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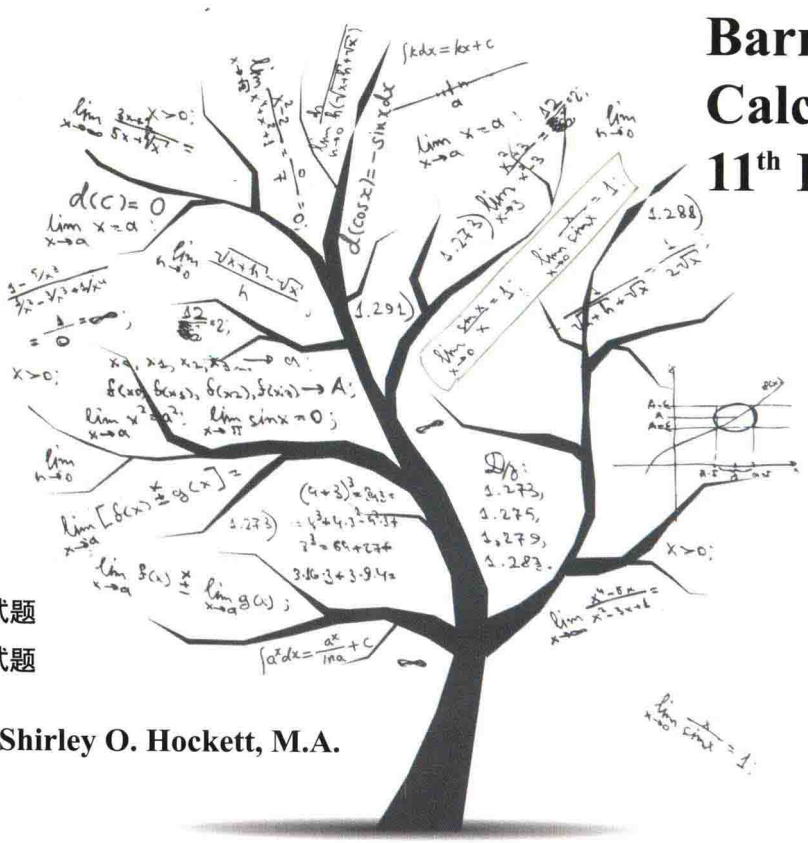
Barron's AP

微积分（第11版）

Barron's AP Calculus 11th Edition



:: 考点透析　应试技巧
:: 4套微积分AB全真测试题
:: 4套微积分BC全真测试题

David Bock, M.S. and Shirley O. Hockett, M.A.

世界图书出版公司

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目 录

巴朗五大要点提示 xi

绪论 1

课程 1

微积分AB考试中可能考查的知识点 1

微积分BC考试中可能考查的知识点 2

考试 3

图形计算器：在AP考试中使用您的图形计算器 4

考试成绩评级 9

CLEP微积分考试 10

本书内容 10

记忆卡 11

诊断测试

微积分AB 17

微积分BC 43

专题复习和习题

1 函数 67

A. 定义 67

B. 特殊函数 70

C. 多项式函数和其他有理函数 73

D. 三角函数 73

E. 指数函数和对数函数 76

F. 参变量函数 77

习题 80

2 极限和连续性 87

A. 定义和例析 87

B. 渐近线 92

C. 极限定理 93

D. 多项式商的极限 95

E. 其他基本极限 96

F. 连续性 97

习题 102

3 微分 111

A. 导数的定义 111

B. 公式 113

C. 链式法则；复合函数的导数 114

D. 可微性和连续性 118

E. 导数的近似求法 119

E1. 数值法 119

E2. 图示法 122

F. 参变量函数的导数 123

G. 隐微分法 124

H. 反函数的导数 126

I. 中值定理 128

J. 不定式和洛必达法则 129

K. 认定一个给定的极限作为其导数 132

习题 134

4 微分学的应用 159

A. 斜率；驻点 159

B. 切线和法线 161

C. 增函数和减函数 162

情形一：其导数连续的函数 162

情形二：其导数不连续的函数 163

D. 最大值、最小值和拐点：定义 163

E. 最大值、最小值和拐点：曲线图 164

情形一：处处可微的函数 164

情形二：存在不可微点的函数 168
F. 全局最大值或最小值 169
情形一：可微函数 169
情形二：存在不可微点的函数 170
G. 作图贴士 170
H. 最优化：涉及最大值和最小值的问题 172
I. 函数和其导数的图示关系 176
J. 直线运动 179
K. 曲线运动：速度和加速度矢量 181
L. 局部线性近似 185
M. 相关速率 188
N. 极曲线的斜率 190
习题 192

5 不定积分 215
A. 不定积分 215
B. 基本公式 215
C. 部分分数积分法 223
D. 分部积分法 224
E. 不定积分的应用；微分方程 227
习题 229

6 定积分 249
A. 微积分的基本定理(FTC)；定积分的定义 249
B. 定积分的性质 249
C. 参变量函数的定积分 254
D. 求和极限的定积分的定义：另一个基本定理 255
E. 定积分的近似计算；黎曼求和 257
E1. 矩形法 257
E2. 梯形法 258
比较近似求和 260
根据导数作出其函数的图像：另一种方法 262
F. $\ln x$所表示的面积 269
G. 平均值 270
习题 277

7 积分在几何学中的应用 291
A. 面积 291
A1. 曲线间的面积 293
A2. 利用对称性 293
B. 体积 298
B1. 已知截面面积的立体 298
B2. 旋转体 300
C. 弧长 305
D. 广义积分 307
习题 317

8 积分的更多应用 345
A. 直线运动 345
B. 平面曲线运动 347
C. 黎曼求和的其他应用 350
D. FTC：比率的定积分是净变化量 352
习题 356

9 微分方程 367
A. 基本定义 367
B. 斜率场 369
C. 欧拉方法 373
D. 一阶微分方程的求解 377
E. 指数增长和衰减 379
情形一：指数增长 379
情形二：约束增长 383
情形三：Logistic增长 386
习题 391

10 序列和级数 409
A. 实数序列 409
B. 无穷级数 410
B1. 定义 410
B2. 无穷级数的收敛和发散定理 412
B3. 无穷级数的收敛判别法 413
B4. 正项级数的收敛判别法 414
B5. 交错级数和绝对收敛 418
C. 幂级数 421

试读结束：需要全本请在线购买：www.ertongbook.com

C1. 定义；收敛 421
C2. 幂级数定义的函数 423
C3. 函数幂级数的展开：泰勒级数和麦克劳林级数 425
C4. 泰勒多项式和麦克劳林多项式的近似函数 429
C5. 带余项的泰勒公式；拉格朗日误差界 433
C6. 幂级数的计算 436
C7. 复幂级数 439
习题 440

11 选择题集锦 455

12 开放式题目集锦 491

AB测试题

AB 测试题1 519
AB 测试题2 545
AB 测试题3 573

BC测试题

BC 测试题1 603
BC 测试题2 625
BC 测试题3 649

附录：参考公式和定理 673

索引 683

Contents

Barron's Essential 5 xi

Introduction 1

The Courses 1
Topics That May Be Tested on the Calculus AB Exam 1
Topics That May Be Tested on the Calculus BC Exam 2
The Examinations 3
The Graphing Calculator: Using Your Graphing Calculator on the AP Exam 4
Grading the Examinations 9
The CLEP Calculus Examination 10
This Review Book 10
Flash Cards 11

DIAGNOSTIC TESTS

Calculus AB 17
Calculus BC 43

TOPICAL REVIEW AND PRACTICE

1 Functions 67

A. Definitions 67
B. Special Functions 70
C. Polynomial and Other Rational Functions 73
D. Trigonometric Functions 73
E. Exponential and Logarithmic Functions 76
BC ONLY | F. Parametrically Defined Functions 77
Practice Exercises 80

2 Limits and Continuity 87

A. Definitions and Examples 87
B. Asymptotes 92
C. Theorems on Limits 93
D. Limit of a Quotient of Polynomials 95
E. Other Basic Limits 96
F. Continuity 97
Practice Exercises 102

3 Differentiation 111

A. Definition of Derivative 111
B. Formulas 113
C. The Chain Rule; the Derivative of a Composite Function 114
D. Differentiability and Continuity 118
E. Estimating a Derivative 119
E1. Numerically 119
E2. Graphically 122
BC ONLY | F. Derivatives of Parametrically Defined Functions 123
G. Implicit Differentiation 124
H. Derivative of the Inverse of a Function 126
I. The Mean Value Theorem 128
BC ONLY | J. Indeterminate Forms and L'Hôpital's Rule 129
K. Recognizing a Given Limit as a Derivative 132
Practice Exercises 134

4 Applications of Differential Calculus 159

A. Slope; Critical Points 159
B. Tangents and Normals 161
C. Increasing and Decreasing Functions 162
Case I. Functions with Continuous Derivatives 162
Case II. Functions Whose Derivatives Have Discontinuities 163
D. Maximum, Minimum, and Inflection Points: Definitions 163
E. Maximum, Minimum, and Inflection Points: Curve Sketching 164
Case I. Functions That Are Everywhere Differentiable 164
Case II. Functions Whose Derivatives May Not Exist Everywhere 168
F. Global Maximum or Minimum 169
Case I. Differentiable Functions 169
Case II. Functions That Are Not Everywhere Differentiable 170

G. Further Aids in Sketching 170
H. Optimization: Problems Involving Maxima and Minima 172
I. Relating a Function and Its Derivatives Graphically 176
J. Motion Along a Line 179
BC ONLY | K. Motion Along a Curve: Velocity and Acceleration Vectors 181
L. Tangent-Line Approximations 185
M. Related Rates 188
BC ONLY | N. Slope of a Polar Curve 190
Practice Exercises 192

5 Antidifferentiation 215

A. Antiderivatives 215
B. Basic Formulas 215
BC ONLY | C. Integration by Partial Fractions 223
D. Integration by Parts 224
E. Applications of Antiderivatives; Differential Equations 227
Practice Exercises 229

6 Definite Integrals 249

A. Fundamental Theorem of Calculus (FTC); Definition of Definite Integral 249
B. Properties of Definite Integrals 249
BC ONLY | C. Integrals Involving Parametrically Defined Functions 254
D. Definition of Definite Integral as the Limit of a Sum: The Fundamental Theorem Again 255
E. Approximations of the Definite Integral; Riemann Sums 257
E1. Using Rectangles 257
E2. Using Trapezoids 258
Comparing Approximating Sums 260
Graphing a Function from Its Derivative; Another Look 262
F. Interpreting $\ln x$ as an Area 269
G. Average Value 270
Practice Exercises 277

7 Applications of Integration to Geometry 291

A. Area 291
A1. Area Between Curves 293
A2. Using Symmetry 293
B. Volume 298
B1. Solids with Known Cross Sections 298
B2. Solids of Revolution 300
BC ONLY | C. Arc Length 305
D. Improper Integrals 307
Practice Exercises 317

8 Further Applications of Integration 345

A. Motion Along a Straight Line 345
BC ONLY | B. Motion Along a Plane Curve 347
C. Other Applications of Riemann Sums 350
D. FTC: Definite Integral of a Rate Is Net Change 352
Practice Exercises 356

9 Differential Equations 367

A. Basic Definitions 367
B. Slope Fields 369
BC ONLY | C. Euler's Method 373
D. Solving First-Order Differential Equations Analytically 377
E. Exponential Growth and Decay 379
Case I: Exponential Growth 379
Case II: Restricted Growth 383
BC ONLY | Case III: Logistic Growth 386
Practice Exercises 391

10 Sequences and Series 409

BC ONLY

A. Sequences of Real Numbers 409
B. Infinite Series 410
B1. Definitions 410
B2. Theorems About Convergence or Divergence of Infinite Series 412
B3. Tests for Convergence of Infinite Series 413
B4. Tests for Convergence of Nonnegative Series 414
B5. Alternating Series and Absolute Convergence 418
C. Power Series 421
C1. Definitions; Convergence 421
C2. Functions Defined by Power Series 423
C3. Finding a Power Series for a Function: Taylor and Maclaurin Series 425
C4. Approximating Functions with Taylor and Maclaurin Polynomials 429
C5. Taylor's Formula with Remainder; Lagrange Error Bound 433
C6. Computations with Power Series 436
C7. Power Series over Complex Numbers 439
Practice Exercises 440

11 Miscellaneous Multiple-Choice Practice Questions 455

12 Miscellaneous Free-Response Practice Exercises 491

AB PRACTICE EXAMINATIONS

AB One 519
AB Two 545
AB Three 573

BC PRACTICE EXAMINATIONS

BC One 603
BC Two 625
BC Three 649

Appendix: Formulas and Theorems for Reference 673

Index 683

Introduction 绪 论

This book is intended for students who are preparing to take one of the two Advanced Placement Examinations in Mathematics offered by the College Entrance Examination Board, and for their teachers. It is based on the May 2012 course description published by the College Board, and covers the topics listed there for both Calculus AB and Calculus BC.

Candidates who are planning to take the CLEP Examination on Calculus with Elementary Functions are referred to the section of this Introduction on that examination on page 10.

THE COURSES 课程

Calculus AB and BC are both full-year courses in the calculus of functions of a single variable. Both courses emphasize:

(1) student understanding of concepts and applications of calculus over manipulation and memorization;

(2) developing the student's ability to express functions, concepts, problems, and conclusions analytically, graphically, numerically, and verbally, and to understand how these are related; and

(3) using a graphing calculator as a tool for mathematical investigations and problem-solving.

Both courses are intended for students who have already studied college-preparatory mathematics: algebra, geometry, trigonometry, analytic geometry, and elementary functions (linear, polynomial, rational, exponential, logarithmic, trigonometric, inverse trigonometric, and piecewise). The AB topical course outline that follows can be covered in a full high-school academic year even if some time is allotted to studying elementary functions. The BC course assumes that students already have a thorough knowledge of all the topics noted above.

TOPICS THAT MAY BE TESTED ON THE CALCULUS AB EXAM 微积分AB考试中可能考查的知识点

1. **Functions and Graphs**
   Rational, trigonometric, inverse trigonometric, exponential, and logarithmic functions.

2. **Limits and Continuity**
   Intuitive definitions; one-sided limits; functions becoming infinite; asymptotes and graphs; indeterminate limits of the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$; $\lim\limits_{\theta\to 0}\frac{\sin\theta}{\theta}$; estimating limits using tables or graphs.

Definition of continuity; kinds of discontinuities; theorems about continuous functions; Extreme Value and Intermediate Value Theorems.

3. **Differentiation**
   Definition of derivative as the limit of a difference quotient and as instantaneous rate of change; derivatives of power, exponential, logarithmic, trig and inverse trig functions; product, quotient, and chain rules; differentiability and continuity; estimating a derivative numerically and graphically; implicit differentiation; derivative of the inverse of a function; the Mean Value Theorem; recognizing a given limit as a derivative.

4. **Applications of Derivatives**
   Rates of change; slope; critical points; average velocity; tangents and normals; increasing and decreasing functions; using the first and second derivatives for the following: local (relative) max or min, concavity, inflection points, curve sketching, global (absolute) max or min and optimization problems; relating a function and its derivatives graphically; motion along a line; local linearization and its use in approximating a function; related rates; differential equations and slope fields.

5. **The Definite Integral**
   Definite integral as the limit of a Riemann sum; area; definition of definite integral; properties of the definite integral; Riemann sums using rectangles or sums using trapezoids; comparing approximating sums; average value of a function; Fundamental Theorem of Calculus; graphing a function from its derivative; estimating definite integrals from tables and graphs; accumulated change as integral of rate of change.

6. **Integration**
   Antiderivatives and basic formulas; antiderivatives by substitution; applications of antiderivatives; separable differential equations; motion problems.

7. **Applications of Integration to Geometry**
   Area of a region, including between two curves; volume of a solid of known cross section, including a solid of revolution.

8. **Further Applications of Integration and Riemann Sums**
   Velocity and distance problems involving motion along a line; other applications involving the use of integrals of rates as net change or the use of integrals as accumulation functions; average value of a function over an interval.

9. **Differential Equations**
   Basic definitions; geometric interpretations using slope fields; solving first-order separable differential equations analytically; exponential growth and decay.

TOPICS THAT MAY BE TESTED ON THE CALCULUS BC EXAM 微积分BC考试中可能考查的知识点

BC ONLY

Any of the topics listed above for the Calculus AB exam may be tested on the BC exam. The following additional topics are restricted to the BC exam.

1. **Functions and Graphs**
   Parametrically defined functions; polar functions; vector functions.

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BC ONLY

2. **Limits and Continuity**
No additional topics.

3. **Differentiation**
Derivatives of polar, vector, and parametrically defined functions; indeterminate forms; L'Hôpital's rule.

4. **Applications of Derivatives**
Tangents to parametrically defined curves; slopes of polar curves; analysis of curves defined parametrically or in polar or vector form.

5. **The Definite Integral**
Integrals involving parametrically defined functions.

6. **Integration**
By parts; by partial fractions (involving nonrepeating linear factors only); improper integrals.

7. **Applications of Integration to Geometry**
Area of a region bounded by parametrically defined or polar curves; arc length.

8. **Further Applications of Integration and Riemann Sums**
Velocity and distance problems involving motion along a planar curve; velocity and acceleration vectors.

9. **Differential Equations**
Euler's method; applications of differential equations, including logistic growth.

10. **Sequences and Series**
Definition of series as a sequence of partial sums and of its convergence as the limit of that sequence; harmonic, geometric, and p-series; integral, ratio, and comparison tests for convergence; alternating series and error bound; power series, including interval and radius of convergence; Taylor polynomials and graphs; finding a power series for a function; MacLaurin and Taylor series; Lagrange error bound for Taylor polynomials; computations using series.

THE EXAMINATIONS 考试

The Calculus AB and BC Examinations and the course descriptions are prepared by committees of teachers from colleges or universities and from secondary schools. The examinations are intended to determine the extent to which a student has mastered the subject matter of the course.

Each examination is 3 hours and 15 minutes long, as follows:

Section I has two parts. Part A has 28 multiple-choice questions for which 55 minutes are allowed. The use of calculators is *not* permitted in Part A.

Part B has 17 multiple-choice questions for which 50 minutes are allowed. Some of the questions in Part B require the use of a graphing calculator.

Section II, the free-response section, has a total of six questions in two parts:

Part A has two questions, of which some parts *require* the use of a graphing calculator. After 30 minutes, however, *you will no longer be permitted to use a calculator.* If you finish Part A early, you will not be permitted to start work on Part B.

Part B has four questions and you are allotted an additional 60 minutes, but *you are not allowed to use a calculator.* You may work further on the Part A questions (without your calculator).

The section that follows gives important information on its use (and misuse!) of the graphing calculator.

THE GRAPHING CALCULATOR: USING YOUR GRAPHING CALCULATOR ON THE AP EXAM
图形计算器：在AP考试中使用您的图形计算器

The Four Calculator Procedures 计算器使用四步

Each student is expected to bring a graphing calculator to the AP Exam. Different models of calculators vary in their features and capabilities; however, there are four procedures you must be able to perform on your calculator:

C1. Produce the graph of a function within an arbitrary viewing window.

C2. Solve an equation numerically.

C3. Compute the derivative of a function numerically.

C4. Compute definite integrals numerically.

Guidelines for Calculator Use 计算器使用指南

1. On multiple-choice questions in Section I, Part B, *you may use any feature or program on your calculator.* Warning: Don't rely on it too much! Only a few of these questions require the calculator, and in some cases using it may be too time-consuming or otherwise disadvantageous.

2. On the free-response questions of Section II Part A:

(a) You may use the calculator to perform any of the four listed procedures. When you do, you need only write the equation, derivative, or definite integral (called the "setup") that will produce the solution, then write the calculator result to the required degree of accuracy (three places after the decimal point unless otherwise specified). Note especially that a setup must be presented in standard algebraic or calculus notation, not just in calculator syntax. For example, you *must* include in your work the setup $\int_0^{\pi} \cos t \, dt$ even if you use your calculator to evaluate the integral.

(b) For a solution for which you use a calculator capability other than the four listed above, you must write down the mathematical steps that yield the answer. A correct answer alone will not earn full credit.

(c) You must provide *mathematical reasoning* to support your answer. Calculator results alone will not be sufficient.

The Procedures Explained 步骤说明

Here is more detailed guidance for the four allowed procedures.

C1. "Produce the graph of a function within an arbitrary viewing window." Be sure that you create the graph *in the window specified,* then copy it carefully onto your exam paper. If no window is prescribed in the question, clearly indicate the window dimensions you have used.

C2. "Solve an equation numerically" is equivalent to "Find the zeros of a function" or "Find the point of intersection of two curves." Remember: you must first show your setup—write the equation out algebraically; then it is sufficient just to write down the calculator solution.

C3. "Compute the derivative of a function numerically." When you seek the value of the derivative of a function at a specific point, you may use your calculator. First, indicate what you are finding—for example, $f'(6)$—then write the numerical answer obtained from your calculator. Note that if you need to find the derivative itself, rather than its value at a particular point, you must show how you obtained it and what it is, even though some calculators are able to perform symbolic operations.

C4. "Compute definite integrals numerically." If, for example, you need to find the area under a curve, you must first show your setup. Write the complete integral, including the integrand in terms of a single variable and with the limits of integration. You may then simply write the calculator answer; you need not compute an antiderivative.

Sample Solutions of Free-Response Questions
开放式题目的样例解析

The following set of examples illustrates proper use of your calculator on the examination. In all of these examples, the function is

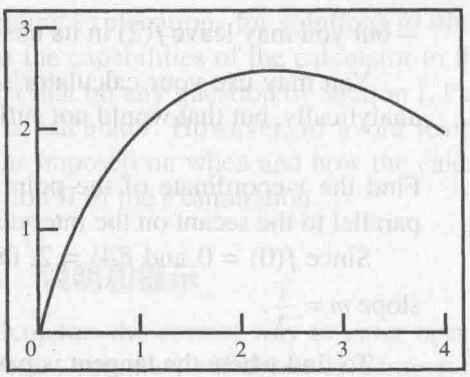
$$f(x)=\frac{10x}{x^2+4} \quad \text{for} \quad 0\leq x\leq 4.$$

1. Graph f in $[0,4]\times[0,3]$.

   Set the calculator window to the dimensions printed in your exam paper.

   Graph $y=\dfrac{10x}{x^2+4}$.

   Copy your graph carefully into the window on the exam paper.



Viewing window $[0,4]\times[0,3]$

2. Write the local linearization for $f(x)$ near $x=1$.

   Note that $f(1)=2$. Then, using your calculator, evaluate the derivative:

$$f'(1)=1.2$$

   Then write the tangent-line (or local linear) approximation

$$f(x)\simeq f(1)+f'(1)(x-1)$$
$$\simeq 2+1.2(x-1)=1.2x+0.8$$

   You need not simplify, as we have, after the last equals sign just above.

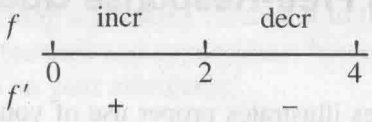
3. Find the coordinates of any maxima of f. Justify your answer.

Since finding a maximum is not one of the four allowed procedures, you must use calculus and show your work, writing the derivative algebraically and setting it equal to zero to find any critical numbers:

$$f'(x) = \frac{(x^2+4)10 - 10x(2x)}{(x^2+4)^2} = \frac{40-10x^2}{(x^2+4)^2}$$
$$= \frac{10(2-x)(2+x)}{(x^2+4)^2}$$

Then $f'(x) = 0$ at $x = 2$ and at $x = -2$; but -2 is not in the specified domain.

We analyze the signs of f' (which is easier here than it would be to use the second-derivative test) to assure that $x = 2$ does yield a maximum for f. (Note that the signs analysis alone is *not* sufficient justification.)



Since f' is positive to the left of $x = 2$ and negative to the right of $x = 2$, f does have a maximum at

$$\left(2, \frac{10(2)}{2^2+4}\right) = \left(2, \frac{5}{2}\right)$$

—but you may leave $f(2)$ in its unsimplified form, without evaluating to $\frac{5}{2}$.

You may use your calculator's maximum-finder to *verify* the result you obtain analytically, but that would not suffice as a solution or justification.

4. Find the x-coordinate of the point where the line tangent to the curve $y = f(x)$ is parallel to the secant on the interval [0,4].

Since $f(0) = 0$ and $f(4) = 2$, the secant passes through (0,0) and (4,2) and has slope $m = \frac{1}{2}$.

To find where the tangent is parallel to the secant, we find $f'(x)$ as in Example 3. We then want to solve the equation

$$f'(x) = \frac{40-10x^2}{(x^2+4)^2} = \frac{1}{2}$$

The last equality above is the setup; we use the calculator to solve the equation: $x = 1.458$ is the desired answer.

5. Estimate the area under the curve $y = f(x)$ using the Trapezoid Rule with four equal subintervals.

$$T = \frac{h}{2}[f(0) + 2f(1) + 2f(2) + 2f(3) + f(4)]$$
$$= \frac{1}{2}\left[0 + 2(2) + 2\left(\frac{5}{2}\right) + 2\left(\frac{30}{13}\right) + 2\right]$$

You may leave the answer in this form or simplify it to 7.808. If your calculator has a program for the Trapezoid Rule, you may use it to *complete* the computation after you have shown the setup as in the two equations above. If you omit them you will lose credit.

6. Find the volume of the solid generated when the curve $y = f(x)$ on [0,4] is rotated about the x-axis.

Using disks, we have

$$\Delta V = \pi R^2 H = \pi y^2 \Delta x\,,$$

$$V = \pi \int_0^4 y^2 dx$$

Note that the equation above is not yet the setup: the definite integral must be in terms of x alone:

$$V = \pi \int_0^4 \left(\frac{10x}{x^2+4}\right)^2 dx$$

Now we have shown the setup. Using the calculator we can evaluate V:

$$V = 55.539$$

A Note About Solutions in This Book 关于本书题解的一点说明

Students should be aware that in this book we sometimes do *not* observe the restrictions cited above on the use of the calculator. In providing explanations for solutions to illustrative examples or to exercises we often exploit the capabilities of the calculator to the fullest. Indeed, students are encouraged to do just that on any question of Section I, Part B, of the AP examination for which they use a calculator. However, to avoid losing credit, you must carefully observe the restrictions imposed on when and how the calculator may be used in answering questions in Section II of the examination.

Additional Notes and Reminders 说明和提示

• **Syntax.** Learn the proper syntax for your calculator: the correct way to enter operations, functions, and other commands. Parentheses, commas, variables, or parameters that are missing or entered in the wrong order can produce error messages, waste time, or (worst of all) yield wrong answers.

• **Radians.** Keep your calculator set in radian mode. Almost all questions about angles and trigonometric functions use radians. If you ever need to change to degrees for a specific calculation, return the calculator to radian mode as soon as that calculation is complete.

• **Trigonometric Functions.** Many calculators do not have keys for the secant, cosecant, or cotangent function. To obtain these functions, use their reciprocals.

For example, $\sec\left(\frac{\pi}{8}\right) = 1/\cos\left(\frac{\pi}{8}\right)$.

Evaluate inverse functions such as arcsin, arccos, and arctan on your calculator. Those function keys are usually denoted as $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$.

Don't confuse reciprocal functions with inverse functions. For example:

$$\cos^{-1}\left(\frac{1}{2}\right) = \arccos\left(\frac{1}{2}\right) = \frac{\pi}{3};$$

$$(\cos 2)^{-1} = \frac{1}{\cos 2} = \sec 2 \approx -2.403;$$

$$\cos 2^{-1} = \cos\left(\frac{1}{2}\right) \approx 0.878;$$

$\cos^{-1}(2) = \arccos 2$, which does not exist.

• **Numerical Derivatives.** You may be misled by your calculator if you ask for the derivative of a function at a point where the function is not differentiable, because the calculator evaluates numerical derivatives using the difference quotient (or the symmetric difference quotient). For example, if $f(x) = |x|$, then $f'(0)$ does not exist. Yet the calculator may find the value of the derivative as

$$\frac{f(x+0.001)-f(0)}{0.001} = 1 \quad \text{or} \quad \frac{f(x+0.001)-f(x-0.001)}{0.002} = 0.$$

Remember: always be sure f is differentiable at a before asking for $f'(a)$.

• **Improper Integrals.** Most calculators can compute only definite integrals. Avoid using yours to obtain an improper integral, such as

$$\int_0^{\pi} \frac{1}{x^2}\,dx \quad \text{or} \quad \int_0^2 \frac{dx}{(x-1)^{2/3}}$$

• **Rounding-Off Errors.** To achieve three-place accuracy in a final answer, do *not* round off numbers at intermediate steps, since this is likely to produce error-accumulations. If necessary, store longer intermediate answers internally in the calculator; do *not* copy them down on paper (storing is faster and avoids transcription errors). Round off only after your calculator produces the final answer.

• **Rounding the Final Answer: Up or Down?** In rounding to three decimal places, remember that whether one rounds down or up depends on the nature of the problem. The mechanical rule followed in accounting (anything less than 0.0005 is rounded down, anything equal to or greater than 0.0005 is rounded up) does not apply.

Suppose, for example, that a problem seeks the largest k, to three decimal places, for which a condition is met, and the unrounded answer is 0.1239 . . . . Then 0.124 is too large: it does not meet the condition. The rounded answer must be 0.123. However, suppose that an otherwise identical problem seeks the smallest k for which a condition is met. In this case 0.1239 meets the condition but 0.1238 does not, so the rounding must be up, to 0.124.

• **Final Answers to Section II Questions.** Although we usually express a final answer in this book in simplest form (often evaluating it on the calculator), this is hardly ever necessary on Section II questions of the AP Examination. According to the directions printed on the exam, "unless otherwise specified" (1) you need not simplify algebraic or numerical answers; (2) answers involving decimals should be correct to three places after the decimal point. However, be aware that if you try to simplify, you must do so correctly or you will lose credit.

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