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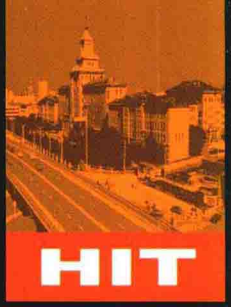

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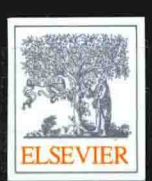
运输过程的统一非局部理论——广义波尔兹曼物理动力学（第2版）

[俄罗斯] Alexeev, B. V.（阿列克谢耶夫）著



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Preface

We are in front of the tremendous catastrophe in modern theoretical physics. Moreover, we have reached the revolutionary situation not only in physics but also in natural philosophy on the whole. Practically we are in front of the new challenge since Newton's *Mathematical Principles of Natural Philosophy* was first published in 1687. It is impossible to believe that in more than 300 years after Newton, we have the situation when 96% of matter and energy is of unknown origin. As it is shown in this monograph, the origin of difficulties consists in the total oversimplification inherent in local physics of the dissipative processes.

In the latter part of twentieth century, two very important results were obtained:

(1) The Irish physicist John Stewart Bell (1928–1990) was to show that all local statistical theories of dissipative processes are wrong in principal.

(2) The Russian physicist Boris V. Alexeev was to show that the derivation of kinetic equation with respect to one-particle distribution function from the BBGKY equations (prior to introducing any approximation destined to break the Bogolyubov chain) leads to additional terms of the non-local origin, generally of the same order of magnitude, appear in the Boltzmann equation. Then the passage to the Boltzmann equation means the neglect of non-local effects. These additional terms cannot be omitted even in the limit cases of kinetic theory, therefore Boltzmann equation is only a plausible equation.

Therefore, the case in point is of unprecedented situation in physics, when the fundamental physical equation is revised. During my stay in Marseille as invited professor, A.J.A. Favre reminds me Henri Poincaré's phrase after the death of a great Austrian physicist—"Boltzmann was wrong, but his mistake is equal to zero." It is a pity, but the situation in kinetic theory is much more serious.

The scientific community was convinced that the mentioned results could lead only to rather small corrections in the modern theoretical physics. So to speak—4% corrections to 96% of the known results, but not quite the reverse! Many scientists are aware that some way out will be achieved after creation of the unified theory of transport processes working from the structure of so-called elementary particles to the Universe evolution. This theory is in front of you.

This book reflects the scales of these alterations. One is safe to say that—as the main result of the non-local physical kinetics—this theory has showed it to be a highly effective tool for solving many physical problems in areas where the classical theory runs into difficulties.

Author is deeply indebted to V.L. Ginzburg and F. Uhlig for their interest in this work and in the subject in general. I am thankful to V. Mikhailov, I. Ovchinnikova, and A. Fedoseyev for cooperation.

This book is devoted to the memory of my mother.

January 2014

Historical Introduction and the Problem Formulation

"Alles Vergängliche
ist nur ein Gleichniss!"
Boltzmann's epigraph
for his "Vorlesungen über Gastheorie"

«Сотри случайные черты -
и ты увидишь — мир прекрасен»
(Obliterate the accidental features
And you will see: the world is splendid.)
Alexander Blok
"The Retribution"

In 1872, L Boltzmann, then a mere 28-year-old, published his famous kinetic equation for the one-particle distribution function $f(\mathbf{r},\mathbf{v},t)$ [1]. He expressed the equation in the form

$$\frac{\mathrm{D}f}{\mathrm{D}t}=J^{\mathrm{st}}(f), \tag{I.1}$$

where J^{st} is the collision (stoß) integral, and

$$\frac{\mathrm{D}}{\mathrm{D}t}=\frac{\partial}{\partial t}+\mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}+\mathbf{F}\cdot\frac{\partial}{\partial \mathbf{v}} \tag{I.2}$$

is the substantial (particle) derivative, $\mathbf{v}$ and $\mathbf{r}$ being the velocity and radius-vector of the particle, respectively.

Equation (I.1) governs (in local approximation) the transport processes in a one-component gas which is sufficiently rarefied that only binary collisions between particles are of importance. While we are not concerned here with the explicit form of the collision integral (which determines the change of the distribution function f in binary collisions), note that it should satisfy conservation laws. For the simplest case of elastic collisions in a one-component gas we have

$$\int J^{\mathrm{st}}\psi_i \mathrm{d}\mathbf{v}=0,(i=1,2,3),\mathrm{d}\mathbf{v}=\mathrm{d}v_x\mathrm{d}v_y\mathrm{d}v_z, \tag{I.3}$$

where ψ_i are the collisional invariants ($\psi_1=m$, $\psi_2=m\mathbf{v}$, $\psi_3=mv^2/_2$, m is the mass of the particle) related to the laws of conservation of mass, momentum, and energy.

Integrals of the distribution function (i.e. its moments) determine the macroscopic hydrodynamic characteristics of the system, in particular the number density of particles

$$n=\int f\mathrm{d}\mathbf{v} \tag{I.4}$$

and the temperature T:

$$\frac{3}{2}k_{\mathrm{B}}nT=\frac{1}{2}m\int f(\mathbf{v}-\mathbf{v}_0)^2\mathrm{d}\mathbf{v}. \tag{I.5}$$

Here k_{B} is the Boltzmann constant and v_0 is the hydrodynamic flow velocity. It follows then that multiplying the Boltzmann integro-differential equation term by term by collisional invariants ψ_i, integrating over all particle velocities, and using the conservation laws (I.3) we arrive at the differential equations of fluid dynamics, whose general form is known as the hydrodynamic Enskog equations.

The Boltzmann equation (BE) is not of course as simple as its symbolic form above might suggest, and it is in only a few special cases that it is amenable to a solution. One example is that of a Maxwellian distribution in a locally, thermodynamically equilibrium gas in the event when no external forces are present. In this case, the equality

$$J^{st} = 0 \tag{I.6}$$

is met, giving the Maxwellian distribution function

$$f^{(0)} = n\left(\frac{m}{2\pi k_B T}\right)^{3/2} \exp\left(-\frac{m\mathbf{V}^2}{2k_B T}\right), \tag{I.7}$$

where $\mathbf{V} = \mathbf{v} - \mathbf{v_0}$ is the thermal velocity.

It was much later, years after Boltzmann's death in 1906, that an analytic method for solving the Boltzmann equation was developed for the purpose of calculating transport coefficients. This method, developed in 1916-1917 by Chapman and Enskog [2–5], led to explicit expressions for the coefficients of viscosity, thermal conductivity, diffusion, and later thermal diffusion in a system with a small parameter (which for Chapman and Enskog's particular problem of a nonreacting gas was the Knudsen number, the ratio of the particle's mean free path to a characteristic hydrodynamic dimension).

However, even in Boltzmann's days there was a complete awareness that his equation acquires a fundamental importance for physics and that its range of validity stretches from transport processes and hydrodynamics all the way to cosmology—thus fully justifying the keen attention it attracted and debates it provoked.

Of the many results L Boltzmann derived from his kinetic equation, one of the most impressive is the molecular-kinetic interpretation of the second principle of thermodynamics and in particular of the statistical meaning of the concept of entropy. It turned out that it is possible to define the function

$$H = \int f \ln f d\mathbf{v}, \tag{I.8}$$

(H is the first letter in the English word *heat* and German word *Heizung*) which behaves monotonically in a closed system. If the relation between S, the entropy per unit volume of an ideal gas, and the H-function is written in the form

$$S = -k_B H + \text{const}, \tag{I.9}$$

then one can prove the inequality

$$\frac{\partial S}{\partial t} \geq 0. \tag{I.10}$$

The laconic formula

$$S = k \ln W, \tag{I.11}$$

connecting the entropy S and the thermodynamic probability W, is inscribed on Boltzmann's tombstone.

Ever since their creation, Boltzmann's physical kinetics and the Boltzmann equation have received severe criticism, much of which remains of interest even today. Let us elaborate on this.

To begin with, Boltzmann's contemporaries were very much in the dark regarding the relation between the Boltzmann equation and classical mechanics—in particular, with the Newton equation. The Boltzmann equation was obtained in a phenomenological manner based on convincing physical arguments and reflects the fact that the distribution function does not change along the particle's trajectory between collisions but rather changes as a result of an "instantaneous" interaction between colliding particles.

J Loschmidt noted in 1876 that the Boltzmann equation underlying the H-theorem includes only the first time derivative whereas the Newton equation contains the second (square of time) and hence the equations of motion are reversible in time. This means that if a system of hard-sphere particles starts a "backward" motion due to the particles reversing their direction of motion at some instant of time, it passes through all its preceding states up to the initial one, and this will increase the H-function whose variation is originally governed by reversible equations of motion. The essential point to be made here is that the observer cannot prefer one of the situations under study, the "forward" motion of the system in time, in favor of the second situation, its "backward" motion. In other words, the problem of the reversibility of time arises here.

Although somewhat differently formulated, essentially the same objection was made in 1896 by Planck's student E Zermelo, who noted that the H-theorem is inconsistent with Poincare's "recurrence" theorem proved in 1890 and stated that any physical system, even with irreversible thermodynamic processes operating in it, has a nonzero probability of returning to its original state. Boltzmann, himself fully aware of this possibility, wrote in the second part of his Lectures on the Theory of Gases (see Ref. [6], p. 251): "As a result of the motion of gas molecules, the H-function always decreases. The unidirectional nature of this process does not follow from the equations of motion, which the molecules obey. Indeed, these equations do not change if time changes sign."

There is a well-known example from probability theory, which Boltzmann employed as an argument in his discussions—sometimes very heated ones—with Zermelo, Planck and Ostwald. If a six-sided die is thrown 6000 times, one expects each

side to turn up about 1000 times. The probability of, say, a six turning up 6000 times in a succession has a vanishing small value of $(1/6)^{6000}$. This example does not clear up the matter, however. Nor do the two papers which Boltzmann's student P Ehrenfest wrote in co-authorship with T Afanas'eva-Ehrenfest after the death of the great Austrian physicist.

Their first model, reported by Afanas'eva-Ehrenfest at the February 12, 1908 meeting of the Russian Physical-Chemical Society, involved the application of the H-theorem to the "plane" motion of a gas [7]. Suppose P-molecules, nontransparent to one another, start moving normally to axis y and travel with the same velocity in the direction of axis x. Suppose further that in doing so they undergo elastic collisions with Q-particles, squares with sides at an angle of 45° to axis y, which are nontransparent to the molecules and are all at rest.

It is readily shown that shortly after, all the molecules will divide themselves into four groups, and it is a simple matter to write down the change in the number of molecules P in each group in a certain time Δt and then to define a "planar-gas" H -function

$$H=\sum_{i=1}^{4} f_i \ln f_i, \tag{I.12}$$

where f_i is the number of molecules of the i-th kind, i.e., of those moving in one of the four possible directions. If all the velocities reverse their direction, the H-function starts to increase and reverts to the value it had when the P-molecules started their motion from the y axis. While this simple model confirms the Poincare-Zermelo theorem, it does not at all guarantee that the H-function will decrease when the far more complicated Boltzmann model is used.

P and T Ehrenfest's second model [8], known as the lottery's model, features two boxes, A and B, and N numbered balls to which there correspond "lottery tickets" placed in a certain box and which are all in box A initially. The balls are then taken one by one from A and transferred to B according to the number of a lottery ticket, drawn randomly. Importantly, the ticket is not eliminated after that but rather is returned to the box. In the event that the newly drawn ticket corresponds to a ball contained in B, the ball is returned to A. As a result, there will be approximately $N/2$ balls in either box.

Now suppose one of the boxes contains n balls—and the other accordingly $N-n$ balls—at a certain step s in the drawing process. We can then define Δ, a function, which determines the difference in the number of balls between the two boxes: $\Delta=n-(N-n)=2n-N$. In "statistical" equilibrium, $\Delta=0$ and $n=N/2$, the dependence $\Delta(s)$ will imitate the behavior of the H-function in a Boltzmann gas.

This example is also not convincing enough because this "lottery" game will necessarily lead to a fluctuation in the Δ function, whereas Boltzmann kinetic theory *excludes* completely fluctuations in the H-function. By the end of his life Boltzmann went over to fluctuation theory, in which the decrease of the H-function in time is only treated as the process the system is most likely to follow. This interpretation, however, is not substantiated by his kinetic theory since the origin of the primary fluctuation remains unclear (the galactic scale of such fluctuation included).

One of the first physicists to see that Boltzmann equation must be modified in order to remove the existing contradictions was J Maxwell. Maxwell thought highly of the results of Boltzmann, who in his turn did much to promote Maxwell electrodynamics and its experimental verification.

We may summarize Maxwell's ideas as follows. The equations of fluid dynamics are a consequence of the Boltzmann equation. From the energy equation, limiting ourselves to one dimension for the sake of simplicity and neglecting some energy transfer mechanisms (in particular, convective heat transfer), we obtain the well-known heat conduction equation

$$\frac{\partial T}{\partial t}=a^2\frac{\partial^2 T}{\partial x^2}. \tag{I.13}$$

The fundamental solution of Eqn (1.13) up to the dimensional constant is

$$T(x,t)=\frac{1}{2\sqrt{\pi a^2 t}}\exp\left(-\frac{x^2}{4a^2 t}\right) \tag{I.14}$$

and represents the temperature at point x at instant t provided at time $t=0$ an amount of heat $c\rho$, with ρ the density and a the thermal diffusivity of the medium, is evolved at the origin of coordinates. Defining an argument of a function T as $\theta=a^2 t$ with the dimension of a coordinate squared we obtain

$$T=\frac{1}{2\sqrt{\pi\theta}}\exp\left(-\frac{x^2}{4\theta}\right). \tag{I.15}$$

The temperature distribution given by this equation is unsatisfactory physically. For small values of θ, the temperature at the heat evolution point $x=0$ is indefinitely large. On the other hand, at any arbitrarily distant point x the temperature produced by an instantaneous heat source will be different from zero for arbitrarily small times. While this difference may be small, it is a point of principal importance that it has a finite value.

As Landau and Lifshitz noted in their classical *Course of Theoretical Physics* ([9], p. 283), "The heat conduction process described by the equations obtained here has the property that any thermal perturbation becomes instantaneously felt over all space." This implies an infinitely fast propagation of heat, which is absurd from the point of view of molecular-kinetic theory. In the courses of mathematical physics this result is usually attributed to the fact that the heat conduction equation is derived phenomenologically, neglecting the molecular-kinetic mechanism of heat propagation. However, as has been already noted, the parabolic equation (I.13) follows from the Boltzmann equation. Some of Maxwell's ideas, phenomenological in nature and aimed at the generalization of the Boltzmann equation, are discussed in Woods' monograph [10].

Work on the hyperbolic equation of heat conduction was no longer related directly to the Boltzmann equation but rather was of a phenomenological nature. Without expanding the details of this approach, we only point out that the idea of the improvement of Eq. (I.13) was to introduce the second derivative with respect to time thus turning Eq. (I.13) into the hyperbolic form

$$\tau_r \frac{\partial^2 T}{\partial t^2} + \frac{\partial T}{\partial t} = a^2 \frac{\partial^2 T}{\partial x^2}, \tag{I.16}$$

where τ_r is treated as a certain relaxation kinetic parameter with the dimensions of time. For the first time in modern physics this idea was formulated by B Davydov [11] (see also interesting discussion about priority between C Cattaneo and P Vernotte [12–15]). The wave equation (I.16) leads to final propagation velocities for a thermal perturbation—although it should be remarked parenthetically that the quasi-linear parabolic equations may also produce wave solutions.

A breakthrough period in the history of kinetic theory occurred in the late 1930s and early 1940s, when it was shown through efforts of many scientists—of which Bogolyubov certainly tops the list—how, based on the Liouville equation for the multiparticle distribution function f_N of a system of N interacting particles, one can obtain a one-particle representation by introducing a small parameter $\varepsilon = nv_b$, where n is the number of particles per unit volume and v_b is the interaction volume [16-20]. This hierarchy of equations is usually referred to as the Bogolyubov or BBGKY (Bogolyubov—Born—Green—Kirkwood—Yvon) chain.

We do not present the technical details in Introduction but refer the reader to the classical works cited above or, for example, to Ref. [21]. Some fundamental points of the problem are worth mentioning here, however.

(1) Integrating the Liouville equation

$$\frac{\partial f_N}{\partial t} + \sum_{i=1}^{N} \mathbf{v}_i \cdot \frac{\partial f_N}{\partial \mathbf{r}_i} + \sum_{i=1}^{N} \mathbf{F}_i \cdot \frac{\partial f_N}{\partial \mathbf{v}_i} = 0 \tag{I.17}$$

subsequently over phase volumes $d\Omega_{s+1}, \ldots, d\Omega_N$ ($d\Omega_j \equiv d\mathbf{r}_j d\mathbf{v}_j$), one obtains a kinetic equation for the s-particle distribution function, with the distribution function f_{s+1} in the integral part of the corresponding equation.

In other words, the set of integro-differential equations turns out to be a linked one, so that in the lowest-order approximation the distribution function f_1 depends on f_2. This means formally that, strictly speaking, the solution procedure for such a set should be as follows. First find the distribution function f_N and then solve the set of BBGKY equations subsequently for decreasingly lower-order distributions. But if we know the function f_N, there is no need at all to solve the equations for f_s and it actually suffices to employ the definition of the function

$$f_s = \int f_N(t, \Omega_1, \ldots, \Omega_N) d\Omega_{s+1} \ldots d\Omega_N. \tag{I.18}$$

We thus conclude that the rigorous solution to the set of BBGKY equations is again equivalent to solving Liouville equations. On the other hand, the seemingly illogical solution procedure involving a search for the distribution function f_1 is of great significance in kinetic theory and in non-equilibrium statistical mechanics. This approach involves breaking the BBGKY chain by introducing certain additional assumptions (which have a clear physical meaning, though). These assumptions are discussed in detail below.

(2) For a non-reacting gas, the Boltzmann equation is valid for two time scales of distribution functions: one of the order of the mean free time of the particles, and the other the hydrodynamic flow time. The Boltzmann equation is invalid for time lengths of the order of the collision times. Notice that a change from the time scale to the length scale can of course be made if desired.

(3) After the BBGKY chain is broken and f_2 represented as a product of one-particle distribution functions (which is quite reasonable for a rarefied gas), the Boltzmann equation cannot be written in a classical form with only one small parameter ε and it reduces instead to the Vlasov equation in a self-consistent field.

(4) Because the Boltzmann equation does not work at distances of the order of the particle interaction radius (or at the r_b scale), Boltzmann particles are pointlike and structureless, and it is one of the inconsistencies of the Boltzmann theory that the resulting collision cross sections of the particles enter the theory by the collision integral.

(5) Usually the one-particle distribution function is normalized to the number of particles per unit volume. For Boltzmann particles the distribution function is "automatically" normalized to an integer because a point-like particle may only be either inside or outside a trial contour in a gas — unlike finite-diameter particles which of course may overlap the boundary of the contour at some instant of time. Another noteworthy point is that the mean free path in Boltzmann kinetic theory is only meaningful for particles modeled by hard elastic spheres. Other models face difficulties related, though, to the level of one-particle description employed. The requirement for the transition to a one-particle model is that molecular chaos should exist prior to a particle collision.

The advent of the BBGKY chain led to the recognition that whatever generalization of Boltzmann kinetic theory is to be made, the logic to be followed should involve all the elements of the chain, i.e. the Liouville equation, the kinetic equations for s-particle distribution functions f_s, and the hydrodynamic equations. This logical construction was not generally adhered to.

In 1951, N Slezkin published two papers [22, 23] on the derivation of alternative equations for describing the motion of gas. The idea was to employ Meshcherskii's variable-mass point dynamics theory [24], well known for its jet propulsion applications.

The assumption of a variable-mass particle implies that at each point a liquid particle, close to this point and moving with a velocity $\mathbf{v}$, adds or loses a certain mass, whose absolute velocity vector $\mathbf{U}$ differs, as Slezkin puts it, by a certain appreciable amount from the velocity vector $\mathbf{v}$ of the particle itself. Since there are different directions for this mass to come or go off, the associated mass flux density vector $\mathbf{Q}$ is introduced.

By applying the laws of conservation of mass, momentum, and energy in the usual way, Slezkin then proceeds to formulate a set of hydrodynamical equations, of which we will here rewrite the continuity equation for a one-component nonreacting gas:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial \mathbf{r}} \cdot (\rho \mathbf{v} + \mathbf{Q}) = 0. \tag{I.19}$$

The mass flux density $\mathbf{Q}$ is written phenomenologically in terms of the density and temperature gradients.

Thus, the continuity equation is intuitively modified to incorporate a source term giving

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial \mathbf{r}} \cdot \rho \mathbf{v} = \frac{\partial}{\partial \mathbf{r}} \cdot \left(D \frac{\partial \rho}{\partial \mathbf{r}} + \beta \frac{\partial T}{\partial \mathbf{r}} \right), \tag{I.20}$$

where the coefficient D is that of self-diffusion, and β is related to thermal diffusion. Thus, we now have fluctuation terms on the right-hand side of Eq. (I.20), which are generally proportional to the mean free time τ_{mt} and, hence, after Eq. (I.20) is made dimensionless, to the Knudsen number which is small in the hydrodynamic limit.

At very nearly the time of the publication of Slezkin's first paper [23], Vallander [25] argued that the standard equations of motion are ill grounded physically and should therefore be replaced by other equations based on the introduction of additional mass Q_i and energy t_i fluxes $(i = 1,2,3)$ $Q_i = D_1 \frac{\partial \rho}{\partial r_i} + D_2 \frac{\partial T}{\partial r_i}$, $t_i = k_1 \frac{\partial \rho}{\partial r_i} + k_2 \frac{\partial T}{\partial r_i}$, where, to quote, "$D_1$ is the density self-diffusion coefficient, D_2 is the thermal self-diffusion coefficient, k_1 is the density heat conductivity, and k_2, the temperature heat conductivity."

Heuristic and inconsistent with Boltzmann's theory, the work of Siezkin and Vallander came under sufficiently severe criticism. Shaposhnikov [26] noted that in these papers, "which are almost identical in content ... the essential point is that instead of the conventional expression $\rho \mathbf{v}_0$, additional effects—'concentration self-diffusion' and 'thermal self-diffusion'—are introduced into the mass flux density which, in addition to the macroscopic mass transfer, cause a molecular mass transfer, much as the macroscopic energy and momentum transfer in a moving fluid goes in parallel with analogous molecular transport (heat conduction and viscosity)." Shaposhnikov then proceeds to derive the equation of continuity from the Boltzmann equation for a one-component gas and shows that the hydrodynamic equations of Siezkin and Vallander are in conflict with the Boltzmann kinetic theory.

Note that Siezkin and Vallander also modified the equations of motion and energy for a one-component gas in a similar way (by including self-diffusion effects). Possible consequences of additional mass transfer mechanisms for the Boltzmann kinetic theory were not analyzed by these authors.

Boltzmann's "fluctuation hypothesis" was repeatedly addressed by Ya Terletskii (see, for instance, Refs. [27, 28]) whose idea was to estimate fluctuations by using the expression the general theorems of Gibbs (see, for example, Ref. [29], pp. 85-88) yield for the mean-square deviation of an arbitrary generalized coordinate. To secure that fluctuations in statistical equilibrium be noticeable, Terletskii modifies the equation of perfect gas state by introducing a gravitational term, which immediately extends his analysis beyond the Boltzmann kinetic theory leaving the question about the irreversible change of the Boltzmann H -function unanswered.

Some comments concerning terminology should be done. In recent years, possible generalizations of the Boltzmann equation have been discussed widely in the scientific literature. Since the term "generalized Boltzmann equation" (GBE) has usually been given to any new modification published, we will only apply this term to the particular kinetic equation derived by me (for example) in Refs. [30–32] to avoid confusion. The corresponding equation is known also in the literature as Alexeev equation. Obviously it is not convenient for me to apply this term. Moreover in the following this kinetic equation will be transformed in the *basic equations of the unified theory* of transport processes (BEUT) valid in the tremendous diapason of scales—from the internal structures of so called "elementary particles" to the Universe expansion. Then GBE is only a particular case of BEUT which can be discussed in the subsequent chapters.

L Woods (see, e.g., Ref. [33]), following ideas dating back to Maxwell [34], introduces in his theory a phenomenological correction to the substantial first derivative on the left-hand side of the Boltzmann equation to take account of the further influence of pressure on transport processes. It is argued that the equation of motion of a liquid particle may be written as $\dot{\mathbf{v}} = \mathbf{F} + \mathbf{P}$, where $\mathbf{P}$ is a certain additional force, proportional to the pressure gradient: $\mathbf{P} = -\rho^{-1}\partial p/\partial \mathbf{r}$, with the result that the left-hand side of the Boltzmann equation becomes

$$\frac{\mathrm{D}f}{\mathrm{D}t} \equiv \frac{\partial f}{\partial t} + \mathbf{v}\cdot\frac{\partial f}{\partial \mathbf{r}} + \left(\mathbf{F} - \frac{1}{\rho}\frac{\partial p}{\partial \mathbf{r}}\right)\cdot\frac{\partial f}{\partial \mathbf{v}}, \tag{I.21}$$

whereas the collisional term remains unchanged. The phenomenological equation (I.21) has no solid foundation and does not fall into the hierarchy of Bogolyubov kinetic equations.

A weak point of the classical Boltzmann kinetic theory is the way it treats the dynamic properties of interacting particles. On the one hand, as the so-called "physical" derivation of the BE suggests [1, 6, 35, 36], Boltzmann particles are treated as material points; on the other hand, the collision integral in the BE brings into existence the cross sections for collisions between particles. A rigorous approach to the derivation of the kinetic equation for f_1 (KE_{f_1}) is based on the hierarchy of the Bogolyubov—Born—Green—Kirkwood—Yvon (BBGKY) equations. A KE_{f_1} obtained by the multi-scale method turns into the BE if one ignores the change of the distribution function (DF) over a time of the order of the collision time (or, equivalently, over a length of the order of the particle interaction radius). It is important to note [30–32] that accounting for the third of the scales mentioned above has the consequence that, prior to introducing any approximations destined to break the Bogolyubov chain, additional terms, generally of the same order of magnitude, appear in the BE. If the method of correlation functions is used to derive KE_{f_1} from the BBGKY equations, then a passage to the BE implies the neglect of non-local effects it time and space. Given the above difficulties of the Boltzmann kinetic theory (BKT), the following clearly interrelated questions arise.

First, what is a physically infinitesimal volume and how does its introduction (and, as a consequence, the unavoidable smoothing out of the DF) affect the kinetic equation [30]?

And second, how does a systematic account for the proper diameter of the particle in the derivation of the KE_{f_1} affect the Boltzmann equation? As it was mentioned before, we will refer to the corresponding KE_{f_1} as the generalized Boltzmann equation, or GBE.

Accordingly, our purpose in this introduction is first to explain the essence of the physical generalization of the BE and then to take a look at the specifics of the derivation of the GBE, when (as is the case in plasma physics) the self-consistent field of forces must of necessity be introduced. As the Boltzmann equation is the centerpiece of the theory of transport processes (TTP), the introduction of an alternative KE_{f_1} leads in fact to an overhaul of the entire theory, including its macroscopic (for example, hydrodynamic) aspects. Conversely, a change in the macroscopic description will inevitably affect the kinetic level of description. Because of the complexity of the problem, this interrelation is not always easy to trace when solving a particular TTP problem. The important point to emphasize is that at issue here is not how to modify the classical equations of physical kinetics and hydrodynamics to include additional transport mechanisms (in reacting media, for example); rather we face a situation in which, those involved believe, we must go beyond the classical picture if we wish the revised theory to describe experiment adequately. The alternative TTPs can be grouped conventionally into the following categories:

(1) theories that modify the macroscopic (hydrodynamic) description and neglect the possible changes of the kinetic description,

(2) those changing the kinetic description at the KE_{f_1} level without bothering much whether these changes are consistent with the structure of the entire BBGKY chain, and

(3) kinetic and hydrodynamic alternative theories consistent with the BBGKY hierarchy.

One of the pioneering efforts in the first line of research was a paper by Davydov [11], which stimulated a variety of studies (see, for instance, [37]) on the hyperbolic equation of thermal conductivity. Introducing the second derivative of temperature with respect to time permitted a passage from the parabolic to the hyperbolic heat conduction equation, thus allowing for a finite heat propagation velocity. However, already in his 1935 paper B I Davydov points out that his method "cannot be extended to the three-dimensional case"' and that "here the assumption that all the particles move at the same velocity would separate out a

five-dimensional manifold from the six-dimensional phase space, suggesting that the problem cannot be limited to the coordinate space alone." We note, however, that also quasi-linear parabolic equations can produce wave solutions.

Therefore, to hyperbolize the heat conduction equation phenomenologically [13] is not valid unless a rigorous kinetic justification is given. The hyperbolic heat conduction equation appears when the BE is solved by the Grad method [38] retaining a term which involves a derivative of the heat flow with respect to time and to which, in the context of the Chapman-Enskog method, no particular order of approximation can be ascribed. Following its introduction, stable and high-precision computational schemes were developed for the hyperbolic equation of heat conduction [39], whose applications included, for example, two-temperature non-local heat conduction models and the study of the telegraph equation as a paradigm for possible generalized hydrodynamics [37, 40].

Major difficulties arose when the question of existence and uniqueness of solutions of the Navier–Stokes equations was addressed. O A Ladyzhenskaya has shown for three-dimensional flows that under smooth initial conditions a unique solution is only possible over a finite time interval. Ladyzhenskaya even introduced a "correction" into the Navier–Stokes equations in order that its unique solvability could be proved (see discussion in [41]). It turned out that in this case the viscosity coefficient should be dependent on transverse flow-velocity gradients—with the result that the very idea of introducing kinetic coefficients should be overhauled.

We shall now turn to approaches in which the KE_{f_1} can be changed in a way which is generally inconsistent with the BBGKY hierarchy. It has been repeatedly pointed out that using a wrong distribution function (DF) for charged particles may have a catastrophic effect on the macro-parameters of a weakly ionized gas.

Let us have a look at some examples of this. As is well known, the temperature dependence of the density of atoms ionized in plasma to various degrees was first studied by Saha [42] and Eggert [43]. For a system in thermodynamic equilibrium they obtained the equation

$$\frac{n_{j+1}n_{\mathrm{e}}}{n_j}=\frac{s_{j+1}}{s_j}\frac{(2\pi m_{\mathrm{e}}k_{\mathrm{B}}T)^{3/2}}{h^3}\exp\left(-\frac{\varepsilon_j}{k_{\mathrm{B}}T}\right), \tag{I.22}$$

where n_j is the number density of j-fold ionized atoms, n_{e} is the number density of free electrons, m_{e} is the electron mass, k_{B} is the Boltzmann constant, h is the Planck constant, s_j is the statistical weight for a j-fold ionized atom [44], and ε_j is the jth ionization potential. The Saha equation (I.22) is derived for the Maxwellian distribution and should necessarily be modified if another velocity distribution of particles exists in the plasma. This problem was studied in work [45], in which, for illustrative purposes, the values of $n_{j+1}n_{\mathrm{e}}/n_j$ calculated with the Maxwell distribution function are compared with those obtained with the Druyvesteyn distribution function, the average energies for both distributions being assumed equal. Let $T=10^4K$, $n_{\mathrm{e}}=10^{14}\mathrm{cm}^{-3}$, $\varepsilon_j=10\,\mathrm{eV}$, the charge number $Z=1, s_{j+1}/s_j=1$. Then one arrives at [45]: $n_{j+1}n_{\mathrm{e}}/n_j=6\times10^2$ (calculation using the Druyvesteyn distribution), $n_{j+1}n_{\mathrm{e}}/n_j=4.53\times10^{16}$ (calculation using the Maxwellian distribution function by the Saha formula).

As E Dewan explained, "the discrepancy in fourteen orders of magnitude obtained above is clearly due to the fact that, unlike Maxwellian distribution, the Druyvesteyn distribution does not have a 'tail'."

In our second example, two quantities—the ionization rate constant and the ionization cross section—were calculated by Gryzinski et al. [46] using the two above-mentioned distributions. The ionization cross section σ_i is defined by the following interpolation formula known to match satisfactorily the experimental data

$$\sigma_i=\sigma_0 G_i(\xi,\varsigma)/\varepsilon_i^2, \tag{I.23}$$

where $\sigma_0=6.56\times10^{-14}\mathrm{cm}^2(e\mathrm{V})^2$, ε_i is the ionization potential of the atom, and ξ is a dimensionless parameter characterizing the atomic electron shell:

$$\xi=W/\varepsilon_i, \tag{I.24}$$

where W is the average kinetic energy of the atomic electrons, given by the formula

$$W=\frac{1}{N_{\mathrm{e}}}\sum_{j=1}^{N_{\mathrm{e}}}\varepsilon_j, \tag{I.25}$$

in which N_{e} is the number of electrons in the atom, and ε_j are the ionization potentials for the atom successively stripped of its electrons. The parameter ς is defined by the expression

$$\varsigma=U_{\mathrm{e}}/\varepsilon_i, \tag{I.26}$$

where U_{e} is the energy of the electrons bombarding the atom. The neutral particle velocities are assumed to be much lower than the average electron velocity, and the plasma is taken to be uniform. The average value of the ionization cross section is then given by

$$\overline{\sigma_i} = \int_0^\infty \sigma_i(v_e) f(v_e) \mathrm{d}v_e, \tag{I.27}$$

and the ionization rates are evaluated by the formula

$$\overline{\sigma_i v_e} = \int_0^\infty \sigma_i(v_e) v_e f(v_e) \mathrm{d}v_e, \tag{I.28}$$

provided the function $G_i(\xi, \varsigma)$ defined as [46]

$$G_i(\xi, \varsigma) = \frac{(\varsigma - 1)\left(1 + {}^2/_3\xi\right)}{(\varsigma + 1)(1 + \xi + \varsigma)}. \tag{I.29}$$

is known.

Table I.1 illustrates the calculated values of $\overline{\sigma}_i$ and $\overline{\sigma_i v_e}$ for $\xi = 1$ and various $\hat{T} = k_B T_e / \varepsilon_i$. It can be seen that the results obtained with different DFs can differ widely, indeed catastrophically so even for relatively small values of $\hat{T}$. Thus, the reliable computation of DFs remains a topic of intense current interest in plasma physics problems, the weak effect of the DF form on its moments being rather an exception than the rule. The use of collision cross sections which are "self-consistent" with kinetic equations is also suggested by the well-known Enskog theory of moderately dense gases [47]. Enskog's idea was to describe the properties of such gases by separating the *non-local part* out of the *essentially*

TABLE I.1 Comparison of ionization cross sections $\overline{\sigma}_i$ and ionization rates $\overline{\sigma_i v_e}$ calculated with the Maxwellian and Druyvesteyn DF

$\hat{T}$	Maxwellian DF		Druyvesteyn DF	
	$\overline{\sigma_i}$	$\overline{\sigma_i v_e}$	$\overline{\sigma_i}$	$\overline{\sigma_i v_e}$
0.1	4.206×10^{-6}	1.184×10^{-5}	1.278×10^{-27}	4.077×10^{-27}
0.2	8.262×10^{-4}	1.184×10^{-3}	4.382×10^{-9}	1.011×10^{-8}
0.3	5.029×10^{-3}	9.251×10^{-3}	2.128×10^{-5}	4.135×10^{-5}
0.4	1.259×10^{-2}	2.103×10^{-2}	5.403×10^{-4}	9.405×10^{-4}
0.5	2.194×10^{-2}	3.415×10^{-2}	2.773×10^{-3}	4.466×10^{-3}
0.6	3.180×10^{-2}	4.687×10^{-2}	7.305×10^{-3}	1.110×10^{-2}
0.7	4.143×10^{-2}	5.842×10^{-2}	1.376×10^{-2}	1.998×10^{-2}
0.8	5.047×10^{-2}	6.857×10^{-2}	2.145×10^{-2}	3.001×10^{-2}
0.9	5.875×10^{-2}	7.733×10^{-2}	2.973×10^{-2}	4.033×10^{-2}
1	6.624×10^{-2}	8.482×10^{-2}	3.813×10^{-2}	5.039×10^{-2}
2	1.079×10^{-1}	1.171×10^{-1}	9.918×10^{-2}	1.132×10^{-1}
3	1.195×10^{-1}	1.190×10^{-1}	1.233×10^{-1}	1.312×10^{-1}
4	1.209×10^{-1}	1.137×10^{-1}	1.311×10^{-1}	1.717×10^{-1}
5	1.185×10^{-1}	1.069×10^{-1}	1.320×10^{-1}	1.298×10^{-1}
6	1.146×10^{-1}	9.992×10^{-2}	1.299×10^{-1}	1.243×10^{-1}
7	1.102×10^{-1}	9.326×10^{-2}	1.263×10^{-1}	1.184×10^{-1}
8	1.056×10^{-1}	8.704×10^{-2}	1.222×10^{-1}	1.125×10^{-1}
9	1.010×10^{-1}	8.123×10^{-2}	1.179×10^{-1}	1.069×10^{-1}
10	9.662×10^{-2}	7.589×10^{-2}	1.137×10^{-1}	1.017×10^{-1}

local (!) Boltzmann collision integral. The transport coefficients obtained in this way for the hard-sphere model yielded an incorrect temperature dependence for the system's kinetic coefficients. To remedy this situation, the model of "soft" spheres was introduced to fit the experimental data (see, for instance, Ref. [48]).

In the theory of the so-called kinetically consistent difference schemes [49], the DF is expanded in a power series of time, which corresponds to using an incomplete second approximation in the "physical" derivation of the Boltzmann equation (see discussion in Refs. [41, 50]). The result is that the difference schemes obtained contain only an artificial ad hoc viscosity chosen specially for the problem at hand. Some workers followed the steps of Davydov by adding the term $\partial^2 f/\partial t^2$ to the kinetic equations for fast processes.

Bakai and Sigov [51] suggest using such a term in the equation for describing DF fluctuations in turbulent plasma. The so-called ordering parameter they introduce alters the very type of the equation. To describe spatial non-locality, Bakai and Sigov complement the kinetic equation by the $\partial^2 f/\partial x^2$ term and higher derivatives, including mixed time-coordinate partial derivatives—a modification which can possibly describe non-Gaussian random sources in the Langevin equations [52]. Vlasov [53] attempted to eliminate the inconsistencies of the Boltzmann theory through the inclusion of additional dynamical variables (derivatives of the velocity) in the one-particle distribution function $f(\mathbf{r}, \mathbf{v}, \dot{\mathbf{v}}, \ddot{\mathbf{v}}, \dddot{\mathbf{v}}, \ldots, t)$. However—due primarily to the reasonable complexity requirement which should be met for a theory to be useful in practice—this approach is, in our view, too early to try until all traditional resources for describing the DF are exhausted. The reader is referred to review [54] of some other theories of transport properties.

Clearly, approaches to the modification of the KE_{f_1} must be based on certain principles, and it is appropriate to outline these in brief here. Of the approaches we have mentioned above, the most consistent one is the third, which clearly reveals the relation between alternative KE_{f_1}'s and the BBGKY hierarchy. There are general requirements to which the generalized KE_{f_1} must satisfy.

(1) Since the artificial breaking of the BBGKY hierarchy is unavoidable in changing to a one-particle description, the generalized KE_{f_1} should be obtainable with the known methods of the theory of kinetic equations, such as the multi-scale approach, correlation function method, iterative methods, and so forth, or combinations of them. In each of these, some specific features of the particular alternative KE_{f_1} are highlighted.
(2) There must be an explicit link between the KE_{f_1} and the way we introduce the physically infinitesimal volume—and hence with the way the moments in the reference contour with transparent boundaries fluctuate due to the finite size of the particles.
(3) In the non-relativistic case, the KE_{f_1} must satisfy the Galileo transformation.
(4) The KE_{f_1} must ensure a connection with the classical H-theorem and its generalizations.
(5) The KE_{f_1} should not lead to unreasonable complexities in the theory.

Although the examples above are purely illustrative and the exhaustive list of difficulties faced by Boltzmann kinetic theory would of course be much longer, it should be recognized that after the intense debates of the early twentieth century, the search for an alternative kinetic equation for a one-particle distribution function has gradually leveled off or, perhaps more precisely, has become of marginal physical importance. Both sides of the dispute have exhausted their arguments. On the other hand, the Boltzmann equation has proven to be successful in solving a variety of problems, particularly in the calculation of kinetic coefficients. Thus, the development of Boltzmann kinetic theory has turned out to be typical for any revolutionary physical theory—from rejection to recognition and further to a kind of "canonization."

In the latter part of twentieth century two very important results were obtained:

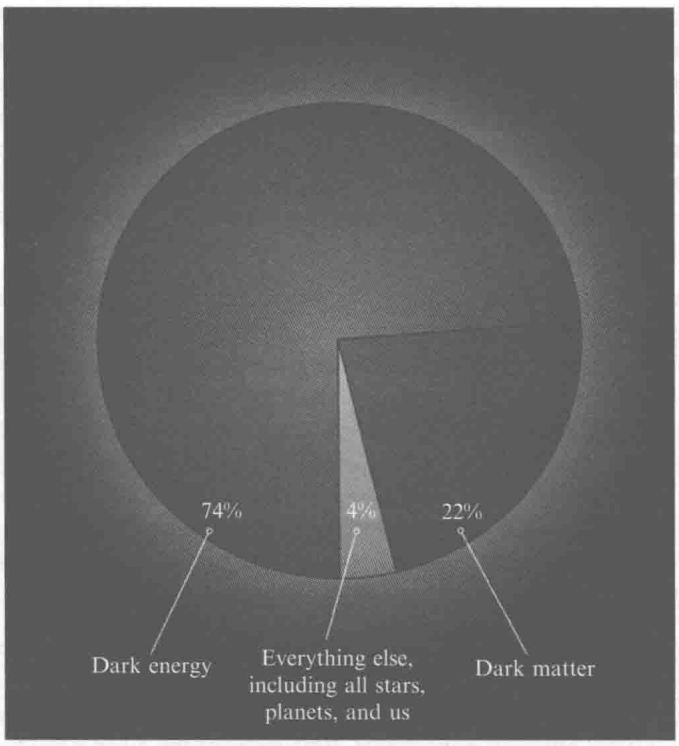
(1) The Irish physicist John Stewart Bell (1928–1990) was to show (Bell's theorems [55]) that all local statistical theories of dissipative processes are wrong in principal.
(2) The Russian physicist Boris V Alexeev was to show that the derivation of KE_{f_1} from the BBGKY equations (*prior* to introducing any approximation destined to break the Bogolyubov chain) leads to additional terms of the non-local origin, generally of the same order of magnitude, appear in the BE. Then the passage to the BE means the neglect of non-local effects. These additional terms cannot be omitted even in the limit cases of kinetic theory, therefore BE is only a plausible equation.

It could be anticipated that we will obtain small corrections to the existing results. But in reality we are in front of the tremendous catastrophe in modern theoretical physics. Several extremely significant problems challenge modern fundamental physics, which can be titled as "Non-solved problems of the fundamental physics" or more precisely of *local physical kinetics* of dissipative processes, namely:

(1) Kinetic theory of entropy and the problem of the initial perturbation;
(2) Strict theory of turbulence;

(3) Quantum non-relativistic and relativistic hydrodynamics, theory of charges separation in the atom structure;
(4) Theory of ball lightning;
(5) Theory of dark matter;
(6) Theory of dark energy, Hubble expansion of the Universe;
(7) The destiny of anti-matter after the Big Bang;
(8) A unified theory of dissipative structures—from atom structure to cosmology.

In appearance these old and new problems (including the problems 5 and 6 in the list) mean that we have reached the revolutionary situation not only in physics but in natural philosophy on the whole. Practically we are in front of the new challenge since Newton's *Mathematical Principles of Natural Philosophy* was first published in 1687. It's impossible to believe that in more that three hundred years after Newton we have the following diagram obtained in astrophysics:



As result we have the Internet reaction like that: It is humbling, perhaps even humiliating, that we know almost nothing about 96% of what is "out there"!! But this situation is not so bad on its own. We have the worst case from position of local physics—dark matter and dark energy cannot be diagnosed by the known experimental methods. WHERE IS THE MAIN MISTAKE?

The origin of difficulties consists in the total oversimplification following from the principles of local physics and reflects the general shortcomings of the local kinetic transport theory.

Let us consider the fundamental methodic aspects of non-local physics from the qualitative standpoint, avoiding excessively cumbersome formulas.

Transport processes in open dissipative systems are considered in physical kinetics. Therefore, the kinetic description is inevitably related to the system diagnostics. Such an element of diagnostics in the case of theoretical description in physical kinetics is the concept of the physically infinitely small volume (PhSV). The correlation between theoretical description and system diagnostics is well known in physics. Suffice it to recall the part played by test charge in electrostatics or by test circuit in the physics of magnetic phenomena.

The traditional definition of PhSV contains the statement to the effect that the PhSV contains a sufficient number of particles for introducing a statistical description; however, at the same time, the PhSV is much smaller than the volume V of the physical system under consideration.

In a first approximation, this leads to local approach in investigating the transport processes. It is assumed in classical hydrodynamics that local thermodynamic equilibrium is first established within the PhSV, and only after that the transition occurs to global thermodynamic equilibrium if it is at all possible for the system under study.

Let us consider the hydrodynamic description in more detail from this point of view. Assume that we have two neighboring physically infinitely small volumes PhSV_1 and PhSV_2 in a non-equilibrium system. The one-particle distribution function (DF) $f_{\mathrm{sm},1}(\mathbf{r}_1,\mathbf{v},t)$ corresponds to the volume PhSV_1, and the function $f_{\mathrm{sm},2}(\mathbf{r}_2,\mathbf{v},t)$—to the volume PhSV_2. It is assumed in a first approximation that $f_{\mathrm{sm},1}(\mathbf{r}_1,\mathbf{v},t)$ does not vary within PhSV_1, same as $f_{\mathrm{sm},2}(\mathbf{r}_2,\mathbf{v},t)$ does not vary within the neighboring volume PhSV_2. It is this assumption of locality that is implicitly contained in the Boltzmann equation (BE). *However, the assumption is too crude.*

Indeed, a particle on the boundary between two volumes, which experienced the last collision in PhSV_1 and moves toward PhSV_2, introduces information about the $f_{\mathrm{sm},1}(\mathbf{r}_1,\mathbf{v},t)$ into the neighboring volume PhSV_2. Similarly, a particle on the boundary between two volumes, which experienced the last collision in PhSV_2 and moves toward PhSV_1, introduces information about the DF $f_{\mathrm{sm},2}(\mathbf{r}_2,\mathbf{v},t)$ into the neighboring volume PhSV_1. The relaxation over translational degrees of freedom of particles of like masses occurs during several collisions. As a result, "Knudsen layers" are formed on the boundary between neighboring physically infinitely small volumes, the characteristic dimension of which is of the order of path length.

Then a correction must be introduced into the DF in the PhSV, which is proportional to the mean time between collisions and to the substantive derivative of the DF being measured. Rigorous derivation is given for example in [21, 30–32, 56, 57].

Let a particle of finite radius be characterized as before by the position $\mathbf{r}$ at the instant of time t of its center of mass moving at velocity $\mathbf{v}$. Then, the situation is possible where, at some instant of time t, the particle is located on the interface between two volumes. In so doing, the lead effect is possible (say, for PhSV_2), when the center of mass of particle moving to the neighboring volume PhSV_2 is still in PhSV_1. However, the delay effect takes place as well, when the center of mass of particle moving to the neighboring volume (say, PhSV_2) is already located in PhSV_2 but a part of the particle still belongs to PhSV_1.

Moreover, even the point-like particles (starting after the last collision near the boundary between two mentioned volumes) can change the distribution functions in the neighboring volume. Adjusting of the particles dynamic characteristics for translational degrees of freedom takes several collisions. Therefore we experience a "Knudsen layer" effect between adjacent small volumes. This leads to fluctuations in mass and hence also in other hydrodynamic quantities. The existence of such "Knudsen layers" is not dependent on the choice of spatial nets and is fully defined by the reduced description for ensemble of particles of finite diameters in the conceptual framework of open physically small volumes, *i.e.*, it depends on the chosen method of measurement.

THE MAIN MISTAKE OF LOCAL PHYSICAL KINETICS CAN BE INDICATED AS FOLLOWS:

This entire complex of the mentioned effects defines non-local effects in space and time. The physically infinitely small volume (PhSV) is an *open* thermodynamic system *for any division of macroscopic system by a set of PhSVs.*

However, the Boltzmann equation (BE) fully ignores non-local effects and contains only the local collision integral J^{B}. The foregoing non-local effects are insignificant only in equilibrium systems, where the kinetic approach changes to methods of statistical mechanics.

This is what the difficulties of classical Boltzmann physical kinetics arise from. The rigorous approach to derivation of kinetic equation relative to one-particle DF f (KE_f) is based on employing the hierarchy of Bogoliubov equations. Low index "1" is omitted if it cannot lead to misunderstandings. Generally speaking, the structure of KE_f is as follows

$$\frac{\mathrm{D}f}{\mathrm{D}t}=J^{\mathrm{B}}+J^{\mathrm{nl}}, \tag{I.30}$$

where J^{nl} is the non-local integral term.

An approximation for the second collision integral is suggested by me in *generalized* Boltzmann physical kinetics,

$$J^{\mathrm{nl}}=\frac{\mathrm{D}}{\mathrm{D}t}\left(\tau\frac{\mathrm{D}f}{\mathrm{D}t}\right), \tag{I.31}$$

τ is non-local parameter (coinciding in a gas with the relaxation time τ_{r} proportional to the mean time τ_{mt} *between* collisions of particles); τ_{mt} is related in a hydrodynamic approximation with dynamical viscosity μ and pressure p,

$$\tau_{\mathrm{mt}}p=\Pi\mu, \tag{I.32}$$

where the factor Π is defined by the model of collision of particles; for neutral hard-sphere gas, $\Pi=0.8$, [35, 36]. Obviously in "the simplest version" τ_{mt} can be used in (I.31) instead of τ; it leads only to variety of Π-parameter in (I.32).

All of the known methods of deriving kinetic equation relative to one-particle DF f lead to the approximation (I.31), including the method of many scales, the method of correlation functions, and the iteration method.

We are faced in fact with the "price-quality" problem familiar from economics. That is, what price—in terms of the increased complexity of the kinetic equation—are we ready to pay for the improved quality of the theory? An answer to this question is possible only through experience with practical problems.

Extremely important:

1. Approximation $J^{\mathrm{nl}} = \dfrac{\mathrm{D}}{\mathrm{D}t}\left(\tau \dfrac{\mathrm{D}f}{\mathrm{D}t}\right)$ delivers local approximation of non-local collision integrals.
2. Approximation $J^{\mathrm{nl}} = \dfrac{\mathrm{D}}{\mathrm{D}t}\left(\tau \dfrac{\mathrm{D}f}{\mathrm{D}t}\right)$ return us to two level description (level of hydrodynamic processes + level of transport processes between collisions).
3. The generalized transport theory is not too complicated in applications.

One can draw an analogy with the Bhatnagar-Gross-Krook (BGK) approximation for local integral J^{B},

$$J^{\mathrm{B}} = \frac{f^{(0)} - f}{\tau_{\mathrm{r}}}, \tag{I.33}$$

(in the simplest case $\tau_{\mathrm{r}} \sim \tau$) the popularity of which in the case of Boltzmann collision integral is explained by the colossal simplification attained when using this approximation. The order of magnitude of the ratio between the second and first terms of the right-hand part of Eq. (I.30) is Kn^2, at high values of Knudsen number, these terms come to be of the same order. It would seem that, at low values of Knudsen number corresponding to hydrodynamic description, the contribution by the second term of the right-hand part of Eq. (I.30) could be ignored.

However, this is not the case. Upon transition to hydrodynamic approximation (following the multiplication of the kinetic equation by invariants collision and subsequent integration with respect to velocities), the Boltzmann integral part goes to zero, and the second term of the right-hand part of Eq. (I.30) *does not go to zero* after this integration and produces a contribution of the same order in the case of generalized Navier-Stokes description.

From the mathematical standpoint, disregarding the term containing a small parameter with higher derivative is impermissible. From the physical standpoint, the arising additional terms proportional to viscosity correspond to Kolmogorov small-scale turbulence; the fluctuations are tabulated. It turns out that the integral term J^{nl} is important from the standpoint of the theory of transport processes at both low and high values of Knudsen number.

Note the treatment of GBE from the standpoint of fluctuation theory,

$$\mathrm{D}f^a/\mathrm{D}t = J^{\mathrm{B}}, \tag{I.34}$$

$$f^a = f - \tau \mathrm{D}f/\mathrm{D}t. \tag{1.35}$$

Equations (I.34) and (I.35) have a correct free-molecule limit. Therefore, $\tau Df/Dt$ is a fluctuation of distribution function, and the notation (I.34) disregarding (I.35) renders the BE open. From the standpoint of fluctuation theory, Boltzmann employed the simplest closing procedure

$$f^a = f. \tag{I.36}$$

Fluctuation effects occur in any open thermodynamic system bounded by a control surface transparent to particles. Obviously the mentioned non-local effects can be discussed from viewpoint of breaking of the Bell's inequalities because in the non-local theory the measurement (realized in PhSV_1) has influence on the measurement realized in the adjoining space-time point in PhSV_2 and vice-versa.

The equation (GBE) reads

$$\frac{\mathrm{D}f}{\mathrm{D}t} = J^{\mathrm{B}} + \frac{\mathrm{D}}{\mathrm{D}t}\left(\tau \frac{\mathrm{D}f}{\mathrm{D}t}\right). \tag{I.37}$$

Here τ is non-local parameter, in the simplest case it is the mean time BETWEEN collisions (for plasma τ is mean time between close collisions), for plasma in $\mathrm{D}/\mathrm{D}t$ should be introduced the self consistent force F. It is interesting to note that the GBE also makes it possible to include higher derivatives of the DF (see the approximation (5.8) in Ref. [32]). For a multi species reacting gas, the generalized Boltzmann equation can be rewritten as

$$\frac{\mathrm{D}f_\alpha}{\mathrm{D}t}-\frac{\mathrm{D}}{\mathrm{D}t}\left(\tau_\alpha\frac{\mathrm{D}f_\alpha}{\mathrm{D}t}\right)=J_\alpha^{\mathrm{B,el}}+J_\alpha^{\mathrm{B,r}}, \tag{I.38}$$

where f_α is distribution function for a particle of the α th kind, τ_α is non-local parameter for α species (in the simplest case τ_α is mean free time for a particle of the α th kind), and $J_\alpha^{\mathrm{B,el}}$,$J_\alpha^{\mathrm{B,r}}$ are the Boltzmann collision integrals for elastic and inelastic collisions, respectively. GBE (I.37) was derived in the theory of liquids, in this case τ is connected with the time of the particle residence in the Frenkel cell.

By the way derived by me GBE was presented in my lectures on physical kinetics given in Sophia University, Bulgaria, in the year 1987; the last monographs in Russian [58, 59].

Let us consider now some aspects of GBE application beginning with hydrodynamic aspects of the theory. Therefore, in the first approximation, fluctuations will be proportional to the mean free path λ (or, equivalently, to the mean time between the collisions). We can state that the number of particles in reference volume is proportional to cube of the character length L of volume, the number of particles in the surface layer is proportional to λL^2, and as result all effect of fluctuation can be estimated as ratio of two mentioned values or as $\lambda/L=\mathrm{Kn}$.

The important methodical question to be considered is how classical conservation laws fit into the GBE picture. Continuum mechanics conservation laws are derived on the macroscopic level by considering a certain reference volume within the medium, which is enclosed by an infinitesimally thin surface. Moving material points (gas particles) can be either within or outside the volume, and it is by writing down the corresponding balance equations for mass, momentum flux, and energy that the classical equations of continuity, motion, and energy are obtained.

Obviously the hydrodynamic equations will explicitly involve fluctuations proportional to τ. For example, the continuity equation changes its form and will contain terms proportional to viscosity. On the other hand, if the reference volume extends over the whole cavity with the hard walls, then the classical conservation laws should be obeyed.

However, we will here attempt to "guess" the structure of the generalized continuity equation using the arguments outlined above. Neglecting fluctuations, the continuity equation should have the classical form with

$$\rho^a=\rho-\rho^{\mathrm{fl}}=\rho-\tau A,\ (\rho\mathbf{v}_0)^a=\rho\mathbf{v}_0-(\rho\mathbf{v}_0)^{\mathrm{fl}}=\rho\mathbf{v}_0-\tau\mathbf{B}, \tag{I.39}$$

where ρ is density and $\mathbf{v}_0$ is hydrodynamic velocity. Strictly speaking, the factors A and $\mathbf{B}$ can be obtained from the generalized kinetic equation, in our case, from the GBE. Still, we can guess their form without appeal to the KE_f.

Indeed, let us write the generalized continuity equation

$$\frac{\partial}{\partial t}(\rho-\tau A)+\frac{\partial}{\partial\mathbf{r}}\cdot(\rho\mathbf{v}_0-\tau\mathbf{B})=0 \tag{I.40}$$

in the dimensionless form, using l, the distance from the reference contour to the hard wall (see Fig. I.1), as a length scale.

Then, instead of $\boldsymbol{\tau}$, the (already dimensionless) quantities A and $\mathbf{B}$ will have the Knudsen number $\mathrm{Kn}_l=\lambda/l$ as a coefficient. In the limit $l\to 0, \mathrm{Kn}_l\to\infty$ the contour embraces the entire cavity contained within hard walls, and there are no fluctuations on the walls. In other words, the classical equations of continuity and motion must be satisfied at the wall. Using hydrodynamic terminology, we note that the conditions $A=0$, $\mathbf{B}=0$ correspond to a laminar sub-layer in a turbulent flow.

Now if a local Maxwellian distribution is assumed, then the generalized equation of continuity in the Euler approximation is written as

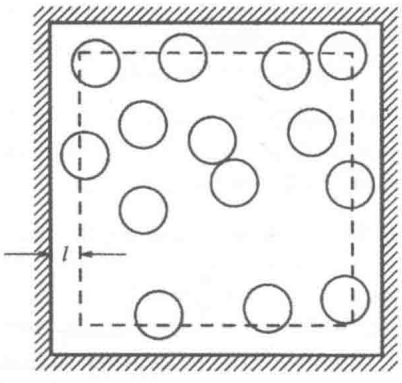


FIG. I.1 Closed cavity and the reference contour containing particles of a finite diameter.