HEP World's Classics

General Investigations of Curved Surfaces of 1827 and 1825

曲面的一般研究

(1827 年和 1825 年)

CARL FRIEDRICH GAUSS

TRANSLATED WITH NOTES AND A BIBLIOGRAPHY BY
JAMES CADDALL MOREHEAD AND
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内容简介

“数学王子”高斯在对大地测量的研究中创立了关于曲面的新的理论，并于 1827 年写成了这一领域的光辉著作《曲面的一般研究》。本书全面阐述了三维空间中的曲面微分几何，并开创了内蕴曲面理论。书中一系列的概念和定理充分而完整地反映了高斯的微分几何观念，远远超越了前辈欧拉在这一领域所作的工作，决定了这一学科以后的发展方向。这一理论后来被黎曼所发展，并成为了爱因斯坦广义相对论的基础。陈省身先生评价道：“微分几何的始祖是 C. F. 高斯。他的曲面论建立了曲面的第一基本形式所奠定的几何，并把欧氏几何推广到曲面上‘弯曲’的几何。”

本书可供所有喜爱数学和数学发展历史的读者阅读，也可供专业研究学者参考。

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HEP World's Classics

There is a Chinese saying: "It is beneficial to open any book." It is even more fruitful to open and read classic books. The world is keeping on changing, but really fundamental and essential things stay the same since there is nothing new under the sun. Great ideas have been discovered and re-discovered, and they should be learnt and re-learnt. Classic books are our inheritance from all the previous generations and contain the best of knowledge and wisdom of all the people before us. They are timeless and universal. We cannot travel back in time, but we can converse with the originators of current theories through reading their books. Classic books have withstood the test of time. They are reliable and contain a wealth of original ideas. More importantly, they are also books which have not finished what they wanted or hoped to say. Consequently, they contain unearthed treasures and hidden seeds of new theories, which are waiting to be discovered. As it is often said: history is today. Proper understanding of the past work of giants is necessary to carry out properly the current and future researches and to make them to be a part of the history of science and mathematics. Reading classics books is not easy, but it is rewarding. Some modern interpretations and beautiful reformulations of the classics often miss the subtle and crucial points. Reading classics is also more than only accumulating knowledge, and the reader can learn from masters on how they asked questions, how they struggled to come up with new notions and theories to overcome problems, and answers to questions. Above all, probably the best reason to open classic books is the curiosity: what did people know, how did they express and communicate them, why did they do what they did? It can simply be fun!

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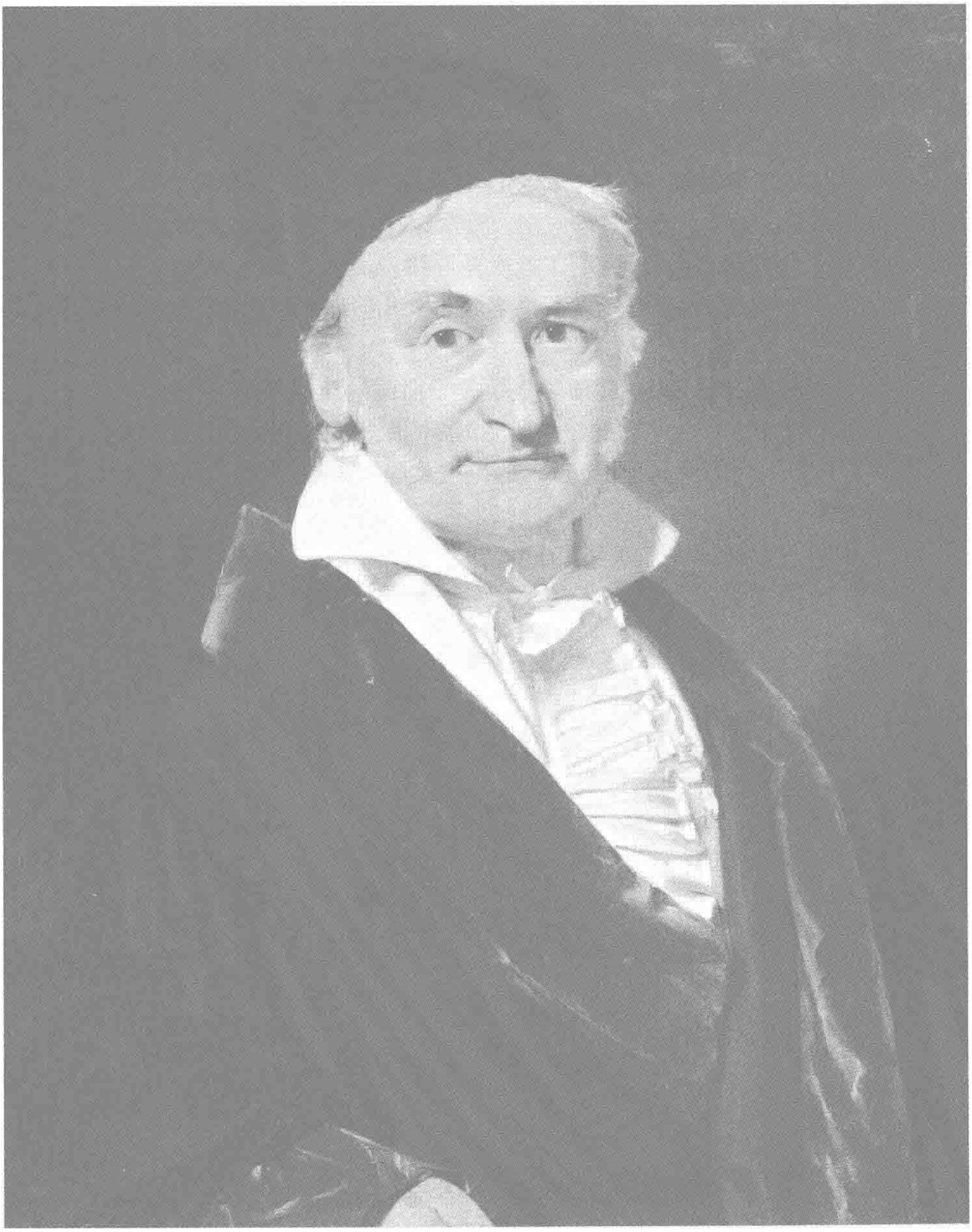

Carl Friedrich Gauss (1777—1855)

高斯 (Carl Friedrich Gauss, 1777—1855), 德国数学家、天文学家和物理学家, 近代数学奠基者之一; 他与阿基米德、牛顿同享盛名, 并列为世界三大数学家, 并享有 “数学王子” 的美誉。

高斯在数论、代数、分析、非欧几何、微分几何、超几何级数、复变函数、椭圆函数等数学的诸多领域都做出了开创性的贡献, 其成就还遍及天文学、大地测量学、地球物理学、力学、光学、静电学、磁学等其他科学分支。高斯的主要数学成就包括证明了代数基本定理, 发明了二次互反律、最小二乘法, 解决了两千年来悬而未决的尺规作图难题, 并且发现了重要的非欧几何, 尽管他在生前并没有发表这一成果。

在对大地测量的研究中, 高斯创立了关于曲面的新的理论, 并于 1827 年写成了这一领域的光辉著作《曲面的一般研究》。本书全面阐述了三维空间中的曲面微分几何, 并开创了内蕴曲面理论; 这一理论被黎曼所发展, 并成为了爱因斯坦广义相对论的基础。

KARL FRIEDRICH GAUSS

GENERAL INVESTIGATIONS

OF

CURVED SURFACES

OF

1827 AND 1825

TRANSLATED WITH NOTES

AND A

BIBLIOGRAPHY

BY

JAMES CADDALL MOREHEAD, A.M., M.S., AND ADAM MILLER HILTEBEITEL, A.M.

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The title page of the original version

INTRODUCTION

In 1827 Gauss presented to the Royal Society of Göttingen his important paper [iii] on the theory of surfaces, which seventy-three years afterward the eminent French geometer, who has done more than any one else to propagate these principles, characterizes as one of Gauss's chief titles to fame, and as still the most finished and useful introduction to the study of infinitesimal geometry[1]. This memoir may be called: General Investigations of Curved Surfaces, or the Paper of 1827, to distinguish it from the original draft written out in 1825, but not published until 1900. A list of the editions and translations of the Paper of 1827 follows. There are three editions in Latin, two translations into French, and two into German. The paper was originally published in Latin under the title:

I*a*. Disquisitiones generales circa superficies curvas
auctore Carolo Friderico Gauss
Societati regiæ oblatæ D. 8. Octob. 1827,

and was printed in: Commentationes societatis regiæ scientiarum Gottingensis recentiores, Commentationes classis mathematicæ. Tom. VI. (ad a. 1823–1827). Gottingæ, 1828, pages 99–146. This sixth volume is rare; so much so, indeed, that the British Museum Catalogue indicates that it is missing in that collection. With the signatures changed, and the paging changed to pages 1–50, I*a* also appears with the title page added:

I*b*. Disquisitiones generales circa superficies curvas
auctore Carolo Friderico Gauss.
Gottingæ. Typis Dieterichianis. 1828.

II. In Monge's Application de l'analyse à la géométrie, fifth edition, edited by Liouville, Paris, 1850, on pages 505–546, is a reprint, added by the Editor, in Latin under the title: Recherches sur la théorie générale des surfaces courbes; Par M. C.-F. Gauss.

III*a*. A third Latin edition of this paper stands in: Gauss, Werke, Her- [iv] ausgegeben von der Königlichen Gesellschaft der Wissenschaften zu Göttingen, Vol. 4, Göttingen, 1873, pages 217–258, without change of the title of the original paper (I*a*).

III*b*. The same, without change, in Vol. 4 of Gauss, Werke, Zweiter Abdruck, Göttingen, 1880.

IV. A French translation was made from Liouville's edition, II, by Captain Tiburce Abadie, ancien élève de l'École Polytechnique, and appears in Nouvelles

[1] G. Darboux, Bulletin des Sciences Math. Ser. 2, vol. 24, page 278, 1900.

Annales de Mathématique, Vol. 11, Paris, 1852, pages 195–252, under the title: Recherches générales sur les surfaces courbes; Par M. Gauss. This latter also appears under its own title.

V*a*. Another French translation is: Recherches Générales sur les Surfaces Courbes. Par M. C.-F. Gauss, traduites en français, suivies de notes et d'études sur divers points de la Théorie des Surfaces et sur certaines classes de Courbes, par M. E. Roger, Paris, 1855.

V*b*. The same. Deuxième Édition. Grenoble (or Paris), 1870 (or 1871), 160 pages.

VI. A German translation is the first portion of the second part, namely, pages 198–232, of: Otto Böklen, Analytische Geometrie des Raumes, Zweite Auflage, Stuttgart, 1884, under the title (on page 198): Untersuchungen über die allgemeine Theorie der krummen Flächen. Von C. F. Gauss. On the title page of the book the second part stands as: Disquisitiones generales circa superficies curvas von C. F. Gauss, ins Deutsche übertragen mit Anwendungen und Zusätzen....

VII*a*. A second German translation is No. 5 of Ostwald's Klassiker der exacten Wissenschaften: Allgemeine Flächentheorie (Disquisitiones generales circa superficies curvas) von Carl Friedrich Gauss (1827). Deutsch herausgegeben von A. Wangerin. Leipzig, 1889. 62 pages.

VII*b*. The same. Zweite revidirte Auflage. Leipzig, 1900. 64 pages.

The English translation of the Paper of 1827 here given is from a copy of the original paper, I*a*; but in the preparation of the translation and the notes all the other editions, except V*a*, were at hand, and were used. The excellent edition of Professor Wangerin, VII, has been used throughout most freely for the text and notes, even when special notice of this is not made. It has been the endeavor of the translators to retain as far as possible the notation, the form and punctuation of the formulæ, and the general style of the original papers. Some changes have been made in order to conform to more recent notations, and the most important of these are mentioned in the notes.

[v] The second paper, the translation of which is here given, is the abstract (Anzeige) which Gauss presented in German to the Royal Society of Göttingen, and which was published in the Göttingische gelehrte Anzeigen. Stück 177. Pages 1761–1768. 1827. November 5. It has been translated into English from pages 341–347 of the fourth volume of Gauss's Works. This abstract is in the nature of a note on the Paper of 1827, and is printed before the notes on that paper.

Recently the eighth volume of Gauss's Works has appeared. This contains on pages 408–442 the paper which Gauss wrote out, but did not publish, in 1825. This paper may be called the New General Investigations of Curved Surfaces, or the Paper of 1825, to distinguish it from the Paper of 1827. The Paper of 1825 shows the manner in which many of the ideas were evolved, and while incomplete and in some cases inconsistent, nevertheless, when taken in connection with the Paper of 1827, shows the development of these ideas in the mind of Gauss. In both papers are found the method of the spherical representation, and, as types,

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the three important theorems: The measure of curvature is equal to the product of the reciprocals of the principal radii of curvature of the surface, The measure of curvature remains unchanged by a mere bending of the surface, The excess of the sum of the angles of a geodesic triangle is measured by the area of the corresponding triangle on the auxiliary sphere. But in the Paper of 1825 the first six sections, more than one-fifth of the whole paper, take up the consideration of theorems on curvature in a plane, as an introduction, before the ideas are used in space; whereas the Paper of 1827 takes up these ideas for space only. Moreover, while Gauss introduces the geodesic polar coordinates in the Paper of 1825, in the Paper of 1827 he uses the general coordinates, p, q, thus introducing a new method, as well as employing the principles used by Monge and others.

The publication of this translation has been made possible by the liberality of the Princeton Library Publishing Association and of the Alumni of the University who founded the Mathematical Seminary.

H. D. THOMPSON

MATHEMATICAL SEMINARY,
PRINCETON UNIVERSITY LIBRARY,
January 29, 1902.

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DISQUISITIONES GENERALES [1]

CIRCA

SUPERFICIES CURVAS

AUCTORE

CAROLO FRIDERICO GAUSS

SOCIETATI REGIAE OBLATAE D. 8. OCTOB. 1827

COMMENTATIONES SOCIETATIS REGIAE SCIENTIARUM
GOTTINGENSIS RECENTIORES. VOL. VI. GOTTINGAE MDCCCXXVIII

GOTTINGAE
TYPIS DIETERICHIANIS
MDCCCXXVIII

GENERAL INVESTIGATIONS OF CURVED SURFACES [3]

BY

CARL FRIEDRICH GAUSS

PRESENTED TO THE ROYAL SOCIETY, OCTOBER 8, 1827

1.

Investigations, in which the directions of various straight lines in space are to be considered, attain a high degree of clearness and simplicity if we employ, as an auxiliary, a sphere of unit radius described about an arbitrary centre, and suppose the different points of the sphere to represent the directions of straight lines parallel to the radii ending at these points. As the position of every point in space is determined by three coordinates, that is to say, the distances of the point from three mutually perpendicular fixed planes, it is necessary to consider, first of all, the directions of the axes perpendicular to these planes. The points on the sphere, which represent these directions, we shall denote by (1), (2), (3). The distance of any one of these points from either of the other two will be a quadrant; and we shall suppose that the directions of the axes are those in which the corresponding coordinates increase.

2.

It will be advantageous to bring together here some propositions which are frequently used in questions of this kind.

I. The angle between two intersecting straight lines is measured by the arc between the points on the sphere which correspond to the directions of the lines.

II. The orientation of any plane whatever can be represented by the great circle on the sphere, the plane of which is parallel to the given plane.

III. The angle between two planes is equal to the spherical angle between the [4]
great circles representing them, and, consequently, is also measured by the arc

intercepted between the poles of these great circles. And, in like manner, the angle of inclination of a straight line to a plane is measured by the arc drawn from the point which corresponds to the direction of the line, perpendicular to the great circle which represents the orientation of the plane.

IV. Letting x, y, z; x', y', z' denote the coordinates of two points, r the distance between them, and L the point on the sphere which represents the direction of the line drawn from the first point to the second, we shall have

$$x' = x + r\cos(1)L,$$
$$y' = y + r\cos(2)L,$$
$$z' = z + r\cos(3)L.$$

V. From this it follows at once that, generally,

$$\cos^2(1)L + \cos^2(2)L + \cos^2(3)L = 1,$$

and also, if L' denote any other point on the sphere,

$$\cos(1)L\cdot\cos(1)L' + \cos(2)L\cdot\cos(2)L' + \cos(3)L\cdot\cos(3)L' = \cos LL'.$$

VI. THEOREM. *If L, L', L'', L''' denote four points on the sphere, and A the angle which the arcs LL', $L''L'''$ make at their point of intersection, then we shall have*

$$\cos LL''\cdot\cos L'L''' - \cos LL'''\cdot\cos L'L'' = \sin LL'\cdot\sin L''L'''\cdot\cos A.$$

Demonstration. Let A denote also the point of intersection itself, and set

$$AL = t,\quad AL' = t',\quad AL'' = t'',\quad AL''' = t'''.$$

Then we shall have

$$\begin{aligned}
\cos LL'' &= \cos t\,\cos t'' + \sin t\,\sin t''\cos A,\\
\cos L'L''' &= \cos t'\cos t''' + \sin t'\sin t'''\cos A,\\
\cos LL''' &= \cos t\,\cos t''' + \sin t\,\sin t'''\cos A,\\
\cos L'L'' &= \cos t'\cos t'' + \sin t'\sin t''\cos A;
\end{aligned}$$

and consequently,

$$\begin{aligned}
&\cos LL''\cdot\cos L'L''' - \cos LL'''\cdot\cos L'L''\\
&= \cos A(\cos t\cos t''\sin t'\sin t''' + \cos t'\cos t'''\sin t\sin t''\\
&\quad - \cos t\cos t'''\sin t'\sin t'' - \cos t'\cos t''\sin t\sin t''')\\
&= \cos A(\cos t\sin t' - \sin t\cos t')(\cos t''\sin t''' - \sin t''\cos t''')\\
&= \cos A\cdot\sin(t'-t)\cdot\sin(t'''-t'')\\
&= \cos A\cdot\sin LL'\cdot\sin L''L'''.
\end{aligned}$$

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