高等学校教材 26

微积分（英文版）

（第2卷）

CALCULUS（II）

张宇 黄艳 主编

哈爾濱工業大學出版社
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内 容 简 介

本书为《微积分》一书的第二卷，适用于工科院校非数学专业本科新生，亦可作为工程技术人员的参考书籍. 本卷包含四个章节，内容涵盖多元函数微分学，多元函数积分学，第二型曲线积分、第二型曲面积分及无穷级数. 本书包含大量例题及习题.

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Preface

Mathematics is one of the most important and wide applied science. Calculus is the branch of mathematics which studies the change of quantities. Modern calculus was developed in 17th century Europe by Newton and Leibniz. It is one of the most fundamental and well studied branch of mathematics. Calculus has comprehensive application in Engineering, economics, business and social and life sciences. Calculus is a part of the modern mathematics education and is important to the students for their future career pursuing.

There are a lot of excellent calculus text books in Chinese which are suitable for the newly college students major in engineering, economics and so on. The authors of this book are very experienced in teaching college calculus, especially in teaching in English for classy students and oversea students. The authors had some trouble in finding a suitable text book in English. Although there are some famous and popular calculus textbooks in English, but because of the uniqueness of our education system, none of them fits our requirement. The publishment of this book will fill in this blank.

This book is dedicate to college freshmen major in engineering, business, economic and so on. It can also be used as a reference for technicians. Some of the key features of this book are:

1. Abundant theories. This book not only contains necessary theories for students of technology, but also contains some classic theories of mathematical analysis for science students. It gives the students more solid foundation of mathematics.

2. Comprehensive examples and exercises. This book contains plentiful of examples and exercises. The examples and exercises are carefully graded, progressing from basic conceptual exercises and skill-development problems to more challenging problems involving applications and proofs. Many of the exercises and examples are related to real-world phenomena.

3. Easy to look up. This book is well organised and easy to look up for theories, it can be used as a reference.

During the preparation of this book, a lot of people gave us a great deal of help. Our colleagues who are all very experienced gave us a lot of helpful suggestions, the editors offered us a lot of assistance. Great thanks to all of these people who helped us to make this book possible.

Authors

January 15th, 2016

Contents

Chapter 8 Differential Calculus of Multivariable Functions

Functions with two or more independent variables appear more often in science than functions of a single variable. In this chapter we extend the basic ideas of one variable differential calculus to such functions. With functions of several independent variables, we work with partial derivatives, which, in turn, give rise to directional derivatives and the gradient, some fundamental concepts in calculus. Partial derivatives allow us to find maximum and minimum values of multivariable functions. We define tangent planes, rather than tangent lines, that allow us to make linear approximations.

8.1 Limits and Continuity of Multivariable Functions

8.1.1 The n -Dimensional Space

If we introduce the rectangular coordinate system in space, then we have a one-to-one correspondence between points P in space and ordered triples (x, y, z). The set of all points described by the triples (x, y, z) is called the three-dimensional space. We denote this space by $\mathbf{R}^3$. Generally, we can consider an ordered n -tuple of real numbers $(x_1, x_2, \cdots, x_n)$ for any integer $n \geqslant 1$. Such an n -tuple is called an n -dimensional point, the individual numbers $x_1, x_2, \cdots, x_n$ being referred to as coordinates or components of the point. The set of all n -dimensional points is called the n -dimensional space. We denote this space by $\mathbf{R}^n$. We shall usually denote points by capital letters A, $B, C, \cdots$, and components by the corresponding small letters $a, b, c, \cdots$

The familiar formula for the distance between two points in $\mathbf{R}^3$ is easily extended to the n -dimentional formula. The distance $\rho(A, B)$ between the points $A(a_1, a_2, \cdots, a_n)$ and $B(b_1, b_2, \cdots, b_n)$ in $\mathbf{R}^n$ is

$$\rho(A,B) = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + \cdots + (a_n - b_n)^2}$$

Let $P_0 \in \mathbf{R}^n$ and $\delta > 0$, a set $\{P \mid \rho(P,P_0) < \delta, P \in \mathbf{R}^n\}$ is called a δ -neighborhood of the point P_0, denoted by $U_\delta(P_0)$ or $U(P_0,\delta)$. If we don't care about the size of δ , we often use "neighborhood" instead of " δ -neighborhood" of P_0, denoted by $U(P_0)$.

A point P_0 in a set E in $\mathbf{R}^n$ is an interior point of E if there exists a number $\delta > 0$ such that the δ -neighborhood $U_\delta(P_0)$ lies entirely in E . A point P_0 is a boundary point of E if every δ -neighborhood $U_\delta(P_0)$ contains points that lie outside of E as well as points that lie in E (Fig. 8.1). An interior point is necessarily a point of E . A boundary point of E needs not belong to E .

The interior points of a set make up the interior of the set. The set's boundary points make up its boundary. A set is open if it consists entirely of interior points. A set is closed if it contains all of its boundary points. A set is connected if every point can be connected to every other point by a smooth curve that lies entirely in the set. A connected open set is called an open region. The union of an open region and its boundary is called a closed region.

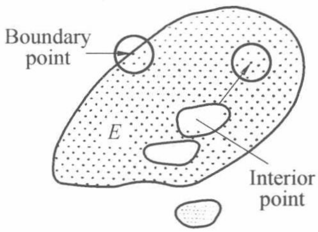

Fig. 8.1

A region in $\mathbf{R}^n$ is bounded if it lies inside a neighborhood of fixed radius. A region is unbounded if it is not bounded. For example, the set $\{(x,y,z) \mid x^2 + y^2 + z^2 \leqslant 1\}$ is a bounded region in $\mathbf{R}^3$, but the set $\{(x,y) \mid x + y > 0\}$ is an unbounded region in $\mathbf{R}^2$.

8.1.2 Functions of Several Variables

Many functions depend on more than one independent variable. The volume V of a right circular cylinder depends on its radius r and its height h . In fact, we know that $V = \pi r^2 h$. The temperature T at a point on the surface of the earth depends on the longitude x and latitude y of the point and on the time t , so we could write $T = f(x,y,t)$.

Definition 8.1 (Functions of Two Variables) A function f of two variables is a rule that assigns to each ordered pair of real numbers (x,y) in a set D a unique real number z , denoted by

$$z = f(x, y), (x, y) \in D$$

where x and y are called the independent variables and z is called the dependent variable. The set D is the domain of f and its range is the set of the values that f takes on, that is, $\{f(x,y) \mid (x,y) \in D\}$.

In general, a function f of n variables is a rule that assigns to each n-tuple $(x_1, x_2, \cdots, x_n)$ of real numbers a unique real number $u = f(x_1, x_2, \cdots, x_n)$. The variables x_1 to x_n are the independent variables and u is the dependent variable.

As with functions of one variable, a function of several variables may have a domain that is restricted by the context of the problem. For example, if the independent variables correspond to price or length or population, they take only nonnegative values, even though the associated function may be defined for negative values of the variables. If not stated otherwise, the domain D is the set of points for which the function is defined.

Example 1 The domain of the function $z = \ln(x + y)$ is $\{(x,y) \mid x + y > 0\}$, which is the set of the points that lie above the line $y = -x$. The domain is an unbounded open region in $\mathbf{R}^2$ (Fig. 8.2).

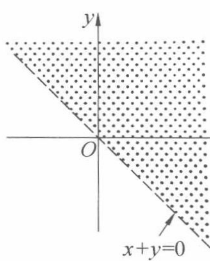

Fig. 8.2

Example 2 The domain of the function $z = \dfrac{\sqrt{2x - x^2 - y^2}}{\sqrt{x^2 + y^2 - 1}}$ is

$$\{(x,y) \mid (x-1)^2 + y^2 \leqslant 1,\ x^2 + y^2 > 1\}$$

which is the set of the points on or within the circle $(x-1)^2 + y^2 = 1$ and outside the circle $x^2 + y^2 = 1$. The domain is a bounded set in $\mathbf{R}^2$ (Fig. 8.3).

Example 3 The domain of the function $u = \sqrt{z - x^2 - y^2} + \arcsin(x^2 + y^2 + z^2)$ is

$$\{(x,y,z) \mid x^2 + y^2 \leqslant z,\ x^2 + y^2 + z^2 \leqslant 1\}$$

which is the set of the points that lie on or above the paraboloid $z = x^2 + y^2$ and on or within the sphere $x^2 + y^2 + z^2 = 1$. The domain is a bounded closed region in $\mathbf{R}^3$ (Fig. 8.4).

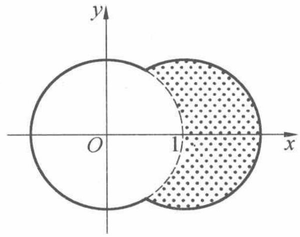

Fig. 8.3

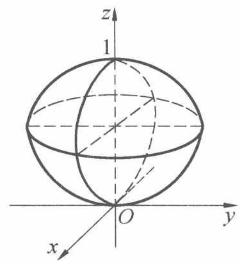

Fig. 8.4

The graph of a function f of two variables is the set of points (x,y,z) that satisfy the equation $z=f(x,y)$ i. e. the surface $z=f(x,y)$. But for functions of three or more independent variables, no geometric picture is available.

8.1.3 Limits and Continuity

This subsection deals with limits and continuity for multivariable functions. There are a number of differences between the calculus of one and two variables. However, the calculus of functions of three or more variables differs only slightly from that of functions of two variables. The study here will be limited largely to functions of two variables.

The following definition of limit for functions of two variables is analogous to the limit definition for functions of one variable.

Definition 8.2 Let f be a function of two variables whose domain D includes points arbitrarily close to $P_0(x_0,y_0)$. Then we say that the limit of $f(x,y)$ as $P(x,y)$ approaches $P_0(x_0,y_0)$ is A and we write

$$\lim_{(x,y)\to(x_0,y_0)} f(x,y) = \lim_{P\to P_0} f(P) = A$$

if for every number $\varepsilon > 0$, there is a corresponding number $\delta > 0$ such that

$$|f(x,y) - A| < \varepsilon$$

whenever $(x,y) \in D$ and $0 < \rho(P,P_0) = \sqrt{(x-x_0)^2 + (y-y_0)^2} < \delta$.

Other notations for the limit in Definition 8.2 are

$$\lim_{\substack{x\to x_0\\ y\to y_0}} f(x,y) = A \quad \text{and} \quad f(x,y) \to A \text{ as } (x,y) \to (x_0,y_0)$$

Example 4 Prove that $\lim\limits_{(x,y)\to(0,0)} (x^2+y^2)\sin\dfrac{1}{xy} = 0$.

Solution Let $\varepsilon > 0$ be given. We want to find $\delta > 0$ such that

$$\left|(x^2+y^2)\sin\frac{1}{xy}-0\right|<\varepsilon \quad \text{whenever} \quad 0<\sqrt{x^2+y^2}<\delta\ (xy\neq 0)$$

But

$$\left|(x^2+y^2)\sin\frac{1}{xy}-0\right|=(x^2+y^2)\left|\sin\frac{1}{xy}\right|\leqslant x^2+y^2$$

Thus, if we choose $\delta=\sqrt{\varepsilon}$ and let $0<\sqrt{x^2+y^2}<\delta\ (xy\neq 0)$, then

$$\left|(x^2+y^2)\sin\frac{1}{xy}-0\right|\leqslant x^2+y^2<\delta^2=\varepsilon$$

Thus, by Definition 8.2

$$\lim_{(x,y)\to(0,0)}(x^2+y^2)\sin\frac{1}{xy}=0$$

The condition $\rho(P,P_0)<\delta$ in Definition 8.2 means that the distance between $P(x,y)$ and $P_0(x_0,y_0)$ is less than δ as P approaches P_0 from all possible directions. Therefore, the limit exists only if $f(x,y)$ approaches A as P approaches P_0 along all possible paths in the domain of f. Thus, if we can find two different paths of approach along which the function $f(x,y)$ has different limits, then it follows that $\lim\limits_{(x,y)\to(x_0,y_0)}f(x,y)$ does not exist.

Example 5 Show that $\lim\limits_{(x,y)\to(0,0)}\dfrac{xy^2}{x^2+y^4}$ does not exist.

Solution Let $f(x,y)=\dfrac{xy^2}{x^2+y^4}$. First let $(x,y)\to(0,0)$ along any nonvertical line through the origin. Then $y=kx$, where k is the slope, and

$$f(x,y)=f(x,kx)=\frac{x\,(kx)^2}{x^2+(kx)^4}=\frac{k^2x}{1+k^4x^2}$$

So

$$\lim_{\substack{(x,y)\to(0,0)\\ y=kx}}f(x,y)=\lim_{x\to0}f(x,kx)=\lim_{x\to0}\frac{k^2x}{1+k^4x^2}=0$$

We now let $(x,y)\to(0,0)$ along the y-axis. Then $x=0$ and $f(0,y)=0$. So

$$\lim_{\substack{(x,y)\to(0,0)\\ x=0}}f(x,y)=\lim_{y\to0}f(0,y)=\lim_{y\to0}0=0$$

Thus, we have obtained identical limits along every lines through the origin. But that does not show that the given limit is 0. If we now let $(x,y)\to(0,0)$ along the parabola $x=y^2$, then we have

$$f(x,y)=f(y^2,y)=\frac{y^2\cdot y^2}{(y^2)^2+y^4}=\frac{1}{2}$$

So

$$\lim_{\substack{(x,y)\to(0,0)\\ x=y^2}} f(x,y) = \lim_{y\to 0} f(y^2,y) = \lim_{y\to 0}\frac{1}{2} = \frac{1}{2}$$

Since different paths lead to different limiting values, the given limit does not exist.

Just as for functions of one variable, the calculation of limits for functions of two (or more) variables can be greatly simplified by the use of properties of limits. The limit laws for functions of one variable can be extended to functions of two (or more) variables. The limit of a sum is the sum of the limits, the limit of a product is the product of the limits, and so on. The Squeeze Theorem also holds.

Example 6 $\lim\limits_{\substack{x\to 0\\ y\to a}} \frac{\sin(xy)}{x} = \lim\limits_{\substack{x\to 0\\ y\to a}} y \cdot \frac{\sin(xy)}{xy} = a \cdot 1 = a.$

Example 7 Evaluate $\lim\limits_{\substack{x\to 0\\ y\to 0}} \frac{2xy^2}{x^2+y^2-y^4}$.

Solution We use polar coordinates to find this limit. Let $x = r\cos\theta, y = r\sin\theta$. Then $(x,y)\to(0,0)$ is equivalent to $r\to 0$ and

$$\frac{2xy^2}{x^2+y^2-y^4} = \frac{2r\cos\theta\sin^2\theta}{1-r^2\sin^4\theta}$$

Since $\left|\frac{2r\cos\theta\sin^2\theta}{1-r^2\sin^4\theta}\right| < \frac{2r}{1-r^2}$ $(0<r<1)$ and $\lim\limits_{r\to 0}\frac{2r}{1-r^2} = 0$, by the Squeeze Theorem we have

$$\lim_{\substack{x\to 0\\ y\to 0}} \frac{2xy^2}{x^2+y^2-y^4} = \lim_{r\to 0}\frac{2r\cos\theta\sin^2\theta}{1-r^2\sin^4\theta} = 0$$

The definition of continuity for functions of two variables is essentially the same as for functions of one variable.

Definition 8.3 Let f be a function of two variables and assume that f is defined at the point $P_0(x_0,y_0)$ and there are points $P(x,y)$ in the domain of f arbitrarily close to (x_0,y_0). Then f is continuous at (x_0,y_0) if

$$\lim_{(x,y)\to(x_0,y_0)} f(x,y) = f(x_0,y_0)$$

If we write $\Delta z = f(P) - f(P_0)$ and $\rho = \rho(P,P_0)$, then the function $z = f(P)$ being continuous at $P_0(x_0,y_0)$ is equivalent to

$$\lim_{\rho\to 0}\Delta z = 0$$

We say that f is continuous on a set E if f is continuous at each point of E. A function f is continuous if it is continuous at every point of its domain.

The intuitive meaning of continuity is that if the point (x,y) changes by a small amount, then the value of $f(x,y)$ changes by a small amount. This means that a surface that is the graph of a continuous function has no hole or break.

Just as for functions of one variable, the sums, differences, products, quotients, and compositions of continuous functions of two (or more) variables are continuous on their domains.

A function of several variables built from a finite number of basic elementary functions of each independent variable through combinations and compositions, which may be represented by a single formula, is called an elementary function of several variables. Elementary functions of several variables are continuous on the interior of its domain. For example, the function $f(x,y)=\frac{xy}{1+x^2+y^2}$ is continuous on $\mathbf{R}^2$; the function $f(x,y)=\frac{xy^2}{x^2+y^4}$ is continuous except at $(0,0)$; the function $f(x,y)=\sin\frac{1}{1-x^2-y^2}$ is continuous except on the circle $x^2+y^2=1$.

Example 8 Determine the points at which the following function

$$f(x,y)=\begin{cases}(1+x^2+y^2)^{\frac{1}{x^2+y^2}}, & (x,y)\neq(0,0)\\ \mathrm{e}, & (x,y)=(0,0)\end{cases}$$

is continuous.

Solution The function f is continuous at any point $(x,y)\neq(0,0)$ since it is equal to an elementary function there. Also, we have

$$\lim_{(x,y)\to(0,0)}f(x,y)=\lim_{(x,y)\to(0,0)}(1+x^2+y^2)^{\frac{1}{x^2+y^2}}=\mathrm{e}=f(0,0)$$

Therefore, f is continuous at $(0,0)$, and so it is continuous on $\mathbf{R}^2$.

Example 9 Where is the following function

$$f(x,y)=\begin{cases}\frac{xy}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$$

continuous?

Solution The function f is continuous for $(x,y)\neq(0,0)$ because it is equal to an elementary function there.

At $(0,0)$, the value of f is defined, but f has no limit as $(x,y) \to (0,0)$. In fact, for every value of k, if we let $(x,y) \to (0,0)$ along the line $y = kx$, then we have

$$\lim_{\substack{(x,y)\to(0,0)\\ y=kx}} f(x,y) = \lim_{x\to 0} f(x,kx) = \lim_{x\to 0} \frac{x(kx)}{x^2 + (kx)^2} = \frac{k}{1+k^2}$$

This limit changes with k, so the limit $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ fails to exist, and the function f is not continuous at $(0,0)$.

Therefore, f is continuous on $\mathbf{R}^2$ except $(0,0)$.

Continuous functions of several variables on closed bounded sets have the following important properties:

1. If a function is continuous on a closed bounded set, then it attains an absolute maximum value and an absolute minimum value at some points in that set.

2. If a function is continuous on a closed bounded set, then it must take all values between its absolute minimum and absolute maximum values on that set.

8.2 Partial Derivatives and Higher-Order Partial Derivatives

8.2.1 Partial Derivatives

Derivatives may be defined for functions of several variables with respect to any of the independent variables. The resulting derivatives are called partial derivatives.

If f is a function of two variables x and y, suppose that we let only x vary while keeping y fixed, say $y = y_0$, where y_0 is a constant. If the function of one variable x, namely, $g(x) = f(x, y_0)$, has a derivative at x_0, then by the definition of a derivative, we have

$$g'(x_0) = \lim_{\Delta x\to 0} \frac{g(x_0+\Delta x) - g(x_0)}{\Delta x} = \lim_{\Delta x\to 0} \frac{f(x_0+\Delta x, y_0) - f(x_0,y_0)}{\Delta x}$$

We define the partial derivative of f with respect to x at the point (x_0, y_0) as the ordinary derivative of g at x_0.

Definition 8.4 The partial derivative of f with respect to x at the point (x_0, y_0), denoted by $f'_x(x_0, y_0)$, is

$$f'_x(x_0,y_0)=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x,y_0)-f(x_0,y_0)}{\Delta x}$$

provided this limit exists.

Similarly, the partial derivative of f with respect to y at the point (x_0,y_0), denoted by $f'_y(x_0,y_0)$, is defined by

$$f'_y(x_0,y_0)=\lim_{\Delta y\to 0}\frac{f(x_0,y_0+\Delta y)-f(x_0,y_0)}{\Delta y}$$

provided this limit exists.

If we now let the point (x_0,y_0) vary, then f'_x and f'_y become functions of two variables.

Definition 8.5 (Partial Derivative Functions) If f is a function of two variables, its partial derivatives are the functions f'_x and f'_y defined by

$$f'_x(x,y)=\lim_{\Delta x\to 0}\frac{f(x+\Delta x,y)-f(x,y)}{\Delta x}$$

$$f'_y(x,y)=\lim_{\Delta y\to 0}\frac{f(x,y+\Delta y)-f(x,y)}{\Delta y}$$

provided these limits exist.

The partial derivative of f with respect to x at the point (x_0,y_0) is the value of the function $f'_x(x,y)$ at (x_0,y_0), that is

$$f'_x(x_0,y_0)=f'_x(x,y)\big|_{(x_0,y_0)}$$

There are many alternative notations for partial derivatives. If $z=f(x,y)$, we write

$$f'_x(x,y)=z'_x=f'_1(x,y)=\frac{\partial f}{\partial x}=\frac{\partial z}{\partial x}=\frac{\partial}{\partial x}f(x,y)$$

$$f'_y(x,y)=z'_y=f'_2(x,y)=\frac{\partial f}{\partial y}=\frac{\partial z}{\partial y}=\frac{\partial}{\partial y}f(x,y)$$

In general, if u is a function of n variables, $u=f(x_1,x_2,\cdots,x_n)$, its partial derivatives with respect to the ith variable x_i is

$$\frac{\partial u}{\partial x_i}=\lim_{\Delta x_i\to 0}\frac{f(x_1,\cdots,x_{i-1},x_i+\Delta x_i,x_{i+1},\cdots,x_n)-f(x_1,\cdots,x_i,\cdots,x_n)}{\Delta x_i}$$

All the rules and results for ordinary derivatives can be used to compute partial derivatives. Specifically, to compute $f'_x(x,y)$, we treat y as a constant and take an ordinary derivative with respect to x. Similarly, to compute $f'_y(x,y)$, we treat x as a constant and take an ordinary derivative with respect to y.

Example 1 Find the two partial derivatives of the function $z = x^2y + \sin y$ at the point $(1,0)$.

Solution Holding y constant and differentiating with respect to x, we get

$$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}(x^2y + \sin y) = 2xy$$

and so

$$\left.\frac{\partial z}{\partial x}\right|_{(1,0)} = 2xy\Big|_{\substack{x=1\\y=0}} = 0$$

Holding x constant and differentiating with respect to y, we get

$$\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}(x^2y + \sin y) = x^2 + \cos y$$

and so

$$\left.\frac{\partial z}{\partial y}\right|_{(1,0)} = (x^2 + \cos y)\Big|_{\substack{x=1\\y=0}} = 2$$

Example 2 Find the three partial derivatives of the function $f(x,y,z) = (z - e^{xy})\sin \ln x^2$ at the point $(1,0,2)$.

Solution It is simpler to calculate the partial derivatives as follows

$$f'_x(1,0,2) = \frac{\mathrm{d}}{\mathrm{d}x}f(x,0,2)\Big|_{x=1} = \frac{\mathrm{d}}{\mathrm{d}x}(\sin \ln x^2)\Big|_{x=1} = \frac{2}{x}\cos \ln x^2\Big|_{x=1} = 2$$

$$f'_y(1,0,2) = \frac{\mathrm{d}}{\mathrm{d}y}f(1,y,2)\Big|_{y=0} = \frac{\mathrm{d}}{\mathrm{d}y}(0)\Big|_{y=0} = 0$$

$$f'_z(1,0,2) = \frac{\mathrm{d}}{\mathrm{d}z}f(1,0,z)\Big|_{z=2} = \frac{\mathrm{d}}{\mathrm{d}z}(0)\Big|_{z=2} = 0$$

Example 3 Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $z = x^y$ $(x > 0)$.

Solution The two partial derivatives are

$$\frac{\partial z}{\partial x} = yx^{y-1}, \frac{\partial z}{\partial y} = x^y \ln x$$

Example 4 If resistors of R_1, R_2 and R_3 ohms are connected in parallel to make an R-ohm resistor, the value of R can be found from the equation

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Find the value of $\frac{\partial R}{\partial R_2}$ when $R_1 = 30$, $R_2 = 45$ and $R_3 = 90$ ohms.