

基础培训

新东方AP考试指定辅导教程

AP 物理C 力学和电磁学

Mechanics & Electromagnetism

- 国内优秀的中英文结合教材 ·
- 易于理解的AP物理C知识体系 ·
- 全面覆盖AP物理C考点 ·



祁恩云 · 编著

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前言

美国大学理事会（College Board）推出的美国大学先修课程AP（Advanced Placement）是面向优秀高中生的大学基础课程。AP物理课程是AP考试项目的一个重要组成部分，分为物理1、物理2和物理C。AP物理C由“力学”和“电磁学”两门课程组成，是为报考理工科的学生设置的。物理C这两门课程都需要微积分的知识，因此算是有一定难度的课程，尤其对于想要自学的学生而言难度更大。作者在加拿大为高中生讲授多年高中物理和AP物理课程，有丰富的SAT物理、AP物理教学和考试经验；在国内有多年的大学物理教学经验和教授高中学生SAT、AP物理的经验。作者参考国外高中、大学物理教材、主流考试辅导教材，结合中国学生的数理基础、英语语言水平，并整理多年的教学案例，编著了这本适合中国学生的AP物理C备考教材。教材内容以英文为主，以便考生适应英文考试；同时，配以简要的中文和生僻词汇注释，方便考生快速理解和掌握所学的内容。

教材特点

1. 考试内容全部用英文进行讲解。
2. 侧边页用中文概述对应位置的英文知识点。
3. 每章的练习题和书后的模拟题均与真题的形式相同。
4. 全面覆盖AP物理核心知识点。
5. 用中文总结知识点，中文解答习题。

教材使用说明

本书既可作为国际学校、国际班、考试辅导机构的辅导教材，也很适合考生自学。本教材适合有微积分基础的高中在读学生。英文能力较强的学生要以英文内容学习为主，知识框架学习需看每章后的总结和每页侧边页的中文概述；英文能力中等的学生需要中、英文兼顾掌握知识点和物理英文术语；而对于英文能力不太强、高中物理又很好的学生来说，要重点学习每页侧边页的中文概述和每章后的中文总结，并在英文内容中学习英文术语。书中每章都有适量的英文练习题供考生巩固所学，学完全部知识点后还有三套全真模拟题来检验学习成果，最终找到知识弱点后加以强化。之后就可到官网下载历年真题进行最后的考前训练。

AP物理C考试介绍

AP物理C力学和电磁学是两个独立的考试科目，依据准备报考的专业，学生可以只参加力学部分或电磁学部分的考试，也可以两个科目的考试都参加。两科考试独立评分，分别给出成绩。AP物理考试采用5分制，一般情况下，美国大学要求4分以上可以抵学分，而名牌大学则多数要求5分。AP成绩是影响大学录取的重要因素。

AP物理C力学：考试时间为90分钟。由两部分组成，多项选择题（multiple-choice question）部分与问答题（free-response question）部分，各占总成绩的50%，考试时间各占45分钟。第一部分共有35道选择题；第

二部分有三道问答计算题，每道题通常又包含几个小问。两部分都可以使用计算器，还会给考生提供一个常用物理公式和常数表。

力学作为物理学的一部分，是研究自然界最基本的运动形式的一门科学。力学的考试内容基于AP物理1的力学内容，融入向量和微积分的知识，给出力学概念的最基本定义，基本物理定律的积分、微分形式。难点在简谐振动、力矩、转动惯量、角动量和刚体转动。

AP物理C电磁学：电磁学也是物理学中一门重要的基础学科，是经典物理学的基本组成部分之一，与近代自然科学、技术的许多领域有着密切的联系，是大学工程学院和理学院不可缺少的基础课程之一。考试时间与形式同物理C力学。电磁学是一门较难的课程，不但要用到向量、微积分运算，还有一些新的知识。难点在高斯定理、环路定理的理解和应用，以及场强、电势、电磁感应和暂态电路的计算。

力学考试内容和所占比例		电磁学考试内容和所占比例	
内容	比例	内容	比例
运动学	18%	静电场	30%
牛顿定律	20%	静电场中的导体、电解质及电容	14%
功和能	14%	电路	20%
动量	12%	磁场	20%
圆周运动和转动	18%	电磁感应和麦克斯韦方程组	16%
振动和万有引力	18%		

考试指南

AP物理C考试中选择题有A、B、C、D和E五个选项，其中只有一个是正确答案，选错不扣分。选择题主要考查对概念的理解和评估知识的广度，通常计算比较简单，有些题目甚至不用计算，依据分析判断即可得出答案。

问答题考查考生深入理解基本物理概念和原理及其在复杂问题中的应用，利用数学工具，如微积分，给出物理概念和原理的公式表达，并用之来解决复杂的物理问题。按评分标准，问答题每一正确步骤都可以得到相应的分数，解答问答题的应试策略如下：

1. 做问答题前，通读所有的题目，先做有把握的题。
2. 做好时间分配，不要在一道题上耗费太多时间，因而导致没有时间完成其他题目。
3. 解题过程中，尽量写下所有的步骤，以免遗漏采分点。标清楚题号，以免漏题。
4. 通常问答题会有几个部分，要尽量回答每一个问题，因为每部分的分数都是独立的，即使前面的题目没有答对，也不影响后面题目的得分。
5. 在计算过程中，要注意单位，避免因单位错误或遗漏而丢分。
6. 要清楚恰当地标出所有图形图表标记，如图形图表的名称、 x 和 y 坐标轴和单位等。

从2015年开始，AP物理C整个考试过程中都可以使用公式表和计算器（之前只在问答题中可以使用），这意味着对定量计算的要求在提高，选择题中也会出现较难的题目；同时实验部分的题目也在加强。考试总体趋势是：难度在增加。

希望考生通过本教材的学习，全面、快速掌握AP物理C考试内容，尽早熟悉和融入美国教育体系，在考试中取得理想成绩，成功迈进美国名校的大门。

祁恩云

致 谢

教材的编写和出版得到了俞敏洪老师的大力支持，在此深表谢意。感谢王国明先生在图表制作和编审工作中所做的大量工作。同时感谢我的女儿，王雨桐，根据自己对AP课程学习和考试的切身体验，对本书编写形式和内容提出的有益建议，并参与本书的编审工作。本书在编写过程中难免有疏漏与不妥之处，还望各位同行和读者指正，不胜感谢！

祁恩云

作者简介

祁恩云：于哈尔滨理工大学任教，从事大学物理教学多年，具有丰富的物理教学经验；曾作为公派访问学者在加拿大阿尔伯塔大学进行合作研究；曾任温哥华功力数理学院（Vancouver Power Math and Science Academy, Canada）教师。有多年加拿大数学、物理教学经验，精通北美标准化考试SAT II和AP数理课程教学。

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PART ONE

力学

Mechanics

1.1 Scalar and Vector

1.2 Addition and Subtraction of Vectors

1.3 Components and Resultants of Vectors

1.4 Dot Product (Scalar Product) of Vectors

1.5 Cross Product (Vector Product) of Vectors

1.6 Derivative of Vectors

1.1 标量和向量

1.2 向量加减法

1.3 向量分解与合成

1.4 向量点积

1.5 叉积（向量积）

1.6 向量导数

Introduction: Vectors will show up almost all places in our study of physics and obey certain rules for vector operations. The principal objective of this chapter is to learn the rules for the arithmetic, algebra and calculus of vectors. It is important to deal with them.

1.1 Scalar and Vector

There are two kinds of physical quantities—**scalar and vector**. A scalar is a quantity which has magnitude but no associated direction, such as mass, time and energy. They are specified completely by giving a number and units. A vector is a quantity which has direction as well as magnitude, such as displacement, velocity, force... etc.

Scalars can be equated with scalars, and vectors can be equated with vectors, but scalars can never be equated with vectors.

Drawing a diagram of a particular physical situation is always helpful in physics, and this is especially true when dealing with vectors. On a diagram, a vector is generally represented by an arrow whose direction is in the direction of the vector and whose length is proportional to the vector's magnitude.

When we write the symbol for a vector, we always use boldface type. Thus for velocity we write **v**. In handwritten work, the symbol for a vector can be indicated by putting an arrow over it, a \vec{F} for force.

Vector notation: Vector **A**

Its magnitude and direction may be represented by a line OP directed from the initial point O to the terminal point P and denoted by \overrightarrow{OP} .

1.1 标量和向量

物理量有两种：标量和向量。

标量是只有大小的物理量。向量是既有大小又有方向的物理量。

标量和向量是两类不同的物理量，永远不会相等。

向量用有方向的线段图示。在印刷体中用黑体字符表示向量，**F**。手写时则在相应的物理量符号上方加上小箭头， \vec{F} 。

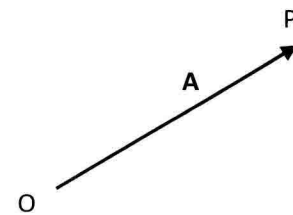


Figure 1-1 Vector notation

1.2 Addition and Subtraction of Vectors

1.2 向量加减法

The two vectors **A** and **B** are added in following way: Triangle method and Parallelogram method.

向量加法遵循三角形法则和平行四边形法则。

To add **A** and **B**, the tip of **A** is placed at the tail of **B**. The arrow drawn from the tail of the first vector to the tip of the second represents the sum, or **resultant**, of the two vectors. This method is known as the tail-to-tip method of adding vectors.

向量平移后不变。

Note that vectors can be translated parallel to themselves to accomplish these manipulations.

A second way to add two vectors is the parallelogram method. It is fully equivalent to the tail-to-tip method.

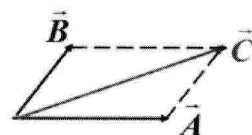
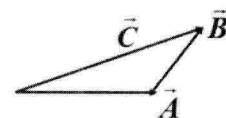


Figure 1-2 Addition of two vectors

Subtraction of vectors, and Multiplication of a Vector by a Scalar:

Given a vector **V**, we define the negative of this vector (**-V**) to be a vector with the same magnitude as **V** but opposite in direction.

$$\mathbf{V} + (-\mathbf{V}) = 0$$

The difference between two vectors, **A - B** is defined as

$$\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$$

Thus our rules for addition of vectors can be applied as shown in Figure 1-3.

两向量差等于第一个向量加上第二个向量的相反向量,由此向量减法转化成向量加法。

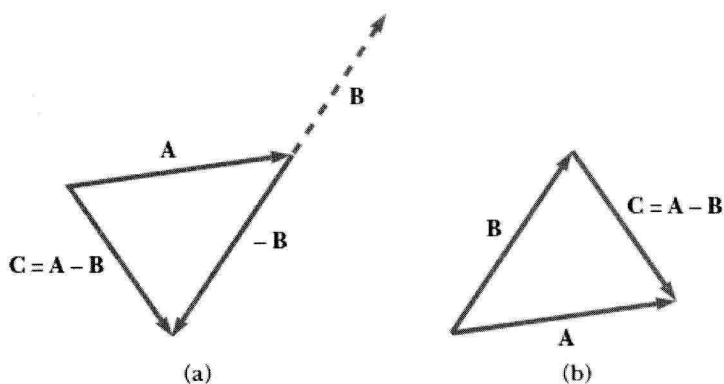


Figure 1-3 Subtracting two vectors: **A - B**

The subtraction between two vectors is the vector that extends from the head of the 2nd vector to the head of the 1st vector.

两向量相减: 两向量顶端相连指向被减向量。

A vector \mathbf{A} can be multiplied by a scalar c . We define $c\mathbf{A}$ to be a vector whose magnitude is $|c|\mathbf{A}$ and whose direction is same as \mathbf{A} if $c > 0$, but is opposite to \mathbf{A} if $c < 0$.

向量数值乘法：方向不变或反向。

1.3 Components and Resultants of Vectors

An advantage of using vector components is that complicated vector problems can be replaced by algebraic manipulation.

There are generally two ways to write vectors: using polar coordinates and using Cartesian coordinates.

Unit vector \vec{A}_0 is a vector with magnitude 1 and in same direction

with \vec{A} , denoted $\vec{A}_0 = \frac{\vec{A}}{A}$ or $\vec{A} = A \vec{A}_0$.

For a fixed Cartesian system, unit vectors \vec{i} , \vec{j} and \vec{k} which have unit magnitude and only indicate directions, are unit vectors along OX, OY, OZ and are mutually perpendicular to one another.

A component of a vector is the projection of the vector on an axis. Components may be in two or three dimensional coordinate system.

The components of a vector \mathbf{A} in three dimension are a_x, a_y, a_z .

$$\vec{A} = a_x \vec{i} + a_y \vec{j} + a_z \vec{k}$$

$$\text{magnitude: } |\vec{A}| = A = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

$$\text{direction: } \cos \alpha = \frac{a_x}{A} \quad \cos \beta = \frac{a_y}{A} \quad \cos \gamma = \frac{a_z}{A}$$

α, β, γ are angles formed by the vector and three axes.

A vector in two dimensions can be represented as

$$\mathbf{A} = A_x \vec{i} + A_y \vec{j} \quad \text{or} \quad \mathbf{A} = \langle A_x, A_y \rangle$$

Components are $A_x = A \cos \theta$ and $A_y = A \sin \theta$

1.3 向量分解与合成

解决复杂向量合成问题。

向量的两种表达方式：

极坐标和直角坐标。

单位向量：在某一方向上大小为 1 的向量，等于向量与向量大小相除。

直角坐标系中三个坐标轴方向单位向量是： \mathbf{i} , \mathbf{j} , \mathbf{k} 。

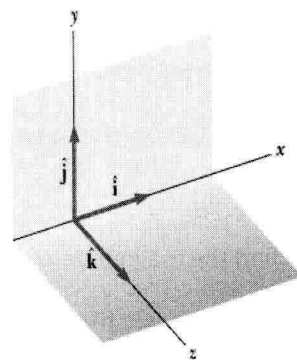


Figure 1-4 直角坐标系单位

向量

三维向量分量、大小、方向表示。

二维向量分量、大小及方向表示。

Magnitude of A is: $A = \sqrt{A_x^2 + A_y^2}$

Direction of A is: $\tan \theta = \frac{A_y}{A_x}$

θ : the angle formed by the vector **A** and positive x axis.

Resultants of vectors: If two vectors **A + B = C**, and

$$\mathbf{A} = A_x \vec{i} + A_y \vec{j} \quad \mathbf{B} = B_x \vec{i} + B_y \vec{j}$$

$$\text{We have } \mathbf{C} = C_x \vec{i} + C_y \vec{j} = \mathbf{B} + \mathbf{A} = (A_x + B_x) \vec{i} + (A_y + B_y) \vec{j}$$

And obviously $C_x = A_x + B_x$ and $C_y = A_y + B_y$

Example 1.1: Suppose vectors **A**=<-3,-1>, **B**=<-1,4>, find the magnitude of **A+B**

Solution:

$$\mathbf{A+B} = <-3+(-1), -1+4> = <-4, 3>$$

Example 1.2: Given the two dimensional position vector

$$\mathbf{R} = (2.5\vec{i} - 3\vec{j}) \text{ (m)},$$

- find the magnitude of R.
- find the angle that R makes with the positive x-direction.
- if R is parallel transported to another location in the plane, what happens to the answers to parts (a) and (b)?

Solution:

$$\text{a. } R = \sqrt{2.5^2 + (-3)^2} = 3.91\text{m}$$

$$\text{b. } \theta = \tan^{-1}\left(\frac{-3}{2.5}\right) = -50.2^\circ$$

- If the vector is parallel transported to another location in the plan, its magnitude and direction are unchanged. The answers to parts (a) and (b) thus remain the same.

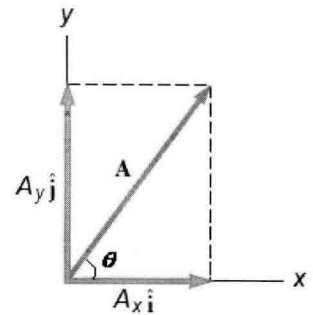


Figure 1-5 向量在二维直角坐标中的分量

向量直角坐标表达: <a,b>

向量直角坐标表达: ai+bj

1.4 Dot Product (Scalar Product) of Vectors

1.4 向量点积

The dot product is often used in physic problems associated with work and power. It describes the projection of one vector onto another and results in a **scalar** product.

两向量点积的结果是标量。
是一个向量对另一个投影。

1.4.1 Definition of Dot Product

1.4.1 点积定义

The scalar product of vectors **A** and **B** (denoted by **A·B**) is defined as

$$\mathbf{A} \cdot \mathbf{B} = AB \cos \theta$$

$$\mathbf{A} \cdot \mathbf{B} = AB \cos \theta$$

点积符合交换律。

Where θ is the angle between **A** and **B**. By definition, **A·B** is a scalar.

$$\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$$

According to the definition, **A·B** = **B·A**, we have: **A·A** = A^2 .

相同单位向量的点积。

In particular, $\vec{i} \cdot \vec{i} = \vec{j} \cdot \vec{j} = \vec{k} \cdot \vec{k} = 1$

$$\vec{i} \cdot \vec{i} = \vec{j} \cdot \vec{j} = \vec{k} \cdot \vec{k} = 1$$

If **A** and **B** are parallel vectors in the same direction, then **A·B** = AB. If **A** and **B** are parallel vectors in the opposite direction, then

两平行向量点积。

方向相同: **A·B** = AB

$$\mathbf{A} \cdot \mathbf{B} = -AB.$$

方向相反: **A·B** = -AB

According to the definition, **A·B** = $AB \cos \theta = 0$, means $A = 0$, or $B = 0$, or $\cos \theta = 0$.

两向量相互垂直点积为零。

A and **B** are perpendicular if **A·B** = 0, given that **A** and **B** are none zero vectors.

$$\mathbf{A} \perp \mathbf{B}: \mathbf{A} \cdot \mathbf{B} = 0$$

两不同单位向量点积为零。

In particular, $\vec{i} \cdot \vec{j} = \vec{j} \cdot \vec{k} = \vec{i} \cdot \vec{k} = 0$

$$\vec{i} \cdot \vec{j} = \vec{j} \cdot \vec{k} = \vec{i} \cdot \vec{k} = 0$$

1.4.2 Dot Product in Coordinate Form

1.4.2 向量点积坐标表示

In rectangular coordinate system, we have

两个不为零的向量点积的结果可能为正、负、或零。

$$\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z$$

From the definition of scalar product **A·B** = $AB \cos \theta$, we see that:
The angle θ between **A** and **B** can be found using

两向量的夹角公式

$$\cos \theta = \frac{\mathbf{A} \cdot \mathbf{B}}{AB}$$

$$\cos \theta = \frac{\mathbf{A} \cdot \mathbf{B}}{AB}$$

If $\mathbf{A} \cdot \mathbf{B} > 0$, θ is acute; if $\mathbf{A} \cdot \mathbf{B} < 0$, θ is obtuse.

Several physical concepts (work, electric and magnetic flux) require dot product we multiply the magnitude of one vector by the magnitude of the component of the other vector that is parallel to the first. That is why the dot product was invented.

力学中的功, 电磁学中的电通量和磁通量都由点积来定义。

Example 1.3

a. Find the angle between the two vectors $\mathbf{A} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k} = (1, -2, 2)$,

$\mathbf{B} = 2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k} = (2, 3, -6)$.

b. Points A, B and C have coordinates (2, 3, 4), (-2, 1, 0) and (4, 0, 2)

respectively. Calculate $\angle BAC$.

Solution:

$$\text{a. } \cos \theta = \frac{\mathbf{A} \cdot \mathbf{B}}{AB} = \frac{(1, -2, 2) \cdot (2, 3, -6)}{\sqrt{1^2 + (-2)^2 + 2^2} \cdot \sqrt{2^2 + 3^2 + (-6)^2}} = \frac{-16}{21}$$

$$\theta = 139.6^\circ$$

$$\text{b. } \overline{AB} = (-2 - 2, 1 - 3, 0 - 4) = (-4, -2, -4)$$

$$\overline{AC} = (4 - 2, 0 - 3, 2 - 4) = (2, -3, -2)$$

$$\text{Use } \angle BAC = \frac{\overline{AB} \cdot \overline{AC}}{AB \cdot AC} = \frac{(-4, -2, -4) \cdot (2, -3, -2)}{\sqrt{36} \cdot \sqrt{17}} = \frac{1}{\sqrt{17}}$$

$$\angle BAC = 76^\circ$$

1.5 Cross Product (Vector Product) of Vectors

1.5 叉积 (向量积)

1.5.1 Definition of Cross Product

1.5.1 叉积定义

The vector product of vectors \mathbf{A} and \mathbf{B} (denoted by $\mathbf{C} = \mathbf{A} \times \mathbf{B}$) is defined as a vector whose direction is perpendicular to both \mathbf{A} and \mathbf{B} and followed the right-hand rule, and whose magnitude is $A B \sin \alpha$ where α is the angle between \mathbf{A} and \mathbf{B} .

两个向量叉积的结果是向量。方向垂直于这两个向量组成的平面, 可用下图右旋法则确定。

Specifically, $\mathbf{A} \times \mathbf{B} = (AB \sin \alpha) \vec{C}_0$

where \vec{C}_0 is a unit vector perpendicular to both \mathbf{A} and \mathbf{B} .

There are two equivalent ways to envision **the vector product right hand rule**.

1) Orient the index finger of your right hand along the direction of the first vector of the product and the middle finger along the direction of the second vector, as in Figure 1-6 a. The extended thumb then indicates the direction of the vector product $\mathbf{A} \times \mathbf{B}$.

2) To find the direction, alternatively, points the four bunched fingers of your right hand in the direction of \mathbf{A} and then curl them in the direction of \mathbf{B} . Your thumb will be pointing in the direction of $\mathbf{A} \times \mathbf{B}$, as shown in Figure 1-6 b. Always choose the smaller of the two possible angles for swinging your fingers from the first vector to the second vector of the product.

Obviously $\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A} \neq \mathbf{B} \times \mathbf{A}$

$$\mathbf{A} \times \mathbf{A} = 0$$

So, if $\mathbf{A} \times \mathbf{B} = 0$ then \mathbf{A} and \mathbf{B} are parallel.

In the three dimensional coordinate system, \mathbf{i} , \mathbf{j} and \mathbf{k} follow that

$$\mathbf{i} \times \mathbf{j} = \mathbf{k}, \mathbf{j} \times \mathbf{k} = \mathbf{i}, \mathbf{k} \times \mathbf{i} = \mathbf{j} \text{ and } \mathbf{i} \times \mathbf{i} = \mathbf{j} \times \mathbf{j} = \mathbf{k} \times \mathbf{k} = 0$$

1.5.2 Cross Product in Matrix Form

If two vectors are defined in unit vector notation, the **cross product** as a vector is equal to the evaluation of the three by three matrix shown. Note that the second row is made up of the components of the *first* vector denoted in the cross product. Note also that you DON'T have to use the *right hand rule* to determine the direction as the direction is determined directly through the matrix evaluation.

$$\vec{A} = a_x \vec{i} + a_y \vec{j} + a_z \vec{k}, \quad \vec{B} = b_x \vec{i} + b_y \vec{j} + b_z \vec{k}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

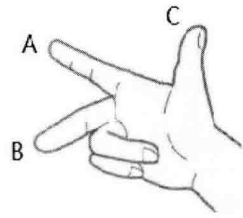


Figure 1-6 a: Right-hand rule 1

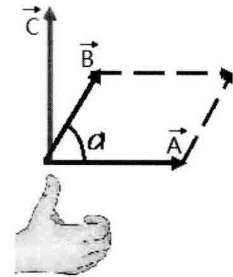


Figure 1-6 b: Right-hand rule 2

叉积不符合交换律。

$$\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A} \neq \mathbf{B} \times \mathbf{A}$$

两向量平行叉积为零。

$$\mathbf{A} \text{ 平行 } \mathbf{B}: \mathbf{A} \times \mathbf{B} = 0$$

单位向量的叉积。

1.5.2 向量叉积矩阵表示

两向量叉积结果可用一个 3×3 矩阵表示。

第二行是第一个向量的分量。

矩阵的行列式直接给出两向量的叉积方向, 不必再用右旋法则判断方向。

Example 1.4: If $\mathbf{A} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$ and $\mathbf{B} = -3\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$,

a. find $\mathbf{A} \times \mathbf{B}$.

b. verify that $\mathbf{A} \times \mathbf{B}$ is perpendicular to both \mathbf{A} and \mathbf{B} .

Solution:

$$\text{a. } \mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 3 & -4 \\ -3 & 2 & -3 \end{vmatrix} = -\mathbf{i} + 18\mathbf{j} + 13\mathbf{k}$$

$$\text{b. } (\mathbf{A} \times \mathbf{B}) \cdot \mathbf{A} = \langle -1, 18, 13 \rangle \cdot \langle 2, 3, -4 \rangle = 0$$

$$\text{And } (\mathbf{A} \times \mathbf{B}) \cdot \mathbf{B} = \langle -1, 18, 13 \rangle \cdot \langle -3, 2, -3 \rangle = 0.$$

Hence $\mathbf{A} \times \mathbf{B}$ is perpendicular to both \mathbf{A} and \mathbf{B} .

1.6 Derivative of Vectors

1.6 向量导数

In our study of motion in the next chapter, we will have frequent need to differentiate vectors which are functions of time.

Differentiation of a vector expressed in Cartesian form proceeds as follows for $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are constant vectors.

直角坐标系中 $\mathbf{i}, \mathbf{j}, \mathbf{k}$ 可看做常数向量。

$$\frac{d\vec{A}}{dt} = \frac{d}{dt}(A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}) = \frac{dA_x}{dt}\mathbf{i} + \frac{dA_y}{dt}\mathbf{j} + \frac{dA_z}{dt}\mathbf{k}$$

The derivative of a vector also is a vector, whose components are the derivatives of the respective scalar component of the original vector. They also follow

向量的导数仍然是向量，遵循加法、乘法运算法则。

$$\frac{d(\vec{A} + \vec{B})}{dt} = \frac{d\vec{A}}{dt} + \frac{d\vec{B}}{dt}$$

$$\frac{d(\vec{A} \cdot \vec{B})}{dt} = \frac{d\vec{A}}{dt} \cdot \vec{B} + \vec{A} \cdot \frac{d\vec{B}}{dt}$$

$$\frac{d(\vec{A} \times \vec{B})}{dt} = \frac{d\vec{A}}{dt} \times \vec{B} + \vec{A} \times \frac{d\vec{B}}{dt}$$