signs of $f'$ (which is easier here than it would be to use the second-derivative test) to assure that $x = 2$ does yield a maximum for $f$. (Note that the signs analysis alone is *not* sufficient justification.)




Since $f'$ is positive to the left of $x = 2$ and negative to the right of $x = 2$, $f$ does have a maximum at

$$\left(2, \frac{10(2)}{2^2+4}\right) = \left(2, \frac{5}{2}\right)$$

—but you may leave $f(2)$ in its unsimplified form, without evaluating to $\frac{5}{2}$.

You may use your calculator's maximum-finder to *verify* the result you obtain analytically, but that would not suffice as a solution or justification.

4. Find the $x$-coordinate of the point where the line tangent to the curve $y = f(x)$ is parallel to the secant on the interval [0,4].

Since $f(0) = 0$ and $f(4) = 2$, the secant passes through (0,0) and (4,2) and has slope $m = \frac{1}{2}$.

To find where the tangent is parallel to the secant, we find $f'(x)$ as in Example 3. We then want to solve the equation

$$f'(x) = \frac{1}{2} \Rightarrow \frac{40-10x^2}{(x^2+4)^2} = \frac{1}{2}$$

The last equality above is the setup; we use the calculator to solve the equation: $x = 1.458$ is the desired answer.

5. Estimate the area under the curve $y = f(x)$ using a Trapezoidal Sum with four equal subintervals:

$$\int_0^4 f(x)\,dx \approx \left(\frac{f(0)+f(1)}{2}\right)(1) + \left(\frac{f(1)+f(2)}{2}\right)(1) + \left(\frac{f(2)+f(3)}{2}\right)(1) + \left(\frac{f(3)+f(4)}{2}\right)(1)$$
$$= \left(\frac{0+2}{2}\right)(1) + \left(\frac{2+(5/2)}{2}\right)(1) + \left(\frac{(5/2)+(30/13)}{2}\right)(1) + \left(\frac{(30/13)+2}{2}\right)(1)$$

You may leave the answer in this form or simplify it to 7.808.