

Philip Phillips

Advanced Solid State Physics

Second Edition

高等固体物理学 第2版

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Advanced Solid State Physics

Second Edition

Providing an up-to-date and lucid presentation of phenomena across modern advanced-level solid state physics, this new edition builds on an elementary understanding to introduce students to the key research topics with the minimum of mathematics. It covers cutting-edge topics, including electron transport and magnetism in solids. It is the first book to explain topological insulators and strongly correlated electrons.

Explaining solid state physics in a clear and detailed way, it also has over 50 exercises for students to test their knowledge. In addition to the extensive discussion of magnetic impurity problems, bosonization, quantum phase transitions, and disordered systems from the first edition, the new edition includes such topics as topological insulators, high-temperature superconductivity and Mott insulators, renormalization group for Fermi liquids, spontaneous symmetry breaking, zero and finite-temperature Green functions, and the Kubo formalism.

Philip Phillips is Professor of Physics at the University of Illinois. As a theoretical condensed matter physicist he has an international reputation for his work on transport in disordered and strongly correlated low-dimensional systems.

Cover illustration: phase diagram of the disordered quantum XY model. The arrows indicate the phase of the Cooper pairs (balls with springs). The yellow region represents a superconductor, the darker blue a glassy phase, and the lighter blue a phase-disordered insulator. The vertical axis represents the temperature while the in-plane axes represent the disorder and magnetic field strengths.

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To Orestes and Angeliki

“The scientists of today think deeply instead of clearly. One must be sane to think clearly, but one can think deeply and be quite insane.”

Nikola Tesla, July 1934.

Preface

In writing the second edition of this text, I have tried to accomplish three things. First, correct all the typos in the first edition. This has turned out to be somewhat harder than I had anticipated. While I am certain my proofreaders and I corrected all mistakes we could find, that might not have been sufficient. As there will undoubtedly be a second printing, simply email me any errors you might find at dimer@illinois.edu. Second, include all the material that should have been in the first edition but that I had given up on writing. This includes Green functions, Luttinger's theorem, renormalization of short-range interactions for Fermi liquids, and symmetry. In keeping with this being a physics rather than a technique or mathematics tract, these subjects are interwoven wherever they are first needed. For example, the section on Green functions is in Chapter 7 where the Anderson impurity problem is treated. For completeness, Luttinger's theorem is also presented in the same chapter but in an appendix. Third, include new material that reflects the fast-moving pace of $\hbar = 1$ research in condensed matter physics. Here I made a judgement based on what I anticipate students would find most useful. Since there are no texts that present the pedagogy of topological insulators (though some excellent review articles exist) and Mott insulators, I chose to focus on those topics. In writing the topological insulator section, I have tried to stick to the formulations that require the fewest definitions and new concepts since the physics of these systems is inherently simple. Regarding the Mott problem, I present what I think is non-controversial but not written down anywhere in a single manuscript. Chapter 16 starts with the band insulator in which the rigid-band picture is valid and then demonstrates that the physics of the Mott problem stands apart because no such rigid-band picture applies. While tomes have been written about rigid-band models, no text deals with the breakdown of the rigid-band picture in strongly correlated electron problems. The cuprate problem is discussed in this context. I had also intended to write a chapter on quantum computing and extend the discussion in Chapter 14 to include the Bose-Hubbard model. However, including such topics would have pushed the page count well over 600 pages, thereby making the book unwieldy. Further, such topics are not, in my estimation, particularly suited to a core second-semester graduate class but rather to a more specialized course. Perhaps I will think differently in a few years.

I have benefitted from much input in the final editing of the current manuscript. Babak Seradjeh, Juan Jottar, and Taylor Hughes offered invaluable critiques of the topological insulator section. Wei-Cheng Lee, Mohammad Edalati, and Taylor Hughes also read the Mott chapter and caught several typos and inaccuracies. I also thank Mohammad for reading and correcting the chapter on symmetries and Robert Leigh for his characteristically level-headed and incisive remarks on strong coupling physics and symmetry. Wade deGottardi offered numerous suggestions on the bosonization chapter. While I received emails from

several students around the world detailing the typographical errors they have caught, I would especially like to thank Wei Han who found two key typos in two figures from the first edition. Many thanks to Taylor Hughes for redrawing these figures. The duty of proof-reading fell on my research group and other members of the ICMT group at Urbana whose arms are still recovering from the non-adiabatic distortions I applied to them. These include Wei-Cheng Lee, Mohammad Edalati, Seungmin Hong, Wade deGottardi, Rodrigo Garrido, and Kiaran Dave. In addition, at the proof stage, Kiaran Dave, Ka Wai Lo, and Huihuo Zheng read the entire manuscript and corrected it assiduously, in their relentless drive to eliminate all typographical errors. I would like to thank Matthew Feickert for converting the LaTeX files to the Cambridge style and for spotting several typographical errors along the way, and the Cambridge staff, Mike Nugent, Simon Capelin, Claire Poole, Abigail Jones and Frances Nex for their dedication to this project. Early influences without which this book might not have been possible include my high school English teacher, Duane Kusler, who encouraged me to write and my twin sisters Andi and Lyndi from whom I learned many math tricks. My endearing thanks go to my family for their support and calming presence.

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Solid state physics grew out of applications of quantum mechanics to the problem of electron conduction in solids. This seemingly simple problem defied solution because the presence of an ion at each lattice site seemed to present an obvious impediment to conduction. How the electrons avoid the ions was thus the basic question. Although the answer to this question is well known, it does serve to illuminate the very essence of solid state physics: there is organization in the many. Each electron adjusts its wavelength to take advantage of the periodicity of the lattice. In the absence of impurities, conduction is perfect. Hence, by understanding this simple fact that periodicity implies perfect conduction, it became clear that the experimentally observed resistivity in a metal came not from electrons running into each of the ions but rather from dirt (disorder), thermal effects mediated by dynamical motion of the ions, or electron–electron interactions. This book examines each of these effects with an eye for identifying underlying organizing principles that simplify the physics of such interactions.

1.1 Spontaneously broken symmetry

The search for organizing principles that help simplify the physics of many-body systems is at the heart of modern solid state or, more generally, condensed matter physics. One such tool is symmetry. Consider the simple case of permutation symmetry typically taught in a first class in quantum mechanics. This symmetry was introduced into quantum mechanics by W. Heisenberg in the context of the indistinguishability of identical particles. The permutation group has a finite number of elements and hence is associated with a discrete symmetry. Permutation symmetry allows us to classify fundamental particles into two groups. Bosons are even with respect to interchange of two particles and fermions odd. This symmetry can be generalized to include a non-integer phase when two particles are interchanged, as we will see in the context of the fractional quantum Hall effect.

To a large extent, the symmetries that are most relevant in condensed matter systems are typically continuous, for example rotational symmetry. Spontaneously breaking a continuous symmetry has a fundamental consequence. For example, the existence of phonons in a solid or spin waves in a magnet follows from the spontaneous breaking of a continuous symmetry. By spontaneous, we mean without the application of an external field. A periodic arrangement of ions in a crystal breaks continuous translational and rotational symmetry. Such spontaneous breaking of a continuous symmetry by the very existence of the lattice is necessarily accompanied by a massless spinless bosonic excitation. That such massless

spinless bosons, known as Nambu–Goldstone bosons (G1961; N1960), necessarily accompany the breaking of a continuous symmetry can easily be deduced from the following considerations. We consider a system with a Lagrangian

$$\mathcal{L} = T - V(\phi), \quad (1.1)$$

consisting of a kinetic energy, T , and a potential energy, $V(\phi)$, where we are allowing for ϕ to be a complex function. The claim that such a system is invariant under a symmetry operation is captured by

$$V(\phi) = V(\phi + \epsilon \delta\phi), \quad (1.2)$$

where $\epsilon \delta\phi$ is the generator of the symmetry operation. Here ϵ is an infinitesimal. We have assumed for the moment that $\delta\phi$ is independent of space. To illustrate what is meant by this identity, consider a potential of the form $V(\phi) = \epsilon_0 |\phi|^2$. This potential is invariant under transformations of the form $\phi \rightarrow \phi e^{i\theta}$. Let θ be a small quantity completely independent of space. Then we can expand the exponential and retain only the first-order term. Consequently, $\phi \rightarrow \phi(1 + i\theta)$ and we identify $\epsilon \delta\phi$ as $i\theta\phi$; that is, $\epsilon = \theta$ and $\delta\phi = i\phi$. This symmetry, known as $U(1)$, is present in models that preserve charge conservation. Expansion of $V(\phi)$ to linear order in ϵ implies that

$$\delta\phi \frac{\delta V}{\delta\phi} = 0, \quad (1.3)$$

assuming that the symmetry is intact. Now assume explicitly that the symmetry is broken such that $V \rightarrow V(\phi_0 + \chi)$, where ϕ_0 minimizes the potential and χ cannot be written as a generator of a symmetry operation as in Eq. (1.2). Since the potential has a minimum, it makes sense to expand

$$V(\phi_0 + \chi) = V(\phi_0) + \frac{1}{2} \chi^2 \left. \frac{\partial^2 V}{\partial \phi^2} \right|_{\phi=\phi_0} = V(\phi_0) + \frac{1}{2} \chi^2 m^2, \quad (1.4)$$

truncating at the restoring term at second order. The second term, which can be used to define the mass (m) in a standard harmonic expansion, is inherently positive semi-definite since we have expanded about the minimum. With this equation in hand, we differentiate Eq. (1.3),

$$\frac{\partial \delta\phi}{\partial \phi} \frac{\delta V}{\delta\phi} + \delta\phi \frac{\partial^2 V}{\partial \phi^2} = 0, \quad (1.5)$$

with respect to ϕ . The first term vanishes when evaluated at the minimum, implying that

$$\delta\phi \left. \frac{\partial^2 V}{\partial \phi^2} \right|_{\phi=\phi_0} = 0 \quad (1.6)$$

must identically vanish for any variation of ϕ in the broken symmetry state. Since $\delta\phi$ is non-zero, Eq. (1.6) is satisfied only if the second-order-derivative term vanishes or equivalently if $m^2 = 0$. That is, the mass vanishes. This is Goldstone's theorem (G1961). A zero mode exists for each generator of a continuously broken symmetry. As a result of this theorem, symmetry occupies a central place in all areas of physics, in particular particle and condensed matter physics. Typically, the massless bosons that arise in condensed matter systems represent collective excitations of the entire many-body system. In fluids, phonons are purely longitudinal and arise from spontaneous breaking of Galilean invariance. In solids, phonons are both transverse and longitudinal, though with no simple correspondence with the spontaneous breaking of Galilean, translational, and rotational symmetry. In magnets, spin waves or magnons are the collective gapless excitations that emerge from the spontaneous breaking of rotational symmetry.

We can of course relax the constraint that θ be independent of space. In so doing, we can entertain what happens under local rather than global (θ independent of space) transformations. While our analysis on the potential energy remains the same, the kinetic energy,

$$T \rightarrow \frac{1}{2}(\partial_\mu \phi^*)(\partial^\mu \phi) + \frac{1}{2}|\phi|^2(\partial_\mu \theta(x))^2, \quad (1.7)$$

does acquire a new term describing the spatial variation of the phase. If the U(1) symmetry is not broken by this transformation, then the second term must vanish. Demanding that

$$\partial_\mu \theta = 0 \quad (1.8)$$

requires that θ be spatially homogeneous for the symmetry to be preserved. As a result, a consequence of breaking the continuous U(1) symmetry is that θ must be spatially non-uniform. This is the situation in a superconductor. In fact, the current inside a superconductor arises entirely from the spatial variation of the phase, as can be seen from the quantum mechanical equation for the current,

$$j_\mu = \frac{e\hbar}{m} \text{Im} \psi^\dagger \partial_\mu \psi = \frac{e^* \hbar}{m} |\Delta|^2 \partial_\mu \theta, \quad (1.9)$$

if we interpret ψ as the wavefunction for the superconducting state; that is, $\psi = \Delta e^{i\theta}$. We will see in the chapter on superconductivity precisely how this state of affairs arises. We will interpret ψ as the order parameter of a superconducting state. While the Bardeen–Cooper–Schrieffer (BCS) theory of superconductivity was certainly not formulated as an example of a broken continuous symmetry, this is the basic principle that underlies this theory. In fact, the key ingredients of superconductivity, charge $2e$ carriers and a supercurrent, all follow from breaking U(1) symmetry.

Massless bosons that emerge from broken symmetry typically generate new unexpected physics. For example, phonons mediate pairing between electrons, thereby driving the onset of superconductivity in metals such as Hg and more complicated systems, for example MgB₂. However, strict rules determine how such Nambu–Goldstone bosons can affect

any system. As shown by Adler (A1965), the interactions induced by massless bosons arising from the breaking of a continuous symmetry must be proportional to the transferred momentum. More formally, interactions mediated by the exchange of a Nambu–Goldstone boson can only obtain through derivative couplings. Consequently, the interaction vanishes for zero exchanged momentum. This principle implies that the electron–phonon interaction which mediates pairing in elemental superconductors is inherently dynamical in nature. We will verify this important principle in the context of the electron–phonon coupling through an explicit derivation. Hence, entirely from the existence of a lattice, phonons and the kinds of interactions they mediate can be easily deduced.

1.2 Tracking broken symmetry: order parameter

The idea of an order parameter is another powerful concept in condensed matter physics. Order parameters track broken symmetry. That is, they are non-zero in the broken symmetry phase and zero otherwise. Consider a ferromagnet. Locally each spin can point along any direction. This is the case at high temperature in which no symmetry is broken. In a phase transition controlled by thermal fluctuations, typically it is the high-temperature phase that has the higher symmetry. To quantify the order in a collection of spins, we sum the z -component of each of the spin operators,

$$M = \frac{1}{N} \sum_i \langle S_i^z \rangle, \quad (1.10)$$

scaled by the number of spins, N . Here S_i^z is the z -component of the spin of the atom on site i and the angle brackets indicate a thermal average over the states of the system. M is the magnetization. At high temperature before any symmetry is broken, the magnetization is identically zero. At sufficiently low temperatures, the spins order and the magnetization acquires a non-zero value. Consider iron for which the Curie or ordering temperature is 1340 K. It turns out that most parts of a block of iron below the magnetization temperature have vanishing magnetization. This state of affairs obtains because the magnetization is in general a function of space. As a result, a block of iron does not break the symmetry uniformly. In fact, the actual magnetization in bulk magnets is not acquired spontaneously but rather by some external means to align all of the individual magnetic domains. At the boundary of a domain, the magnetization changes sign, creating a domain wall. Typical domain sizes in iron are roughly 300 ions. Placing a chunk of Fe in a magnetic field will orient all of the domains in the same direction, a state of affairs that will persist long after the field is turned off. This is important since the re-oriented domain state does not constitute a minimum energy state of the system. The domains lock into place by becoming pinned to defects. One would expect then that as the magnetizing field is varied, the magnetization would not change continuously but by discontinuous jumps as domain walls de-pin from defects. This is the essence of the Barkhausen effect, the tiny discontinuous jumps the magnetization makes in the presence of an external magnetic field and ultimately the reason why the magnetization curve in a ferromagnet exhibits hysteresis as depicted in Fig. 1.1.