HEP World's Classics

An Investigation of the Laws of Thought

On Which Are Founded the Mathematical Theories of Logic and Probabilities

思维规律的研究

GEORGE BOOLE

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This edition first published 1854, MACMILLAN & CO.

内容简介

乔治·布尔的《思维规律的研究》于1854年出版，是19世纪颇具影响力的一本书，它创立了逻辑和概率的数学理论，也是作者关于代数逻辑的两本专著中的一本。布尔是爱尔兰科克皇后学院的教授。

布尔的著作创立了代数逻辑的基本轮廓，它经常被误读为我们今日所知的布尔代数。但事实上，布尔的代数 (Boole's algebra) 和现代的布尔代数 (Boolean algebra) 并不一样：布尔考虑的代数无法用带有并、交、补运算的集合来解释，而现代布尔代数是可以的。发展现代布尔代数的工作落在研究代数逻辑的布尔的继任者身上，这包括 Jevons (1869)、Peirce (1880)、Jevons (1890)、Schröder (1890) 和 Huntingdon (1904)。

图书在版编目（CIP）数据

思维规律的研究 = An investigation of the laws of thought: 英文 /（英）布尔 (Boole, G.) 著. -- 北京：高等教育出版社，2016.6

ISBN 978-7-04-045300-3

Ⅰ. ①思… Ⅱ. ①布… Ⅲ. ①思维规律－研究－英文 Ⅳ. ①B802

中国版本图书馆 CIP 数据核字 (2016) 第 086536 号

策划编辑 赵天夫　　责任编辑 赵天夫　　封面设计 王凌波　　责任校对 陈旭颖
责任印制 韩 刚

出版发行	高等教育出版社	网　址	http://www.hep.edu.cn
社　址	北京市西城区德外大街4号		http://www.hep.com.cn
邮政编码	100120	网上订购	http://www.hepmall.com.cn
印　刷	涿州市星河印刷有限公司		http://www.hepmall.com
开　本	787mm × 1092mm　1/16		http://www.hepmall.cn
印　张	22.25		
字　数	580 千字	版　次	2016 年 6 月第 1 版
购书热线	010-58581118	印　次	2016 年 6 月第 1 次印刷
咨询电话	400-810-0598	定　价	138.00 元

本书如有缺页、倒页、脱页等质量问题，请到所购图书销售部门联系调换

物 料 号　45300-00

HEP World's Classics

There is a Chinese saying: "It is beneficial to open any book." It is even more fruitful to open and read classic books. The world is keeping on changing, but really fundamental and essential things stay the same since there is nothing new under the sun. Great ideas have been discovered and re-discovered, and they should be learnt and re-learnt. Classic books are our inheritance from all the previous generations and contain the best of knowledge and wisdom of all the people before us. They are timeless and universal. We cannot travel back in time, but we can converse with the originators of current theories through reading their books. Classic books have withstood the test of time. They are reliable and contain a wealth of original ideas. More importantly, they are also books which have not finished what they wanted or hoped to say. Consequently, they contain unearthed treasures and hidden seeds of new theories, which are waiting to be discovered. As it is often said: history is today. Proper understanding of the past work of giants is necessary to carry out properly the current and future researches and to make them to be a part of the history of science and mathematics. Reading classic books is not easy, but it is rewarding. Some modern interpretations and beautiful reformulations of the classics often miss the subtle and crucial points. Reading classics is also more than only accumulating knowledge, and the reader can learn from masters on how they asked questions, how they struggled to come up with new notions and theories to overcome problems, and answers to questions. Above all, probably the best reason to open classic books is the curiosity: what did people know, how did they express and communicate them, why did they do what they did? It can simply be fun!

This series of classic books by Higher Education Press contains a selection of best classic books in natural history, mathematics, physics, chemistry, information technology, geography, etc. from the past two thousand years. They contain masterpieces by the great people such Archimedes, Newton, Lavoisier, Dalton, Gauss, Darwin, Maxwell, and hence give a panorama of science and mathematics. They have been typeset in modern fonts for easier and more enjoyable reading. To help the reader understand difficult classics better, some volumes contain introductions and commentaries by experts. Though each classic book can stand in its own, reading them together will help the reader gain a bigger perspective of science and mathematics and understand better interconnection between seemingly unrelated topics and subjects.

Higher Education Press has been the largest publisher in China. Besides the long tradition of providing high quality books for proper education and training of university and graduate students, she has also set out to provide research monographs and references books to people at all levels around the world. Higher Education Press considers it her duty to keep the world science and mathematics community informed of what has been achieved in their subjects in easy and accessible formats. This series of classic books is an integral part of this effort.

George Boole (1815—1864)

乔治·布尔 (George Boole 1815—1864)，英国数理逻辑学家，逻辑代数的创始人。1815 年 11 月 2 日生于英格兰的林肯，1864 年 12 月 8 日卒于爱尔兰的科克。出身于手工业者家庭，通过自学掌握了数学。他运用代数方法研究逻辑学，1844 年发表了著名的论文《关于分析中的一个普遍方法》，1849 年受聘为爱尔兰科克皇后学院教授，并被选为英国皇家学会成员。1847 年出版专著《逻辑的数学分析》，1854 年出版了《思维规律的研究》。

布尔用数学方法研究逻辑问题，成功地建立了第一个逻辑演算。他用等式表示判断，把推理看作等式的变换。这种变换的有效性不依赖人们对符号的解释，只依赖于符号的组合规律。这一逻辑理论，既可以进行公式推演，又可以对命题取作数值；例如，可以把真命题取作 1 值，假命题取作 0 值，由此复杂的命题仅作数值计算就可以求得它为真值还是假值了。这样，把已给的公式中出现的符号的逻辑解释放在一边，把它转变为表示数量的符号，但只能取 0 或 1，对它实现求解的一切必须的步骤；最后再还给它以逻辑的解释。这一理论在布尔之后虽然也有些改进，但它的基本轮廓是布尔建立起来的，因此，人们常称它为布尔代数。20 世纪 30 年代，逻辑代数在电路系统上获得应用，随后，由于电子技术与计算机的发展，出现各种复杂的大系统，它们的变换规律也遵守布尔所揭示的规律。因此，布尔代数的应用日益广泛，它的内容日益普及。

在 19 世纪中叶，布尔研究人类思想活动所揭示的规律，当时既无明显的实际背景，也不可能考虑到它的实际应用。而几十年后，从 20 世纪 30 年代开始，逐步开拓了它的应用，在理论和实践中都发挥了很大的作用。

（摘自《中国大百科全书》数学卷）

AN INVESTIGATION

OF

THE LAWS OF THOUGHT,

ON WHICH ARE FOUNDED

THE MATHEMATICAL THEORIES OF LOGIC AND PROBABILITIES.

BY

GEORGE BOOLE, LL. D.

PROFESSOR OF MATHEMATICS IN QUEEN'S COLLEGE, CORK.

LONDON:

WALTON AND MABERLY,

UPPER GOWER-STREET, AND IVY-LANE, PATERNOSTER-ROW.

CAMBRIDGE: MACMILLAN AND CO.

1854.

The title page of the original version

TO

JOHN RYALL, LL. D..

VICE-PRESIDENT AND PROFESSOR OF GREEK

IN QUEEN'S COLLEGE, CORK,

THIS WORK IS INSCRIBED

IN TESTIMONY OF FRIENDSHIP AND ESTEEM.

PREFACE.

The following work is not a republication of a former treatise by the Author, entitled, "The Mathematical Analysis of Logic." Its earlier portion is indeed devoted to the same object, and it begins by establishing the same system of fundamental laws, but its methods are more general, and its range of applications far wider. It exhibits the results, matured by some years of study and reflection, of a principle of investigation relating to the intellectual operations, the previous exposition of which was written within a few weeks after its idea had been conceived.

That portion of this work which relates to Logic presupposes in its reader a knowledge of the most important terms of the science, as usually treated, and of its general object. On these points there is no better guide than Archbishop Whately's "Elements of Logic," or Mr. Thomson's "Outlines of the Laws of Thought." To the former of these treatises, the present revival of attention to this class of studies seems in a great measure due. Some acquaintance with the principles of Algebra is also requisite, but it is not necessary that this application should have been carried beyond the solution of simple equations. For the study of those chapters which relate to the theory of probabilities, a somewhat larger knowledge of Algebra is required, and especially of the doctrine of Elimination, and of the solution of Equations containing more than one unknown quantity. Preliminary information upon the subject-matter will be found in the special treatises on Probabilities in "Lardner's Cabinet Cyclopædia," and the "Library of Useful Knowledge," the former of these by Professor De Morgan, the latter by Sir John Lubbock; and in an interesting series of Letters translated from the French of M. Quetelet. Other references will be given in the work. On a first perusal the reader may omit at his discretion, Chapters X., XIV., and XIX., together with any of the applications which he may deem uninviting or irrelevant.

In different parts of the work, and especially in the notes to the concluding chapter, will be found references to various writers, ancient and modern, chiefly designed to illustrate a certain view of the history of philosophy. With respect to these, the Author thinks it proper to add, that he has in no instance given a citation which he has not believed upon careful examination to be supported either by parallel authorities, or by the general tenor of the work from which it was taken. While he would gladly have avoided the introduction of anything which might by possibility be construed into the parade of learning, he felt it to be due both to his subject and to the truth, that the statements in the text should be accompanied by the means of verification. And if now, in bringing to its close a labour, of the extent of which few persons will be able to judge from its apparent fruits, he may be permitted to speak for a single moment of the feelings with which

he has pursued, and with which he now lays aside, his task, he would say, that he never doubted that it was worthy of his best efforts; that he felt that whatever of truth it might bring to light was not a private or arbitrary thing, not dependent, as to its essence, upon any human opinion. He was fully aware that learned and able men maintained opinions upon the subject of Logic directly opposed to the views upon which the entire argument and procedure of his work rested. While he believed those opinions to be erroneous, he was conscious that his own views might insensibly be warped by an influence of another kind. He felt in an especial manner the danger of that intellectual bias which long attention to a particular aspect of truth tends to produce. But he trusts that out of this conflict of opinions the same truth will but emerge the more free from any personal admixture; that its different parts will be seen in their just proportion; and that none of them will eventually be too highly valued or too lightly regarded because of the prejudices which may attach to the mere form of its exposition.

To his valued friend, the Rev. George Stephens Dickson, of Lincoln, the Author desires to record his obligations for much kind assistance in the revision of this work, and for some important suggestions.

5, Grenville-place, Cork,
Nov. 30*th.* 1853.

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AN INVESTIGATION [1]

OF

THE LAWS OF THOUGHT.

CHAPTER I.

NATURE AND DESIGN OF THIS WORK.

1. The design of the following treatise is to investigate the fundamental laws of those operations of the mind by which reasoning is performed; to give expression to them in the symbolical language of a Calculus, and upon this foundation to establish the science of Logic and construct its method; to make that method itself the basis of a general method for the application of the mathematical doctrine of Probabilities; and, finally, to collect from the various elements of truth brought to view in the course of these inquiries some probable intimations concerning the nature and constitution of the human mind.

2. That this design is not altogether a novel one it is almost needless to remark, and it is well known that to its two main practical divisions of Logic and Probabilities a very considerable share of the attention of philosophers has been directed. In its ancient and scholastic form, indeed, the subject of Logic stands almost exclusively associated with the great name of Aristotle. As it was presented to ancient Greece in the partly technical, partly metaphysical disquisitions of the Organon, such, with scarcely any essential change, it has continued to the present day. The stream of original inquiry has rather been directed towards questions of general philosophy, which, though they have arisen among the disputes of the [2] logicians, have outgrown their origin, and given to successive ages of speculation their peculiar bent and character. The eras of Porphyry and Proclus, of Anselm and Abelard, of Ramus, and of Descartes, together with the final protests of Bacon and Locke, rise up before the mind as examples of the remoter influences of the study upon the course of human thought, partly in suggesting topics fertile of discussion, partly in provoking remonstrance against its own undue pretensions. The history of the theory of Probabilities, on the other hand, has presented far more of that character of steady growth which belongs to science. In its origin the early genius of Pascal,—in its maturer stages of development the most recondite of all the mathematical speculations of Laplace,—were directed to its improvement;

to omit here the mention of other names scarcely less distinguished than these. As the study of Logic has been remarkable for the kindred questions of Metaphysics to which it has given occasion, so that of Probabilities also has been remarkable for the impulse which it has bestowed upon the higher departments of mathematical science. Each of these subjects has, moreover, been justly regarded as having relation to a speculative as well as to a practical end. To enable us to deduce correct inferences from given premises is not the only object of Logic; nor is it the sole claim of the theory of Probabilities that it teaches us how to establish the business of life assurance on a secure basis; and how to condense whatever is valuable in the records of innumerable observations in astronomy, in physics, or in that field of social inquiry which is fast assuming a character of great importance. Both these studies have also an interest of another kind, derived from the light which they shed upon the intellectual powers. They instruct us concerning the mode in which language and number serve as instrumental aids to the processes of reasoning; they reveal to us in some degree the connexion between different powers of our common intellect; they set before us what, in the two domains of demonstrative and of probable knowledge, are the essential standards of truth and correctness,—standards not derived from without, but deeply founded in the constitution of the human faculties. These ends of speculation yield neither in
[3] interest nor in dignity, nor yet, it may be added, in importance, to the practical objects, with the pursuit of which they have been historically associated. To unfold the secret laws and relations of those high faculties of thought by which all beyond the merely perceptive knowledge of the world and of ourselves is attained or matured, is an object which does not stand in need of commendation to a rational mind.

3. But although certain parts of the design of this work have been entertained by others, its general conception, its method, and, to a considerable extent, its results, are believed to be original. For this reason I shall offer, in the present chapter, some preparatory statements and explanations, in order that the real aim of this treatise may be understood, and the treatment of its subject facilitated.

It is designed, in the first place, to investigate the fundamental laws of those operations of the mind by which reasoning is performed. It is unnecessary to enter here into any argument to prove that the operations of the mind are in a certain real sense subject to laws, and that a science of the mind is therefore *possible*. If these are questions which admit of doubt, that doubt is not to be met by an endeavour to settle the point of dispute *à priori*, but by directing the attention of the objector to the evidence of actual laws, by referring him to an actual science. And thus the solution of that doubt would belong not to the introduction to this treatise, but to the treatise itself. Let the assumption be granted, that a science of the intellectual powers is possible, and let us for a moment consider how the knowledge of it is to be obtained.

4. Like all other sciences, that of the intellectual operations must primarily rest upon observation,—the subject of such observation being the very operations and processes of which we desire to determine the laws. But while the necessity of a foundation in experience is thus a condition common to all sciences, there

are some special differences between the modes in which this principle becomes available for the determination of general truths when the subject of inquiry is the mind, and when the subject is external nature. To these it is necessary to direct attention.

The general laws of Nature are not, for the most part, immediate objects [4]
of perception. They are either inductive inferences from a large body of facts, the common truth in which they express, or, in their origin at least, physical hypotheses of a causal nature serving to explain phænomena with undeviating precision, and to enable us to predict new combinations of them. They are in all cases, and in the strictest sense of the term, *probable* conclusions, approaching, indeed, ever and ever nearer to certainty, as they receive more and more of the confirmation of experience. But of the character of probability, in the strict and proper sense of that term, they are never wholly divested. On the other hand, the knowledge of the laws of the mind does not require as its basis any extensive collection of observations. The general truth is seen in the particular instance, and it is not confirmed by the repetition of instances. We may illustrate this position by an obvious example. It may be a question whether that formula of reasoning, which is called the *dictum* of Aristotle, *de omni et nullo*, expresses a primary law of human reasoning or not; but it is no question that it expresses a general truth in Logic. Now that truth is made manifest in all its generality by reflection upon a single instance of its application. And this is both an evidence that the particular principle or formula in question is founded upon some general law or laws of the mind, and an illustration of the doctrine that the perception of such general truths is not derived from an induction from many instances, but is involved in the clear apprehension of a single instance. In connexion with this truth is seen the not less important one that our knowledge of the laws upon which the science of the intellectual powers rests, whatever may be its extent or its deficiency, is not probable knowledge. For we not only see in the particular example the general truth, but we see it also as a certain truth,—a truth, our confidence in which will not continue to increase with increasing experience of its practical verifications.

5. But if the general truths of Logic are of such a nature that when presented to the mind they at once command assent, wherein consists the difficulty of constructing the Science of Logic? Not, it may be answered, in collecting the materials of knowledge, but in discriminating their nature, and determining their [5]
mutual place and relation. All sciences consist of general truths, but of those truths some only are primary and fundamental, others are secondary and derived. The laws of elliptic motion, discovered by Kepler, are general truths in astronomy, but they are not its fundamental truths. And it is so also in the purely mathematical sciences. An almost boundless diversity of theorems, which are known, and an infinite possibility of others, as yet unknown, rest together upon the foundation of a few simple axioms; and yet these are all *general* truths. It may be added, that they are truths which to an intelligence sufficiently refined would shine forth in their own unborrowed light, without the need of those connecting links of thought, those steps of wearisome and often painful deduction, by which the knowledge of

them is actually acquired. Let us define as fundamental those laws and principles from which all other general truths of science may be deduced, and into which they may all be again resolved. Shall we then err in regarding that as the true science of Logic which, laying down certain elementary laws, confirmed by the very testimony of the mind, permits us thence to deduce, by uniform processes, the entire chain of its secondary consequences, and furnishes, for its practical applications, methods of perfect generality? Let it be considered whether in any science, viewed either as a system of truth or as the foundation of a practical art, there can properly be any other test of the completeness and the fundamental character of its laws, than the completeness of its system of derived truths, and the generality of the methods which it serves to establish. Other questions may indeed present themselves. Convenience, prescription, individual preference, may urge their claims and deserve attention. But as respects the question of what constitutes science in its abstract integrity, I apprehend that no other considerations than the above are properly of any value.

6. It is designed, in the next place, to give expression in this treatise to the fundamental laws of reasoning in the symbolical language of a Calculus. Upon this head it will suffice to say, that those laws are such as to suggest this mode of expression, and to give to it a peculiar and exclusive fitness for the ends in view.
[6] There is not only a close analogy between the operations of the mind in general reasoning and its operations in the particular science of Algebra, but there is to a considerable extent an exact agreement in the laws by which the two classes of operations are conducted. Of course the laws must in both cases be determined independently; any formal agreement between them can only be established *a posteriori* by actual comparison. To borrow the notation of the science of Number, and then assume that in its new application the laws by which its use is governed will remain unchanged, would be mere hypothesis. There exist, indeed, certain general principles founded in the very nature of language, by which the use of symbols, which are but the elements of scientific language, is determined. To a certain extent these elements are arbitrary. Their interpretation is purely conventional: we are permitted to employ them in whatever sense we please. But this permission is limited by two indispensable conditions,—first, that from the sense once conventionally established we never, in the same process of reasoning, depart; secondly, that the laws by which the process is conducted be founded exclusively upon the above fixed sense or meaning of the symbols employed. In accordance with these principles, any agreement which may be established between the laws of the symbols of Logic and those of Algebra can but issue in an agreement of processes. The two provinces of interpretation remain apart and independent, each subject to its own laws and conditions.

Now the actual investigations of the following pages exhibit Logic, in its practical aspect, as a system of processes carried on by the aid of symbols having a definite interpretation, and subject to laws founded upon that interpretation alone. But at the same time they exhibit those laws as identical in form with the laws of the general symbols of algebra, with this single addition, viz., that the symbols of Logic are further subject to a special law (Chap. II.), to which the