

Computational Mechanics Series

**Multiphysics Modeling:
Numerical Methods and
Engineering Applications**

**多物理场仿真：
数值方法及工程应用(英文版)**

张群 岑松 著
Qun Zhang Song Cen

清华大学出版社

Computational Mechanics Series

**Multiphysics Modeling:
Numerical Methods and
Engineering Applications**

**多物理场仿真：
数值方法及工程应用（英文版）**

张群 岑松 著

清华大学出版社
北京

内 容 简 介

本书为英文版。书中详细介绍了工程中常见的结构、流体、温度、电磁、噪声等物理场间的耦合问题,论述了强耦合方法、弱耦合方法以及跨程序之间的耦合方法。对不同耦合边界条件及其数值实现方法进行了深入讨论,涉及流-固耦合、流-固-温度场耦合、电磁-流-固-热耦合等复杂耦合问题。书中还列举了汽车工程、航空航天工程、旋转机械、微机电系统、生物医学工程等领域的 20 多个实际耦合分析案例。本书提供了从耦合理论、方法、软件开发到工程实践的较为全面综合的知识和工程实践体系,可供相关学科和行业的科研人员、工程技术人员、软件开发人员阅读参考,也可作为理工科相关课程的研究生教材。

For sale and distribution in the mainland of People's Republic of China exclusively.

此版本仅限于中国大陆地区销售。

本书海外版由清华大学出版社授权 Elsevier 在中国大陆以外地区出版发行:
ISBN 978-0-12-407709-6。

版权所有,侵权必究。侵权举报电话:010-62782989 13701121933

图书在版编目(CIP)数据

多物理场仿真:数值方法及工程应用 = Multiphysics Modeling: Numerical Methods and Engineering Applications: 英文/张群,岑松著. —北京:清华大学出版社,2016 (2016.11 重印)

(Computational Mechanics Series)

ISBN 978-7-302-45076-4

I. ①多… II. ①张… ②岑… III. ①物理模拟—英文 IV. ①O411.3

中国版本图书馆 CIP 数据核字(2016)第 222663 号

责任编辑:石磊

封面设计:常雪影

责任印制:沈露

出版发行:清华大学出版社

网 址: <http://www.tup.com.cn>, <http://www.wqbook.com>

地 址:北京清华大学学研大厦 A 座 邮 编:100084

社总机:010-62770175

邮 购:010-62786544

投稿与读者服务:010-62776969, c-service@tup.tsinghua.edu.cn

质量反馈:010-62772015, zhiliang@tup.tsinghua.edu.cn

印 装 者:虎彩印艺股份有限公司

经 销:全国新华书店

开 本:153mm × 235mm

印 张:27.25

版 次:2016 年 10 月第 1 版

印 次:2016 年 11 月第 2 次印刷

定 价:79.00 元

产品编号:051675-01

Preface

The readers of this book are the researchers and engineers who are interested in the numerical methodology study, code implementation and engineering applications in multidisciplinary problems. This book can also be used as a textbook for graduate students and high level undergraduate students in mechanical engineering, automotive engineering, aerospace engineering, civil engineering, biomechanical engineering, and many other areas.

Research and engineering applications regarding multiphysics simulations have been given great attention over the last decade. This book describes the basic principles and methods for multiphysics modeling, covering related areas of physics, such as structure mechanics, fluid dynamics, heat transfer, electromagnetic, and acoustics fields. Although the fundamental equations for each and every physics model are presented in this book, the main focus will be on the coupling related terms and conditions, as well as the nonlinearity and stabilization issues.

In this book, the coupling problems are classified into different categories, namely: (1) essential coupling; (2) production term coupling; (3) natural boundary condition coupling; (4) constitutive equation coupling; and (5) analysis domain coupling by physical characteristics, which include strong coupling problems and weak coupling problems, by the level of the coupling. All of the possible interface coupling conditions and load transfer conditions among these five physics models are listed, and the resulted coupled equations are presented. The Direct Matrix Assembly (DMA) method, Direct Interface Coupling (DIC) method, Multipoint Constraint (MPC) equation-based, Lagrangian Multiplier (LM) based and Penalty Method (PM) based strong coupling methods are proposed for different types of strongly coupled problems. Background theories, algorithms, key technologies, and code implementation for inter-solver weak coupling methods are also covered and discussed in details.

The challenges and important topics in multiphysics simulation are also presented: the nonlinearities and numerical stabilization in spatial and time domain; multiphysics simulation of rotating machinery; moving boundary problems for nonstructural physics models; parallel computing for large scale multiphysics simulation, etc.

This book systematically discusses about the multiphysics modeling among fluid, structure, thermal, electromagnetic, and acoustics problems. The fundamental equations, numerical schemes as well as the strategies and procedures for code implementation are presented. Most of the technologies presented in this book have been implemented in general purposed multiphysics simulation software INTESIM. More than 20 valuable engineering applications in automotive, aerospace, MEMS device, rotating machinery, and biomedical engineering etc. are presented in this book.

Organization of chapters

In Chapter 1, we briefly review the fundamental equation for fluid dynamics, structure mechanic, thermal, electromagnetic analysis, and acoustics. Special emphasis is put on the coupling terms in each equation, as well as the discretization and stabilization algorithm.

In Chapter 2, we clarify the coupling types and coupling characteristics among different physics models, and also review and discuss about appropriate coupling methods for different coupling problems.

In Chapter 3, we discuss about the coupling methods, which include the strong coupling method, general weak coupling method and intersolver-based weak coupling method.

The morphing and automatic re-meshing scheme is presented in Chapter 4 for the nonstructural physics with moving boundary analysis and coupled physics simulation with deformed structure.

The stabilization method in space and time domain to cope with the coupling non-linearity and convergence issues in each physics model, are discussed in Chapter 5.

The multiphysics coupling problems of rotating machinery are addressed in Chapter 6.

In Chapter 7, the parallel algorithm for strong and weak coupling methods is introduced.

Three general fluid–structure interaction analyses will be presented in Chapter 8.

Multiphysics simulations in automotive engineering, aerospace engineering, MEMS devices, turbine machinery and biomechanical engineering are presented in Chapter 9, Chapter 10, Chapter 11, Chapter 12, and Chapter 13, respectively.

In Chapter 14, the FSI simulation of a sensor device used in civil engineering, and an acoustic-structure coupling problem of a closed cube metal box are presented.

In Chapter 15, an overview of the commercial multiphysics software is given and the features of code implementation for multiphysics coupling is presented.

This book covers five different physics fields, namely: fluid, structure, thermal, electromagnetic, and acoustics fields. We tried our best to use common and consistent styles of statement and symbols throughout this book to make it easy to read.

This is the first edition of the book. Due to the wide coverage of the book and limitation of our knowledge, if you find any mistakes and errors in this edition, please kindly give us the feedback. We will appreciate your tolerance and efforts and correct them in the next edition.

Acknowledgments

Many of our collaborators: Prof Liu Bin and Dr Zhu Baoshan, from Tsinghua University, Prof Guan Zhenqun and Prof Liu Jun from Dalian University of Technology, Prof Li Hongguang from Shanghai Jiaotong University, and Prof Hu Qiya from the Chinese Academy of Science, for their advice and the contributions to the development of INTESIM technology.

We would like to say thanks to all our colleagues from INTESIM (Dalian) Co. Ltd. who have worked hard to prepare this book. Computational technology development and engineering examples: Dr Jiang Peng. Engineering examples: Mr Han Yepeng, Ms Huang Xiaoxiao, Mr Sun Xinghua, and Ms Xu Yu.

Many thanks to our colleagues and students who helped us in checking and verifying the English and prepare graphs, tables, etc: Ms Bai Liang, Dr Li Jianqiao, Mr Wu Xiaoming, Mr Xu Shuoyuan, Mr Zhou Xiaogang, and Mr Zhu Rubin.

We would not have the chapter on multiphysics simulation in aerospace engineering without the help of our collaborator, Prof Liu Jun from Dalian University of Technology.

Finally, many thanks to Mr Shi Lei, from Tsinghua University Press, for helping us to prepare this book.

Contents

Preface	v
Acknowledgments	vii
1 The physics models	1
1.1 Heat flow fundamentals	2
1.2 Fluid dynamics	6
1.3 Structural mechanics	26
1.4 Electromagnetic field	59
1.5 Acoustic analysis	93
2 Physics coupling phenomena and formulations	97
2.1 Introduction to coupling problems	98
2.2 General coupling equations	98
2.3 Types of coupling interfaces	102
2.4 Classification of coupling phenomena	103
2.5 The coupling matrices among physics models	104
2.6 Thermal–stress coupling	104
2.7 Fluid–structure interaction	107
2.8 Conjugate heat transfer problem	112
2.9 Acoustic–structure coupling	113
2.10 Piezoelectric analysis	115
2.11 Electrostatic–structure coupling	116
2.12 Magneto–structure coupling	116
2.13 Magneto–fluid coupling	117
2.14 Electrothermal coupling	118
2.15 Magnetic–thermal coupling	120
2.16 Summary of the coupling types	120
3 The coupling methods	125
3.1 Introduction to coupling methods	125
3.2 The strong coupling method	126
3.3 Weak coupling methods	133
3.4 Comparisons of the strong and weak coupling methods	151
3.5 Time integration scheme for transient multiphysics problems	152

4	Nonstructural physics with moving boundary	157
4.1	The moving domain problem in multiphysics simulation	157
4.2	Advanced morphing method	159
4.3	Automatic remeshing technology	160
4.4	Mesh controls for rotating machinery	164
4.5	Treatment for pinched flow problems	167
4.6	Examples for mesh control	167
5	Stabilization schemes for highly nonlinear problems	177
5.1	An overview of stabilization methods	177
5.2	Stabilization methods in spatial domain	178
5.3	Stabilization in the time integration scheme	191
5.4	Underrelaxation of the solution vector	197
5.5	Capping for the solution	198
5.6	Trade off the stability, accuracy, and efficiency	199
6	Coupling simulation for rotating machines	201
6.1	Reference frames	201
6.2	General coupling boundary conditions	203
6.3	Governing equations in body-attached rotating frame	209
6.4	Multiple frames of references for rotating problems	210
6.5	Morphing technology for rotating problems	219
6.6	Multiphysics simulation for rotating machines	220
7	High-performance computing for multiphysics problems	227
7.1	The challenges in large-scale multiphysics simulation	227
7.2	Parallel algorithm for the strong coupling method	228
7.3	Parallel scheme for weak coupling methods	230
8	General multiphysics study cases	233
8.1	Efficiency studies of strong and weak coupling methods for simple case	233
8.2	Fluid–structure interaction simulation of flow around a cylinder with a flexible flag attached	237
8.3	Fluid–structure simulation of a flapping wing structure in a water channel	245
9	Multiphysics applications in automotive engineering	251
9.1	The study of dynamic characteristics of hydraulic engine mounts by strong coupling finite element model	251
9.2	Weak coupling fluid–solid–thermal analysis of exhaust manifold	267
9.3	Coupling analysis of permanent magnet synchronous motor	273

10	Computational fluid dynamics in aerospace field and CFD-based multidisciplinary simulations	295
10.1	Application and development of computational fluid dynamics simulation in the aerospace field	295
10.2	The research topic and its progress	297
10.3	Example	310
11	Multiphysics simulation of microelectro-mechanical systems devices	329
11.1	Introduction to MEMS	329
11.2	Micropump	329
11.3	Natural convection cooling of a microelectronic chip	334
12	Bidirectional multiphysics simulation of turbine machinery	339
12.1	The fluid–structure–thermal bidirectional coupling analysis on the rotor system of turbo expander	339
12.2	The fluid–structure coupling analysis of the turbine blade	349
13	Multiphysics modeling for biomechanical problems	363
13.1	Numerical analysis of a 3D simplified artificial heart	363
13.2	FSI simulation of a vascular tumor	366
14	Other multiphysics applications	375
14.1	FSI simulation of a sensor device in civil engineering	375
14.2	Acoustic structural coupling case	381
15	Code implementation of multiphysics modeling	387
15.1	Overview of commercial CAE software for multiphysics	387
15.2	Code implementation for multiphysics modeling	389
	References	397
	Index	409

The physics models

1

Chapter Outline

1.1 Heat flow fundamentals 2

- 1.1.1 Basic equations 2
- 1.1.2 Boundary conditions 2
- 1.1.3 Weak forms of the thermal equation 3
- 1.1.4 The shape functions for FEM 3
- 1.1.5 Formulations in matrix form 4
- 1.1.6 The nonlinearity in thermal analysis 5
- 1.1.7 Stabilization method for convection-dominant transport equations 5
- 1.1.8 Penalty-based thermal contact 5

1.2 Fluid dynamics 6

- 1.2.1 Basic equations for fluid flow 6
- 1.2.2 Boundary and initial conditions for fluid flow 7
- 1.2.3 The constitutive equation for fluid flow 8
- 1.2.4 The weak forms 8
- 1.2.5 Finite element equations 9
- 1.2.6 The nonlinearity and numerical challenging in CFD 12
- 1.2.7 The stabilization methods 12
- 1.2.8 Turbulence model in CFD 14
- 1.2.9 The general transport equations 23

1.3 Structural mechanics 26

- 1.3.1 Governing equations for structure analysis 26
- 1.3.2 The equation in matrix form 27
- 1.3.3 The general nonlinearity in structural dynamics 28
- 1.3.4 Incompressible hyperelastic material for the rubber (Watanabe, 1995) 29
- 1.3.5 Formulations for thin structure with large deformation 30
- 1.3.6 Mixed interpolation tensorial component shell element (Bathe, 2006) 32
- 1.3.7 Generalized conforming flat shell element 34
- 1.3.8 Contact problems in structural mechanics 46

1.4 Electromagnetic field 59

- 1.4.1 Fundamental equations 59
- 1.4.2 Classification of Maxwell's equations and potential formulations 60
- 1.4.3 FEM discretization of potential formulations 65
- 1.4.4 Gauge methods for electromagnetic elements 80
- 1.4.5 Output results 83
- 1.4.6 Sliding interface coupling in electromagnetic problems 86

1.5 Acoustic analysis 93

- 1.5.1 Governing equations 93
- 1.5.2 Momentum equation 94
- 1.5.3 The boundary conditions 94
- 1.5.4 The equations in weak form 94
- 1.5.5 Acoustics Equations in Matrix Form 95

1.1 Heat flow fundamentals

1.1.1 Basic equations

The solution for heat flow analysis is to find the temperature distribution $T(X, t) \in \mathcal{S}_T$, so that the following governing Equation (1.1) is satisfied.

$$\frac{\partial(\rho c_v T)}{\partial t} + \frac{\partial}{\partial x_i} \left(\rho c_v T v_i - k \frac{\partial T}{\partial x_i} \right) = \overline{q^B}, \text{ in } \Omega_i^T \quad (1.1)$$

The primary variable is temperature T in the infinite-dimensional space of \mathcal{S}_T . The first part on the left-hand side of Equation (1.1) is the time derivative term, the second term is the convection term caused by fluid flow, and the third term is the diffusion term. The source term of heat generation is on the right-hand side of Equation (1.1).

The material properties needed to be decided are; mass density, ρ ; specific heat, c_v ; and thermal conductivity, k . For fully incompressible flow, ρ is assumed to be constant, but for slightly compressible and low-speed compressible flow, we assume $\rho = \rho(p)$ and $\rho = \rho(p, T)$, respectively. Also, we assume specific heat c_v and conductivity k to be either constant or a function of temperature. v is the convective velocity, and q^B is the heat generation from other physics models in multiphysics simulation.

Remark 1.1: q^B is the source term of heat generation that may come from thermo-elastic damping, fluid viscous heat, or electro or electromagnetic heat in multiphysics simulation. On the other hand, temperature T as output may affect the material properties or other quantities of the corresponding physics models.

Symbol $\overline{\quad}$ indicates the value that is received from other physics model, and $\overleftarrow{\quad}$ represents the value transferred to other physics model in multiphysics simulation. Symbol $\overleftarrow{\quad}$ means the value can be either sent out to or received from other physics model in multiphysics simulation.

1.1.2 Boundary conditions

Three types of boundary conditions are considered:

1. Specified temperature on Γ_g :

$$T = \overline{T^s} \text{ on } \Gamma_g \quad (1.2)$$

Here, T^s is the specified temperature.

2. Specified heat flow on Γ_{q1} :

$$k \cdot \frac{(n_i \partial T)}{(\partial x_i)} = \overline{q^s} \text{ on } \Gamma_{q1} \quad (1.3)$$

where, q^s is the specified heat flow and n_i is the component of the unit normal direction \mathbf{n} .

Remark 1.2: The boundary conditions: Equations (1.2) and (1.3) can be used in conjugate heat transfer coupling.

3. Specified convection surfaces acting over surface Γ_{q2} :

$$k \cdot \frac{(n_i \partial T)}{(\partial x_i)} = h_f (T_B - T_S) \quad \text{on} \quad \Gamma_{q2} \quad (1.4)$$

Where; h_f , is the convective heat transfer coefficient; T_B , is the bulk temperature of the adjacent physics model; and T_S , is the temperature at the surface of the model.

Remark 1.3: The positive specified heat flow is into the boundary.

1.1.3 Weak forms of the thermal equation

The weak form of heat flow equations is given in Equation (1.5) as follows:

find temperature $T \in \mathcal{S}_T$, such that $\forall w_T \in \mathcal{W}_T$:

$$\int_{\Omega} w_T \left[\frac{\partial(\rho c_v T)}{\partial t} + \frac{\partial}{\partial x_i} \left(\rho c_v T v_i - k \frac{\partial T}{\partial x_i} \right) \right] d\Omega = \int_{\Omega} w_T q^B d\Omega. \quad (1.5)$$

Here, w_T is the test function of temperature T in infinite-dimensional space \mathcal{W}_T .

The semi-discrete Galerkin finite element formulation of the heat flow problem is stated as:

find $T^h \in \mathcal{S}_T^h$, such that $\forall w_T^h \in \mathcal{W}_T^h$:

$$\int_{\Omega} w_T^h \left[\frac{\partial(\rho c_v T^h)}{\partial t} + \frac{\partial}{\partial x_i} \left(\rho c_v T^h v_i^h - k \frac{\partial T^h}{\partial x_i} \right) \right] d\Omega = \int_{\Omega} w_T^h q^B d\Omega \quad (1.6)$$

Where, \mathcal{S}_T^h and \mathcal{W}_T^h are the sets of finite-dimensional trial and test functions for temperature, respectively.

1.1.4 The shape functions for FEM

The basic shape function for the coordinate:

$$\mathbf{x}(\xi) = \sum_{i=1}^{n_{en}} N_i^E(\xi) \mathbf{x}_i = N_i^E(\xi) \mathbf{x}_i. \quad (1.7)$$

Here; $N_i^E(\xi)$, is the shape function at node i ; ξ , is the element local coordinate; and \mathbf{x}_i , is the nodal coordinate at node i ; n_{en} , is the number of nodes per element, respectively.

The interpolation for temperature variables:

$$T(\xi) = \sum_{i=1}^{n_{en}} N_i^T T_i = N_i^T T_i. \quad (1.8)$$

Here, N_i^T is the shape function for temperature at node i , which is consistent with N_i^E for the class of isoparametrical element. The standard isoparametrical element can be used for the temperature interpolation, for example, the low order 4-node tetrahedral element and 8-node hexahedral element, are the most commonly used for thermal analysis, although the high order 10-node tetrahedral element and 20-node hexahedral element are also useful in achieving better accuracy with the same number of nodes. The other shapes, for example, wedge and pyramid can be used for the transition region between tetrahedral and hexahedral elements. The details about these shape functions can be found in Bathe (2006) and Wang (2000).

1.1.5 Formulations in matrix form

Integrating of Equation (1.6) by part for the third term on the left-hand side, applying boundary conditions of Equations (1.2)–(1.4), and substituting the shape functions into it, one can obtain assembled global Equation (1.9) in matrix form:

$$CT + (K(v)^{tm} + K^{tb} + K^{tc})T = Q^{\text{flux}} + Q^{\text{conv}} + \overline{Q^g} (q^B). \quad (1.9)$$

Where,

$$C = \sum_{e=1}^{ne} C_e, \quad C_{e(i,j)} = \rho \int_{\Omega_e} c_v N_i^T N_j^T d\Omega \text{ is the element specific heat matrix.}$$

$K^{tm} = \sum_{e=1}^{ne} K_e^{tm}$, $K_{e(i,j)}^{tm} = \rho \int_{\Omega_e} c_v N_i^T v_k N_{j,k}^T d\Omega$ is the element mass transport conductivity matrix.

$K^{tb} = \sum_{e=1}^{ne} K_e^{tb}$, $K_{e(i,j)}^{tb} = \int_{\Omega_e} N_{i,m}^T k_{mm} N_{j,m}^T d\Omega$ is the element diffusion conductivity matrix.

$K^{tc} = \sum_{e=1}^{ne} K_e^{tc}$, $K_{e(i,j)}^{tc} = \int_{\Gamma_{q2}} h_f N_i^T N_j^T d\Gamma$ is element convection surface conductivity matrix.

$$Q^{\text{flux}} = \sum_f Q_f^{\text{flux}}, \quad Q_{f(i)}^{\text{flux}} = \int_{\Gamma_{q1}} N_i^T q^s d\Gamma \text{ is element surface heat flux vector.}$$

$Q^{\text{conv}} = \sum_f Q_f^{\text{conv}}$, $Q_{f(i)}^{\text{conv}} = \int_{\Gamma_{q2}} T_B h_f N_i^T d\Gamma$ is the element convection surface heat flow vector.

$$Q^g = \sum_e Q_e^g, \quad Q_{e(i)}^g = \int_{\Omega_e} q^B N_i^T d\Omega \text{ is the element heat generation vector.}$$

Here, $\sum_{e=1}^{ne}$ represents the assembly operator for element matrix or vector with number of ne elements.

\sum_f represents the assembly operator for element surface loads.

1.1.6 The nonlinearity in thermal analysis

1.1.6.1 Material properties

The material properties depend on the temperature, that is, $k = k(T)$, $c_v = c_v(T)$.

1.1.6.2 Convection term from computational fluid dynamics (CFD) coupling

The definition of Peclet number for convective heat transfer problem is

$$\text{Pe} = \frac{c_v \rho}{k} \|\mathbf{v}\| h^e. \quad (1.10)$$

Where, h^e stands for element size.

For a heat flow problem with $\text{Pe} \geq 2.0$, to avoid oscillation of the solution of the temperature field and to get a better quality matrix equation, the spatial stabilization method may be needed. The streamline upwind Petrov/Galerkin (SUPG) method is presented in the further section.

1.1.7 Stabilization method for convection-dominant transport equations

The SUPG method is used for the convection-dominated heat transfer problem for the case, where Peclet number is higher than 2.0. For details about implementation of the stabilization method for transport equations, please refer Section 1.2.9.3.

1.1.8 Penalty-based thermal contact

The thermal contact problem can be used for thin air gap or other thin-layered problems with different thermal conductivities.

Assuming the thickness of a thin air gap is d , and T_1 and T_2 are the temperatures on the side A and side B of the contact wall, respectively, then the heat flux from T_2 side into T_1 side is:

$$q = k \frac{T_2 - T_1}{d}. \quad (1.11)$$

Here, k is the thermal conductivity, and the heat flux from T_1 side into T_2 side is the negative of q in Equation (1.11). Here we assume the air layer is thin enough, so linear distribution assumption of temperature across the air gap is acceptable.

Then thermal conductance for this gap is:

$$h_f = \frac{k}{d} \quad (1.12)$$

where h_f , as the contact thermal conductance, can be setup as input property.

1.1.8.1 The matrix equation for thermal contact

The heat flow for side A

$$\int_{\Gamma_A} w_A h_f (T_B - T_A) d\Gamma \quad (1.13)$$

The previous equation can be added into the left-hand side of Equation (1.5) to consider the thermal contact affect from the air gap, and the matrix form can be expressed as:

$$k_{AiAj} = \int_{\Gamma_A} N_{Ai} h_f N_{Aj} d\Gamma \quad (1.14)$$

$$k_{AiBj} = - \int_{\Gamma_A} N_{Ai} h_f N_{Bj} d\Gamma \quad (1.15)$$

where, Ai is the surface node i on side A and Bj is the surface node j on side B , respectively. $k_{AiAj} T_A + k_{AiBj} T_B$ needs to be added into the left side of Equation (1.9) for side A , and its negative value will be added in the same way for side B .

1.2 Fluid dynamics

1.2.1 Basic equations for fluid flow

We assume the viscous fluid to be isothermal and barotropic (i.e., $F(p, \rho) = 0$) and that $\partial p / \partial \rho = B / \rho$, in which B , p , and ρ are fluid bulk modulus, pressure, and fluid density, respectively. The Arbitrary Lagrangian Eulerian (ALE) formulation is usually used to handle moving boundary problems of fluid flow in coupling analysis. The fundamental equations of fluid flow (Zhang and Hisada, 2001) are expressed as:

$$\frac{1}{B} \frac{\partial \bar{p}}{\partial t} \Big|_x + \frac{1}{B} c_i \frac{\partial p}{\partial x_i} + \frac{\partial v_i}{\partial x_i} = 0 \quad \text{in } \Omega_t^f \quad (1.16)$$

$$\rho \frac{\partial \bar{v}_i}{\partial t} \Big|_x + \rho c_j \frac{\partial v_i}{\partial x_j} = \frac{\partial \sigma_{ij}}{\partial x_j} + \rho g_i + \bar{f}_i \quad \text{in } \Omega_t^f \quad (1.17)$$

$$\begin{aligned} \frac{\partial(\rho c_v \bar{T})}{\partial t} \Big|_x + \frac{\partial}{\partial x_i} \left(\rho c_v T v_i - k \frac{\partial T}{\partial x_i} \right) = \\ 2\mu D^2 + \frac{\partial v_i}{\partial x_i} \left(-p + \lambda \frac{\partial v_i}{\partial x_i} \right) + \bar{q}^B \quad \text{in } \Omega_t^f. \end{aligned} \quad (1.18)$$

The primary variables are pressure p , velocity vector \mathbf{v} , and temperature T . Here, $\boldsymbol{\sigma}$ is the Cauchy stress tensor, \mathbf{g} is the acceleration of gravity, and $\mathbf{c} = \mathbf{v} - \mathbf{v}_m$ is the relative velocity of fluid particles to the ALE coordinates with \mathbf{v}_m the mesh velocity. Ω_t^f denotes the spatial thermal fluid domain bounded by the boundary Γ_t^f of interest at any instant t . Here the superscript f stands for the fluid component. λ is the second viscosity, and k is the thermal conductivity. And in Equation (1.18), the strain rate related energy term is expressed as:

$$D^2 = e_{ij}e_{ij} \quad \text{with} \quad e_{ij} = \frac{1}{2}(v_{i,j} + v_{j,i}).$$

Remark 1.4: The source term f_i in Equation (1.17) may come from the coupling physics, for example, the electromagnetic force. The outgoing variables are the pressure p and velocity vector \mathbf{v} in the coupled physics simulation.

1.2.2 Boundary and initial conditions for fluid flow

The boundary is composed of Γ_i^g and Γ_i^h corresponding to Dirichlet- and Neumann-type boundary conditions, respectively.

$$v_i = \bar{g}_i \quad \text{on} \quad \Gamma_g^f \quad (1.19)$$

$$\sigma_{ji}n_j = \bar{h}_i \quad \text{on} \quad \Gamma_h^f \quad (1.20)$$

$$\bar{T} = T^s \quad \text{on} \quad \Gamma_g^T \quad (1.21)$$

$$k \cdot n_i \partial T_i / \partial x_i = q^s \quad \text{on} \quad \Gamma_q^T \quad (1.22)$$

And subject to the following initial conditions:

$$v_i(0) = {}^0v_i \quad \text{on} \quad \Omega^f \quad (1.23)$$

$$p(0) = {}^0p \quad \text{on} \quad \Omega^f \quad (1.24)$$

$$T(0) = {}^0T \quad \text{on} \quad \Omega^f \quad (1.25)$$

The boundary conditions for the energy equation are the same as those in previous sections.

Remark 1.5: Equations (1.19) and (1.20) represent the coupling boundaries in the fluid–structure interaction (FSI) problem with g_i received from the structure and h_i sent to the structure.

1.2.3 The constitutive equation for fluid flow

The fluid is assumed to be Newtonian, and the constitutive equation is:

$$\sigma_{ij} = -p \delta_{ij} + \frac{1}{2} \mu (v_{i,j} + v_{j,i}) \quad (1.26)$$

where δ_{ij} is the component of identity tensor and μ is the dynamic viscosity of the fluid. The properties for thermal are provided in the previous section.

1.2.4 The weak forms

1.2.4.1 Galerkin formulation for N-S equations

The weak forms for the fluid equations given in Equations (1.16)–(1.18) by the Galerkin method are written as: find pressure $p \in \mathcal{S}_p$, velocity $\mathbf{v} \in \mathcal{S}_v$, and temperature $T \in \mathcal{S}_T$, such that $\forall w_p \in \mathcal{W}_p, \forall \mathbf{w}_v \in \mathcal{W}_v$, and $\forall w_T \in \mathcal{W}_T$:

$$\int_{\Omega} w_p \left(\frac{1}{B} \frac{\partial p}{\partial t} \Big|_x + \frac{1}{B} c_i \frac{\partial p}{\partial x_i} + \nabla \cdot \mathbf{v} \right) d\Omega = 0 \quad (1.27)$$

$$\int_{\Omega} \mathbf{w}_v \cdot \rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{c} \cdot \nabla \mathbf{v} - \mathbf{f} \right) d\Omega + \int_{\Omega} \boldsymbol{\varepsilon}(\mathbf{w}) : \boldsymbol{\sigma}(\mathbf{v}, p) d\Omega = \int_{\Gamma_b} \mathbf{w}_v \cdot \mathbf{h} d\Gamma \quad (1.28)$$

$$\int_{\Omega} w_T \left[\frac{\partial(\rho c_v T)}{\partial t} + \nabla \cdot (\rho c_v T \mathbf{v}_i - k \nabla T) \right] d\Omega = \int_{\Omega} w_T (2\mu D^2 + q^B + \nabla \cdot \mathbf{v} (-p + \lambda(\nabla \cdot \mathbf{v}))) d\Omega \quad (1.29)$$

Where \mathcal{S}_p , \mathcal{S}_v , and \mathcal{S}_T denote the sets of infinite-dimensional trial functions for pressure, velocity, and temperature, respectively. \mathcal{W}_p , \mathcal{W}_v , and \mathcal{W}_T are the sets of test functions (weighing functions) for the continuity, equilibrium, and energy equations, respectively.

The Galerkin finite element formulation of the fluid equations is stated as follows: find pressure $p^h \in \mathcal{S}_p^h$, velocity $\mathbf{v}^h \in \mathcal{S}_v^h$, and temperature $T^h \in \mathcal{S}_T^h$ such that $\forall w_p^h \in \mathcal{W}_p^h, \forall \mathbf{w}_v^h \in \mathcal{W}_v^h, \forall w_T^h \in \mathcal{W}_T^h$:

$$\int_{\Omega} w_p^h \left(\frac{1}{B} \frac{\partial p^h}{\partial t} \Big|_x + \frac{1}{B} c_i^h \frac{\partial p^h}{\partial x_i} + \nabla \cdot \mathbf{v}^h \right) d\Omega = 0 \quad (1.30)$$

$$\int_{\Omega} \mathbf{w}_v^h \cdot \rho \left(\frac{\partial \mathbf{v}^h}{\partial t} + \mathbf{c}^h \cdot \nabla \mathbf{v}^h - \mathbf{f}^h \right) d\Omega + \int_{\Omega} \boldsymbol{\varepsilon}(\mathbf{w}^h) : \boldsymbol{\sigma}(\mathbf{v}^h, p^h) d\Omega = \int_{\Gamma_b} \mathbf{w}_v^h \cdot \mathbf{h}^h d\Gamma \quad (1.31)$$