放眼世界的数学星空

Mathematicians Born in India

印度数学家

英文版

石雷 张宝义／编

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前　言

入世后，我国经济和社会发展与世界接轨的进程加快，需要大量的国际化的复合型人才。为迎接入世挑战，培养出更多的国际化的复合型人才，进一步深化素质教育，我国实施了新一轮的中小学课程改革。

在此改革中，“双语教学”已成为外语教学改革中一道亮丽的风景线。当前，我国大中城市的部分高校及中小学、一些境外来华办学机构以及有些民办学校已在实施“双语教学”。“双语教学”已成为教育界的热门话题，并呈现出良好的发展前景。

为顺应“双语教学”的新潮流和大趋势，我们出版了《放眼世界的数学星空》丛书，本丛书介绍了法国数学家、俄罗斯数学家、中国数学家、印度数学家，他们的伟大成就吸引着我们，激励着我们去学习、去拼搏。与此同时，还可以使您在英语字母点缀的星空里，轻松领略数学家

们的才华，并且使您真正提高阅读能力、巩固和扩大英语词汇量、增强使用英语的自信心。

本丛书在选编过程中由于涉及面广，时间仓促，有误之处，敬请广大读者朋友们热忱提出批评和建议，以便今后修订完善。

编　者

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Mathematicians Born in India

Apastamba

Born: about 600 **BC in India**
Died: about 600 **BC in India**

To write a biography of Apastamba is essentially impossible since nothing is known of him except that he was the author of a Sulbasutra which is certainly later than the Sulbasutra of Baudhayana. It would also be fair to say that Apastamba's Sulbasutra is the most interesting from a mathematical point of view. We do not know Apastamba's dates accurately enough to even guess at a life span for him, which is why we have given the same approximate birth year as death year.

Apastamba was neither a mathematician in the sense that we would understand it today, nor a scribe who simply copied manuscripts like Ahmes. He would certainly have been a man of very considerable learning but probably not interested in mathematics for its own sake, merely interested in using it for religious purposes. Undoubtedly he wrote the Sulbasutra to provide rules for religious rites and to improve and expand on the rules which had been given by his predecessors. Apastamba would have been a Vedic priest instructing the people in the ways of conducting the religious rites he describes.

The mathematics given in the Sulbasutras is there to enable the accurate construction of altars needed for sacrifices. It is clear from the writing that Apastamba, as well as being a priest and a teacher of religious practices, would have been a skilled craftsman. He must have been himself skilled in the practical use of the mathematics he described as a craftsman who himself constructed sacrificial altars of the highest quality.

The Sulbasutras are discussed in detail in the article Indian Sulbasutras. Below we give one or two details of Apastamba's Sulbasutra. This work is an expanded version of that of Baudhayana. Apastamba's work consisted of six chapters while the earlier work by Baudhayana contained only three.

The general linear equation was solved in the Apastamba's Sulbasutra. He also gives a remarkably accurate value for $\sqrt{2}$ namely

$1+1/3+1/(3\times4)-1/(3\times4\times34)$.

which gives an answer correct to five decimal places. A possible way that Apastamba might have reached this remarkable result is described in the article Indian Sulbasutras.

As well as the problem of squaring the circle, Apastamba considers the problem of dividing a segment into 7 equal parts. The article [3] looks in detail at a reconstruction of Apastamba's version of these two problems.

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Aryabhata the Elder

Born: 476 in Kusumapura (now Patna), India

Died: 550 in India

Aryabhata is also known as Aryabhata I to distinguish him from the later mathematician of the same name who lived about 400 years later. Al—Biruni has not helped in understanding Aryabhata's life, for he seemed to believe that there were two different mathematicians called Aryabhata living at the same time. He therefore created a confusion of two different Aryabhatas which was not clarified until 1926 when B Datta showed that al—Biruni's two Aryabhatas were one and the same person.

We know the year of Aryabhata's birth since he tells us that he was twenty—three years of age when he wrote Aryabhatiya which he finished in 499. We have given Kusumapura, thought to be close to Pataliputra (which was refounded as Patna in Bihar in 1541), as the place of Aryabhata's birth but this is far from certain, as is even the location of Kusumapura itself. As Parameswaran writes in [26]:—

> …*no final verdict can be given regarding the locations of Asmakajanapada and Kusumapura.*

We do know that Aryabhata wrote Aryabhatiya in Kusumapura at the time when Pataliputra was the capital of the Gupta empire and a major centre of learning, but there have been numerous other places proposed by historians as his birthplace. Some conjecture that he was born in south India, perhaps Kerala, Tamil Nadu or Andhra Pradesh, while others conjecture that he was born in the north-east of India, perhaps in Bengal. In [8] it is claimed that Aryabhata was born in the Asmaka region of the Vakataka dynasty in South India although the author accepted that he lived most of his life in Kusumapura in the Gupta empire of the north. However, giving Asmaka as Aryabhata's birthplace rests on a comment made by Nilakantha Somayaji in the late 15th century. It is now thought by most historians that Nilakantha confused Aryabhata with Bhaskara I who was a later commentator on the Aryabhatiya.

We should note that Kusumapura became one of the two major mathematical centres of India, the other being Ujjain. Both are in the north but Kusumapura (assuming it to be close to Pataliputra) is on the Ganges and is the more northerly. Pataliputra, being the capital of the Gupta empire at the time of Aryabhata, was the centre of a communications network which allowed learning from other parts of the world to reach it easily, and also allowed the mathematical and astronomical advances made by Aryabhata and his school to reach across India and also eventually into the Islamic world.

As to the texts written by Aryabhata only one has survived. However Jha claims in [21] that: —

> …*Aryabhata was an author of at least three astronomical texts and wrote some free stanzas as well.*

The surviving text is Aryabhata's masterpiece the Aryabhatiya which is a small astronomical treatise written in 118 verses giving a summary of Hindu mathematics up to that time. Its mathematical section contains 33 verses giving 66 mathematical rules without proof. The Aryabhatiya contains an introduction of 10 verses, followed by a section on mathematics with, as we just mentioned, 33 verses, then a section of 25 verses on the reckoning of time and planetary models, with the final section of 50 verses being on the sphere and eclipses.

Now there is a difficulty with this layout which is discussed in detail by van der Waerden in [35]. Van der Waerden suggests that in fact the 10 verse Introduction was written later than the other three sections. One reason for believing that the two parts were not intended as a whole is that the first section has a different meter to the remaining three sections. However, the problems do not stop there. We said that the first section had ten verses and indeed Aryabhata titles the section Set of ten giti stanzas. But it in fact contains eleven giti stanzas and two arya stanzas. Van der Waerden suggests that three verses have been added and he identifies a small number of verses in the remaining sections which he argues have also been added by a member of Aryabhata's school at Kusumapura.

The mathematical part of the Aryabhatiya covers arithmetic, algebra, plane trigonometry and spherical trigonometry. It also contains continued fractions, quadratic equations, sums of power series and a table of sines. Let us examine some of these in a little more detail.

First we look at the system for representing numbers which Aryabhata invented and used in the Aryabhatiya. It consists of giving numerical values to the 33 consonants of the Indian alphabet to represent 1, 2, 3, …, 25, 30, 40, 50, 60, 70, 80, 90, 100. The higher numbers are denoted by these consonants followed by a vowel to obtain 100, 10000, ⋯In fact the system allows numbers up to 10^{18} to be represented with an alphabetical notation. Ifrah in [3] argues that Aryabhata was also familiar with numeral symbols and the place—value system. He writes in [3]:—

> ⋯*it is extremely likely that Aryabhata knew the sign for zero and the numerals of the place value system. This supposition is based on the following two facts: first, the invention of his alphabetical counting system would have been impossible without zero or the place—value system; secondly, he carries out calculations on square and cubic roots which are impossible if the numbers in question are not written according to the place—value system and zero.*

Next we look briefly at some algebra contained in the Aryabhatiya. This work is the first we are aware of which examines integer solutions to equations of the form $by=ax+c$ and $by=ax-c$, where a, b, c are integers. The problem arose from studying the problem in astronomy of determining the periods of the planets. Aryabhata uses the kuttaka method to solve problems of this type. The word kuttaka means "to pulverise" and the method consisted of breaking the problem down into new problems where the coefficients became smaller and smaller with each step. The method here is essentially the use of the Euclidean algorithm to find the highest common factor of a and b but is also related to continued fractions.

Aryabhata gave an accurate approximation for π. He wrote in the Aryabhatiya the following:—

Add four to one hundred, multiply by eight and then add sixty—two thousand. the result is approximately the circumference of a circle of diameter twenty thousand. By this rule the relation of the circumference to diameter is given.

This gives $\pi = 62832/20000 = 3.1416$ which is a surprisingly accurate value. In fact $\pi = 3.14159265$ correct to 8 places. If obtaining a value this accurate is surprising, it is perhaps even more surprising that Aryabhata does not use his accurate value for π but prefers to use $\sqrt{10} = 3.1622$ in practice. Aryabhata does not explain how he found this accurate value but, for example, Ahmad [5] considers this value as an approximation to half the perimeter of a regular polygon of 256 sides inscribed in the unit circle. However, in [9] Bruins shows that this result cannot be obtained from the doubling of the number of sides. Another interesting paper discussing this accurate value of π by Aryabhata is [22] where Jha writes: —

> *Aryabhata I's value of π is a very close approximation to the modern value and the most accurate among those of the ancients. There are reasons to believe that Aryabhata devised a particular method for finding this value. It is shown with sufficient grounds that Aryabhata himself used it, and several later Indian mathematicians and even the Arabs adopted it. The conjecture that Aryabhata's value of π is of Greek origin is critically examined and is found to be without foundation. Aryabhata discovered this value independently and also realised that π is an irrational number. He had the Indian background, no doubt, but excelled all his predecessors in evaluating π. Thus the credit of discovering this exact value of π may be ascribed to the celebrated mathematician, Aryabhata I.*

We now look at the trigonometry contained in Aryabhata's treatise. He gave a table of sines calculating the approximate values at intervals of $90°/24=3°45'$. In order to do this he used a formula for $\sin(n+1)x-\sin nx$ in terms of $\sin nx$ and $\sin(n-1)x$. He also introduced the versine (versine = 1 − cosine) into