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接触力学与摩擦学中的降维解法

Method of Dimensionality Reduction in Contact Mechanics and Friction

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内 容 简 介

本书完整地描述了一种用于快速计算接触和摩擦问题的仿真方法。与现有的仿真方法不同，降维法(MDR)可将各种三维接触问题精确地转化为一维接触，这样的转化不仅可将三维体系转化为一维体系，而且转化后的系统自由度是独立的。采用这种降维法可以大大减少计算时间，也简化了接触力学中的分析计算过程，适用于采用有限元计算方法解决结构力学和计算流体力学问题。

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序　言

柏林工业大学波波夫(Valentin L. Popov)教授与海斯(Markus Heß)博士编写的《接触力学与摩擦学中的降维解法》(Method of Dimensionality Reduction in Contact Mechanics and Friction)一书致力于解决弹性体和黏弹性体的接触力学与摩擦问题,该书着重考虑了接触界面的摩擦和黏着等因素。与其他接触力学书籍相比,本书的主要不同点在于,作者整体上采用了一种降维(method of dimensionality reduction,MDR)方法。这个看似简单但非常有效的方法使很多研究人员和工程人员,甚至初学者,都能快速地了解接触力学这门学科的丰富内容。同时,在解决实际界面接触问题时,该方法也为理论分析和数值仿真提供了一个有效的工具。

本书提到的降维法是基于轴对称弹性物体在法向接触中的问题求解的,这种方法最早由Galin于1953年、Green和Zerna于1954年分别提出。后来,随着1965年Sneddon的著作出版,这一方法才变得众所周知。本书作者Popov教授发现,Galin-Green-Sneddon方程可以通过一维空间中一系列独立的"弹簧"来诠释。这样的替换提供了一个非常简单的"助记规则",这一规则有助于重现任意轴对称接触问题中的Gallin-Green-Sneddon方程的求解。作者通过类比提出了一个全面的适用于各种接触问题的方法,该方法的基本理论以及进一步的发展来源于接触力学中一系列已经广为人知的定理,这些定理将某些复杂的接触问题降为简单的弹性体的法向接触问题。因此,Cattaneo (1938)、Mindlin (1949)、Jäger (1995)和Ciavarella (1998)的叠加原理才会表述为:摩擦系数为常数的切向接触问题可以转化为相应的法向接触问题。而Lee (1955)和Radok (1957)的经典定理也对任意线性黏弹性接触问题提出了类似的方法,即黏弹性体的法向接触问题可以通过相应的弹性体接触的函数方程来求解。Heß于2011年提出,黏着接触也可以类似地简化到一维模型中求解,这就重现了Borodich和Galanov (2004)以及Yao和Gao (2006)对黏着接触的求解。

降维法有两个主要特点，使其在工程应用中如此有吸引力：

1. 将三维问题降至一维，二者的自由度满足各自空间的需要；

2. 三维接触中的自由度是相互影响的，而对应的一维接触模型中的自由度是相互独立的。

就这两个特点，单独来看，每一点都大大简化了接触问题，不管是做理论分析还是数值计算。综合来说，这两点使得该方法在处理接触问题时实现了大的突破，同时也为界面接触的数值仿真打开了一条全新的道路。

需要注意的是，降维法也有其适用范围。对于某些三维接触问题，降维法给出的是精确的完整解，有些问题给出的是精确的渐近解，在有些情况下降维法的解是精确的，但只适用于特定的物理范畴。对于某些应用领域，降维法的解不是精确解，但给出了非常好的定性分析，比如滚动接触。当然，也有一些问题是不能用降维法来处理的。降维法的适用领域在本书中有详细的描述。

本书适用于很多专业领域，给出的方法是一个在理论计算和数值仿真方面易学易用的工具，这对于求解涉及应力、摩擦、磨损和黏着等因素的力学问题特别适用。此外，这种降维法可以发展为一个标准模块，用于包含界面接触、摩擦和黏着问题的有效系统的动态仿真。因为这种方法简单易懂，特将其推荐给本科生学习及使用。

雒建斌

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2015 年 4 月 8 日

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Preface

The present book, "Method of Dimensionality Reduction in Contact Mechanics and Friction" by Valentin L. Popov and Markus Heß, is devoted to contact mechanics of elastic and viscoelastic bodies with account of friction and adhesion in contact interface. Its main difference to many other publications on contact mechanics is that the whole treatise is consequently based on the so-called Method of Dimensionality Reduction (MDR). This elegant and extremely effective method makes many areas of contact mechanics available to a wide range of even unexperienced scientists and engineers. It provides effective tools for understanding as well as for analytical calculation and numerical simulation of systems with contact interfaces in a broad range of applications.

The MDR is based on the solution of the normal axially-symmetric contact problem of elastic bodies which was first found by Galin (1953) and independently by Green and Zerna (1954) and became widely known through the works of Sneddon (1965). Valentin L. Popov, the author of the book notice that the equations of Galin-Green-Sneddon can be interpreted in terms of a contact with a one-dimensional series of independent "springs". This interpretation provides a very simple "mnemonic rule" enabling easy reproduction of the Gallin-Green-Sneddon relations for any rotationally symmetric profile. Starting with this simple analogy, authors of the book develop a comprehensive method which is applicable to a great variety of contact problems. The theoretical basis for this further development stems from a series of well-known theorems in contact mechanics which reduce some classes of more complicated problems to the simple normal contact problem of elastic bodies. Thus, the well-known superposition principles of Cattaneo (1938), Mindlin (1949), Jäger (1995) and Ciavarella (1998) state that the tangential contact with a constant coefficient of friction can be reduced to the solution of the corresponding normal contact. The classical theorems of Lee (1955) and Radok (1957) provide a similar reduction for viscoelastic contacts with arbitrary linear rheology: The normal contact problem of viscoelastic bodies can be solved with the method of the so-called functional equations proceeding from the corresponding "elastic solution". Finally, Heß (2011) has shown with simple arguments that the adhesive problem can be reduced to the one-dimensional MDR model too, thus reproducing the solutions of Borodich and Galanov (2004) and Yao and Gao (2006) for contacts of arbitrary bodies of revolution.

There are two main features of the MDR which make it so attractive for engineering applications:

1. Firstly, the MDR reduces a contact problem of bodies whose degrees of freedom fill a three-dimensional space by a problem in which degrees of freedom fill a one-dimensional space.

2. Secondly, while in the initial 3D contact problem the degrees of freedom are interacting, the degrees of freedom of the substitute MDR model are independent.

Each of these features alone would greatly simplify the problem, regardless of whether it is handled analytically or numerically. But in combination they provide a real breakthrough for all problems in the realm of validity of the method and open completely new possibilities in numerical simulations of systems with contact interfaces.

Of course, the MDR has its region of validity which has to be carefully observed. For some contact problems, it provides the exact and comprehensive solution of the corresponding three-dimensional problem. For other problems, it gives exact asymptotic solutions. In still other cases, the MDR is exact but applicable only to a restricted set of physical quantities. Then there are areas of application, where the MDR is not exact but provides a very good qualitative solution (e. g. rolling contact). There are, naturally, limits to its validity, some problems cannot be treated with this method. The realm of validity of the MDR is described carefully throughout the book.

The present book is written first of all for practitioners, giving them an easy-to-learn and easy-to-use tool of thinking, analytical calculations and numerical simulations. It will be of interest for engineers optimizing structures with respect to stresses, frictional damping, wear and adhesive strength. The MDR has good chances to become a standard and effective module in dynamic simulation of systems containing contacts and frictional or adhesive interfaces. Due to its simplicity and elegance, the method can be recommended even to undergraduate students.

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Date: April 8, 2015

Foreword

Contact and friction are phenomena that are of extreme importance in uncountable technical applications. Simultaneously, they are phenomena which cause difficulties in their theoretical consideration and numerical simulation. This book presents a method that trivializes two classes of contact problems to such a degree that they become accessible even for first semester engineering students who possess an elementary understanding of mathematics and physics. Furthermore, this method presents a very simple way to numerically simulate contact and frictional forces.

The "trivialization" occurs with the help of the method of dimensionality reduction, which is the primary focus of this book. This method is based on the analogy between certain classes of three-dimensional contacts and contacts with one-dimensional elastic or viscoelastic foundations. Within the framework of the method of dimensionality reduction, three-dimensional contacts are replaced by a series of one-dimensional elastic or viscoelastic elements. In doing this, we would like to strongly accentuate the fact that this *is not an approximation*: Certain macroscopic contact properties correspond *exactly* with those of the original three-dimensional contact.

The method of dimensionality reduction offers a *two-fold* reduction: First, a three-dimensional system is replaced by a one-dimensional system, and second, the resulting degrees of freedom for the one-dimensional system are independent of one another. Both of these properties lead to an enormous simplification in the treatment of contact problems and a qualitative acceleration of numerical simulations.

The method of dimensionality reduction distinguishes itself by four essential properties: It is *powerful*, it is *simple*, it is *proven*, and it is *counterintuitive*. It is difficult to be convinced of its validity. Every expert in the field of contact mechanics who has not yet engaged himself in the detailed proofs of the reduction method would immediately form the opinion that it cannot possibly work. It appears to completely contradict a healthy intuition that a system with another spatial dimension, and furthermore, independent degrees of freedom can correctly agree with a three-dimensional system with spatial interactions. And nevertheless, it works! This book is dedicated to the reasons for and under which limitations this is the case.

In writing this book, we have followed two main goals. The first is the simplest possible presentation of the rules of application of the method. The second is to prove the assertions of the reduction method with strict mathematical evidence, so that even the most rigorous practitioner in contact mechanics may be convinced of the correctness of this method. We have attempted to keep these two goals separated. We attempted to keep the mathematical proofs to a minimum in the chapters in which the fundamentals of practical application are explained. This is primarily in Chap. 3, but also in the immediately following Chaps. (4–7) as well as Chap. 10, which is dedicated to the contact mechanics of rough surfaces. It is clear to us that this has not been possible at every point.

Above all, the method of dimensionality reduction offers the engineer a practical tool. In order to stress the practicality of the method even more, we have included many problems at the end of most chapters, which serve for a better understanding of the use of the reduction method and its areas of application. Therefore, this book can also be used as a textbook in a tribologically oriented course of studies.

Author Biographies

Prof. Dr. rer. nat. Valentin L. Popov studied physics (1976–1982) and obtained his doctorate in 1985 from the Moscow State Lomonosov University. He worked at the Institute of Strength Physics of the Russian Academy of Sciences. After a guest-professorship in the field of theoretical physics at the University of Paderborn (Germany) from 1999 to 2002, he has headed since 2002 the Department of System Dynamics and the Physics of Friction in the Institute of Mechanics at the Berlin University of Technology.

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Dr. Markus Heß studied physical engineering sciences at TU Berlin. He obtained his doctorate in 2011, receiving in the same year the Research Prize from the German Tribological Society for his dissertation. Since 2011, he has worked for the Studienkolleg at TU Berlin, where he teaches engineering mathematics.

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Chapter 10: Normal Contact of Rough Surfaces—together with R. Pohrt
Chapter 11: Frictional Force—together with S. Kürschner
Chapter 12: Frictional Damping—together with E. Teidelt
Chapter 13: The Coupling of Macroscopic Dynamics—together with E. Teidelt
Chapter 14: Acoustic Emission in Rolling Contacts—together with M. Popov und J. Benad
Chapter 15: Coupling on the Microscale—together with R. Pohrt
Chapter 19: Appendix 3: Replacing the Material Properties with Radok's Method of Functional Equations—together with S. Kürschner

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Berlin, May 2014

Valentin L. Popov
Markus Heß

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