Essentials of Calculus（Volume Ⅰ）

高等数学（Ⅰ）

侯书会　刘白羽　编

科学出版社

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高等数学（I）

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内 容 简 介

本书分上、下两册出版. 上册共七章，着重介绍一元微积分学的基础理论知识. 内容包括函数、极限、函数连续性，导数、微分及其应用，不定积分、定积分及其应用；下册共六章，着重介绍多元微分学的基础理论知识. 内容包括无穷级数、向量代数与空间解析几何，多元函数、极限及其连续性，多元函数的微分及应用，重积分、曲线积分、曲面积分及常微分方程.

本书是基于多年教学经验，兼顾国内工科类本科数学基础要求和海外学习的双重需要编写而成的. 与经典的中文微积分教材相比，本书适当降低了难度，突出了微积分学和后续应用型课程中常用的计算和证明方法. 在保证教材内容符合学科要求且不低于本科阶段微积分课程教学标准的前提下，力求语言精准、简练，以适应我国学生的外语水平和学习特点.

本书适于作为工科院校的国际班、双语教学班的高等数学教材和参考书.

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前　　言

微积分历来是大学数学最重要的组成部分，是工科院校非数学专业学生必修的一门数学基础课程. 本课程是运用数学概念、理论或方法去研究现实世界的空间形式和数量关系. 通过本课程的学习，培养学生综合分析、解决问题等逻辑思维能力，使其学会将问题化难为易、化繁为简，激发其创新意识. 本教材分上、下两册出版. 上册共七章，着重介绍一元微积分学的基础理论知识. 内容包括函数、极限、函数连续性，导数、微分及其应用，不定积分、定积分及其应用；下册共六章，着重介绍多元微分学的基础理论知识. 内容包括无穷级数、向量代数与空间解析几何，多元函数、极限及其连续性，多元函数的微分及应用，重积分、曲线积分、曲面积分及常微分方程.

为了使我国的高等教育尽快与国际接轨，国家教育部出台了一系列倡导高校开设英语授课或双语教学的国际班的相关政策. 目前，大部分高校多采用将母语外的另一种外国语言(主要指英语)直接应用于非语言类课程教学，并使外语与学科知识同步获取的一种教学模式. 但是，由于国内外高校授课方式的差异，直接使用外文原版教材根本无法达到国际班的教学目的. 也就是，国际班的教学内容及教学方式仍处于探索阶段. 鉴于此，我们兼顾中文教材的理论严谨性和外文原版教材的重实际应用，适当降低了中文教材的难度，突出了微积分学中实用的计算和证明方法，力求语言简练，通俗易懂，编制了适用于理工科本科国际班的高等数学教材.

在本书的编写过程中，我们严格遵循从直观到抽象、由浅入深、由易到难等循序渐进的原则，概念清晰，内容简练，语言通俗易懂，便于自学与教学. 上、下册内容，各需 60 学时，即可完成全部教学内容.

本书的编写得到了“十二五”期间北京科技大学教材建设经费资助，在此表示感谢. 北京科技大学汪飞星教授与北京理工大学蒋立宁教授审阅了全部书稿，并提出了许多中肯的意见和建议. 伦敦国王学院 Sam Beatson 博士和香港大学 SiumingYiu 副教授分别对书稿上册和下册进行了语言润色与修改. 编者向以上同志致以最诚挚的谢意.

由于编者水平有限，本书中可能会有错漏之处，恳请同行和读者不吝指正.

编　者

2016 年 3 月

Preface

Calculus is a fundamental subject in mathematics. It enables the application of mathematical theories and approaches in exploring real world continuity and is often a compulsory university course. Studying calculus is helpful in the development of analytical ability in addition to practical engineering problem solving skills. Students will learn how to think logically in the reduction of complex systems to simple ones. This coursebook, consisting of two volumes, includes instruction on Spatial Analytic Geometry, Single Variable Calculus (differentiation and integration of functions of a single variable), Multivariable Calculus (differentiation and integration of functions of multiple variables), Infinite Series and Ordinary Differential Equations among other topics.

Higher education becomes more internationalized. In light of this, more and more universities have begun to set up sino-foreign classes, in which both Chinese and foreign students are able to learn together. However, what to teach and how to teach in these classes are still in the exploratory phase. This book is for students in sino-foreign cooperative classes which tend to have fewer class hours than general classes. The sections are deliberately concise while remaining informative by incorporating the authors' teaching experience and utilising both Chinese and foreign teaching materials in constructing the course. We believe that this book will assist students in learning and teachers in teaching the basic knowledge of calculus.

This work was supported by the "Construction Fund for Textbooks of Twelfth Five-Year Plan" of University of Science and Technology, Beijing.

Shuhui Hou received her PhD degree in informatics from Kyoto University, Japan in 2009. She has been with University of Science and Technology Beijing since 1996. Her current research interests include cryptography, computer security and forensics. She has taught courses in Calculus, Linear Algebra, Information

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Security and Cryptology and so on.

Baiyu Liu received her PhD degree in mathematics from Tsinghua University, China in 2010. She has been with University of Science and Technology Beijing since 2010. Her current research interests include geometry analysis, theory and application of partial differential equations. She has taught courses in Calculus, Methods of Mathematical Physics, Probability and Statistics and so on.

Contents

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Chapter 1

Preliminaries

In this chapter, we review some elementary mathematical concepts required for the study of calculus.

1.1 Some Set Theory Notation for the Study of Calculus

1.1.1 Definition of Set

Definition 1.1.1 A set is a collection of well-defined objects (elements), on the proviso that the elements are distinct and there is a rule to decide whether an element is a member of a set.

The elements of a set, also called its members, can be anything: numbers, people, letters of the alphabet, other sets and so on. Sets are conventionally denoted with capital letters.

1.1.2 Descriptions of Set

There are several ways of describing sets or specifying the members of a set.

1. Listing each member of the set.

For example,

$C=\{4,2,1,3\}$; $D=\{\text{blue, white, red}\}$; $N=\{0,1,2,\cdots,n,\cdots\}$;

$A=\{a_1,a_2,\cdots,a_n\}=\{a_i\}_{i=1}^{n}$.

2. Using a rule or semantic description.

For example, rational numbers $\mathbb{Q}=\left\{\frac{p}{q}\middle|p \text{ and } q \text{ are integers with } q\neq 0\right\}$; real numbers $\mathbb{R}=\{$All numbers (rational and irrational) that can measure lengths, together with their negatives and zeros$\}$.

Note that the empty set is the unique set having no members, denoted by $\varnothing$. If something is or is not an element of a particular set then this is symbolized by $\in$ or $\notin$, respectively. Every element of a set must be unique and the order in which the elements of a set are listed is irrelevant, unlike a sequence. For example, $\{6,11\}=\{11,6\}=\{11,11,6,11\}$.

1.1.3 Set Operations

Definition 1.1.2 If every member of set A is also a member of set B (i.e., if $x\in A$, then $x\in B$), then A is said to be a subset of B, written $A\subseteq B$, read as A is included in B or A is contained in B.

Definition 1.1.3 The statement that sets A and B are equal means that they have precisely the same members; $A=B$ if and only if $A\subseteq B$ and $B\subseteq A$.

For example, $\mathbb{N}\subseteq\mathbb{Z}$, $\mathbb{Z}\subseteq\mathbb{Q}$, $\mathbb{Q}\subseteq\mathbb{R}$. Obviously, we have

1. $A\subseteq A$; $A=A$; $\varnothing\subseteq A$;
2. $A\subseteq C$ if $A\subseteq B$ and $B\subseteq C$.

Definition 1.1.4 Given two sets A and B, we define:

(1) Union: the union of A and B is the set of all elements which are members of either A or B, denoted by $A\cup B=\{x|x \in A \text{ or } x \in B\}$ (Figure 1.1 (a)).

(2) Intersection: the intersection of A and B is the set of all elements which are members of both A and B, denoted by $A\cap B=\{x|x \in A \text{ and } x \in B\}$(Figure 1.1(a)). If $A\cap B=\varnothing$, then A and B are said to be disjoint.

(3) Complement: the relative complement of B in A is the set of all elements which are members of A but not members of B, denoted by $A\backslash B=\{x|x \in A \text{ and } x \notin B\}$ (Figure 1.1 (a)). Note that as $B\subseteq A$, $A\backslash B$ can be simply called the complement of B in A, and is denoted by $B_A^C=A\backslash B$ (Figure 1.1 (b)).

(4) Cartesian product: the Cartesian product of two sets A and B is the set of all ordered pairs (x,y) such that x is a member of A and y is a member of B, denoted by $A\times B=\{(x,y)|x \in A, y \in B\}$ (Figure 1.1 (c)).

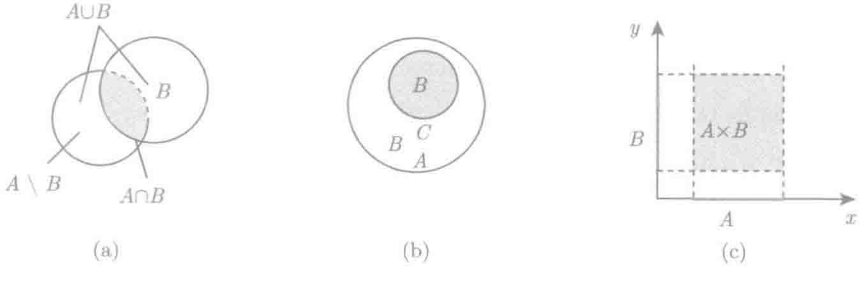


Figure 1.1 Operation on sets

1.1.4 Interval

Intervals can also describe sets and we introduce special terminology and notation for several kinds of intervals.

Open interval consists of all numbers between a and b except the endpoints a and b, denoted by (a, b): $(a, b)=\{x|a < x < b\}$.

Closed interval consists of all numbers between a and b including the endpoints a and b, denoted by $[a, b]$: $[a, b]=\{x|a \leqslant x \leqslant b\}$.

Half open (or closed) interval consists of all numbers between a and b including either endpoint a or b (but not both), denoted by $(a, b]$ or $[a, b)$: $[a, b)=\{x|a \leqslant x < b\}$ or $(a, b]=\{x|a < x \leqslant b\}$.

In addition to the finite intervals above, there also exist infinite intervals. For example, $[a, \infty)=\{x|a \leqslant x\}$, $(-\infty, b]=\{x|x \leqslant b\}$ and $(-\infty, \infty)=\{x|x \in \mathbb{R}\}$.

1.1.5 Neighbourhood

Suppose that δ is an arbitrary positive number. The open interval $(a-\delta,\ a+\delta)$ is termed a neighbourhood of a, where the point a is the center and δ is the radius of the neighbourhood (Figure 1.2).

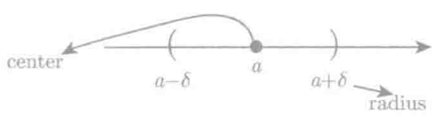


Figure 1.2 δ-neighbourhood of point a

We use $U(a, \delta)$ to denote the δ-neighbourhood of point a, in other words, $U(a, \delta)=\{x|a-\delta < x < a+\delta\}=\{x|\ |x-a| < \delta\}$.

Sometimes, we use a **deleted neighborhood** of a point a. It is a neighbourhood of a without a, denoted by $\mathring{U}(a, \delta)=\{x|0 < |x-a| < \delta\}$. Moreover, $(a-\delta, a)$ is called the left-handed neighborhood and $(a, a+\delta)$ is called the right-handed neighborhood.

1.2 The Rectangular Coordinate System

1.2.1 Cartesian Coordinates

In the plane, draw two real lines, one horizontal and the other vertical, so that they intersect at the zero points of the two lines. The two lines are called **coordinate**

axes; their intersection is labeled O and is called the **origin**. By convention, the horizontal line is called the ***x*-axis** and the vertical line is called the ***y*-axis**. The positive half of the x-axis is to the right; the positive half of the y-axis is upward. The coordinate axes divide the plane into four regions, called quadrants, labeled I, II, III, and IV, as shown in Figure 1.3(a).

Each point P in the plane can now be assigned a pair of numbers, called its **cartesian coordinates**. If the vertical and horizontal lines through P intersect the x- and y-axes at a and b, respectively, then P has the coordinates (a, b) (Figure 1.3(b)). We call (a, b) an **ordered pair** of numbers because it makes a difference which number is first. The first number a is the ***x*-coordinate**; the second number b is the ***y*-coordinate**.

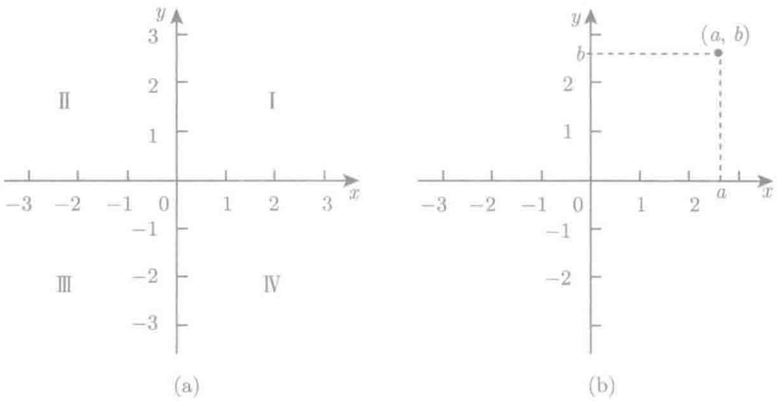


Figure 1.3 The rectangular coordinate system

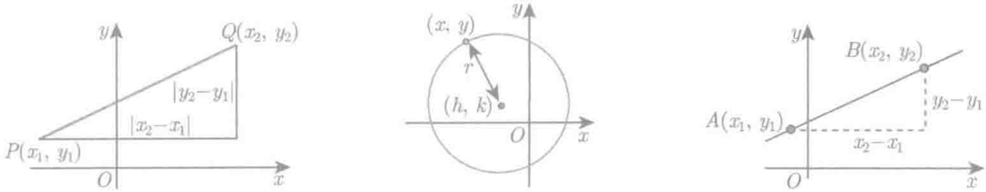
1.2.2 Distance Formula

Given any two points $P(x_1, y_1)$ and $Q(x_2, y_2)$, the distance between the two points (Figure 1.4(a)) can be calculated by the following **distance formula**

$$d(P, Q) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

1.2.3 The Equation of a Circle

A circle is the set of points that lie at a fixed distance (the radius) from a fixed point (the center). Let radius be r and the center be (h, k), The standard equation of a circle is $(x-h)^2+(y-k)^2=r^2$ (Figure 1.4(b)).



(a) Distance between P and Q (b) A circle with radius r and center (h, k) (c) The slope of a line

Figure 1.4 Distance, circle and line

There is a one-to-one corresponding relation between a circle and its equation. To say that $(x-h)^2+(y-k)^2=r^2$ is the equation of the circle with radius r and center (h, k) means two things:

1. If a point is on this circle, then its coordinates (x, y) satisfy the equation.

2. If x and y are numbers that satisfy the equation, then they are the coordinates of a point on the circle.

Example 1.2.1 Find the standard equation of a circle with radius 5 and center $(1, -5)$. Also find the y-coordinates of the two points on this circle with x-coordinate 2.

Solution The desired equation is

$$(x-1)^2+(y+5)^2=25.$$

We substitute $x=2$ in the equation and solve for y.

$$\begin{aligned}(2-1)^2+(y+5)^2&=25,\\(y+5)^2&=24,\\y+5&=\pm\sqrt{24},\\y&=-5\pm\sqrt{24}=-5\pm2\sqrt{6}.\end{aligned}$$

□

Example 1.2.2 Show that the equation

$$x^2-2x+y^2+6y=-6$$

represents a circle, and find its center and radius.

Solution Completing the square, we rewrite the equation

$$x^2 - 2x + 1 + y^2 + 6y + 9 = -6 + 1 + 9,$$
$$(x-1)^2 + (y+3)^2 = 4.$$

The last equation is in standard form. It is the equation of a circle with center $(1, -3)$ and radius 2. □

1.3 The Straight Line

1.3.1 The Slope of a Line

In general, for a line through $A(x_1, y_1)$ and $B(x_2, y_2)$ (Figure 1.4 (c)), where $x_1 \neq x_2$, we define the slope m of that line by

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

The slope m is a measure of the steepness of line. The larger the absolute value of the slope is, the steeper the line is. Please note that a horizontal line has a slope of zero and the concept of the slope of a vertical line makes no sense.

1.3.2 The Equation of a Line

The equation of a line can be expressed in various forms.

The Point-slope Form the line passing through the (fixed) point (x_1, y_1) with slope m has equation

$$y - y_1 = m(x - x_1).$$

We call this the **point-slope** form of the equation of a line.

The Slope-intercept Form suppose that we are given the slope m for a line and the y-intercept b (i.e., the line intersects the y-axis at $(0, b)$), we get the following equation

$$y = mx + b.$$

This is called the **slope-intercept** form. Notice that the equation of any vertical line can be put in the form $x = k$ and the equation of a horizontal line can be written in the form $y = k$, where k is a constant.

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