

CONTEMPORARY MATHEMATICS

698

Dynamical Systems, Ergodic Theory, and Probability: in Memory of Kolya Chernov

Conference Dedicated to the Memory of Nikolai Chernov
Dynamical Systems, Ergodic Theory, and Probability
May 18–20, 2015
University of Alabama at Birmingham, Birmingham,
Alabama

Alexander M. Blokh
Leonid A. Bunimovich
Paul H. Jung
Lex G. Oversteegen
Yakov G. Sinai
Editors



This volume contains the proceedings of the Conference on Dynamical Systems, Ergodic Theory, and Probability, which was dedicated to the memory of Nikolai Chernov, held from May 18–20, 2015, at the University of Alabama at Birmingham, Birmingham, Alabama.

The book is devoted to recent advances in the theory of chaotic and weakly chaotic dynamical systems and its applications to statistical mechanics. The papers present new original results as well as comprehensive surveys.

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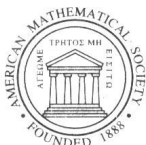
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Dynamical Systems, Ergodic Theory, and Probability: in Memory of Kolya Chernov

Dedicated to the memory of Kolya Chernov

Preface

On August 7, 2014, after a long fight with cancer, our dear friend and colleague Kolya Chernov passed away. Less than a year later, a conference, dedicated to his memory and sponsored by the AMS, convened in Birmingham, AL, where Kolya had worked for the last 20 years. This collection consists of contributions by participants of the conference. We hope that for years to come it will serve as a reflection of Kolya's great influence and remarkable contribution to Mathematics.

Alexander M. Blokh
Leonid A. Bunimovich
Paul H. Jung
Lex G. Oversteegen
Yakov G. Sinai

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N. I. CHERNOV (1956-2014)

Leonid Bunimovich

The Mathematics and Mathematical Physics communities have suffered a heavy and untimely loss. After a long fight with cancer Professor Nikolai Ivanovich (Kolya) Chernov passed away on the seventh of August 2014.

Kolya was born and grew up in the city of Kryvyi Rih in Ukraine (former USSR). His interest in Mathematics appeared early and began to flourish already in Kolya's school years. Kolya Chernov was a winner of the State, the Republic and the International Mathematical Olympiad.

In 1974 Kolya entered the Department of Mechanics and Mathematics of the Moscow State University. He had chosen the field of Probability Theory and started to work under the supervision of Professor Sinai. Kolya graduated at 1979 with Master Degree. His Master thesis was devoted to the study of kneading invariants of one-dimensional maps with an absolutely continuous invariant measure. It was the topic of the first paper out of more than a hundred research papers published by Professor Chernov.

In the same year Kolya became a graduate student in the Department of Mechanics and Mathematics continuing his work with Prof. Sinai. At that time he became interested in the dynamics of chaotic billiards which remained the Love of his scientific life.

The theory of chaotic billiards forms one of the most respected and the most difficult areas in the modern theory of dynamical systems. A billiard dynamics is generated by the motion of a point particle with constant speed in a region Q which is called a billiard table. Upon reaching the boundary of Q the particle gets elastically reflected according to the law "the angle of incidence equals the angle of reflection". Billiards are natural models for many problems in Physics, first of all in Statistical Mechanics. The study of billiards becomes especially difficult when the dimension of the billiard table is greater than two. His PhD thesis, which N. Chernov defended in 1984, was devoted exactly to such high-dimensional billiards. In particular, he constructed there the stable and unstable manifolds for multidimensional semi-dispersing billiards.

After defending his PhD Dr. Chernov was working as a research scientist in the Joint Institute for Nuclear research in Dubna, a small town in the vicinity of Moscow. It allowed Kolya to continue attend Dynamical Systems seminar in the Moscow University. He also never interrupted thinking on billiard dynamics.

As a part of the team in the Information Technology lab in the Dubna Institute Dr. Chernov was working on statistical analysis of data from physics experiments. A problem they faced was how to approximate noisy data by simple curves like straight segments and circles.

It was the main characteristic of Prof. Chernov's scientific style to penetrate deeply into any problem he was dealing with. Its not surprising that he became an expert in this field as well. He published many papers about this topic and several of his students in the University of Birmingham defended PhDs in this area. Kolya also published a very well received book "Circular and Linear Regression: Fitting Circles and Lines by Least Squares", (2010) where he summarized the geometric, algebraic and numerical aspects of fitting lines and circles to data.

However billiards always were the greatest of Prof. Chernov's scientific interests, and a steady flow of deep results in this area was not interrupted by him being busy with his immediate responsibilities at work in the Dubna Institute.

The most important example of a high-dimensional billiard is the celebrated Boltzmann gas of N hard spheres confined in a box (or in a torus). This system is equivalent to a semi-dispersing billiard, where the boundary of the billiard table consists of cylinders. Each such cylinder corresponds to the collisions of some fixed pair of particles. The fundamental problem of ergodicity of the Boltzmann gas with two particles was settled by Sinai. However in dimensions greater than two the situation becomes technically much more complicated.

In Kolya's joint paper with Sinai (1987) a general scheme was presented to prove ergodicity of the Boltzmann gas. This paper contains the celebrated Sinai-Chernov Ansatz which became a cornerstone for proving ergodicity of non-uniformly hyperbolic dynamical systems. Chaotic billiards belongs to this fundamental class of dynamical systems because of singularities which arise from the tangent collisions and/or singularities of the boundary of the billiard table.

In 1991 Dr.Chernov moved to the US. He held visiting positions at UCLA, Georgia Tech and Princeton before joining the Mathematics Department in the University of Alabama in Birmingham at 1994. There Prof. Chernov was actively working with a large number of undergraduate and graduate students. Some of them became visible researchers in the theory of billiards.

Already in 1991 Kolya started a new area of research. Together with Eyink, Lebowitz and Sinai they constructed nonequilibrium steady states and investigated ergodic and statistical properties of the Lorentz gas with a small electric field. Because the speed of the moving particle in this system goes with time to infinity, one needs to introduce some kind of dissipation. This was done with the help of the so called Gaussian Thermostat. Prof. Chernov continued to develop this important area with the same collaborators and with his students till the very end.

Perhaps the most important and groundbreaking of Kolya's papers was the one published in 1998, where he proved a quasi-exponential rate of decay of correlations for Anosov flows. It was well known that Anosov diffeomorphisms have an exponential rate of correlations decay. However in dynamical systems with continuous time (flows) there appears a new (time) dimension and along this dimension there is no exponential instability which one has along the stable and unstable transversal manifolds. It was the first dynamical proof of the fast decay of correlations in systems with continuous time which was based on new insights and a very delicate

new technique. Before that such a result was known only for geodesic flows on compact manifolds with negative curvature. However the proof of this fact was purely algebraic and did not shed any light on the dynamical mechanism behind this phenomenon. In 2002 together with Balint, Szasz, and Toth, Kolya discovered the new types of singularities in the stable and unstable manifolds of semi-dispersing billiards, which was a fundamental result for the theory of such billiards.

Kolya established a very fruitful collaboration with Dima Dolgopyat. They produced several remarkable papers which solved some long standing classical problems. Perhaps the most impressive of their results is a two hundred page proof of the existence and limiting distributions in the classical model of real diffusion, where a heavy particle elastically collides with a gas of noninteracting light particles. This is exactly a model of Brownian motion, as observed by Brown.

For almost forty years the theory of chaotic billiards was virtually non-accessible for beginners. It was contained in research papers which were very hard to read. Kolya together with Markarian beautifully filled in this gap by producing the book "Chaotic Billiards" (2006) which has already made a great impact and allowed many gifted young mathematicians to join in the exiting game of mathematical billiards.

Kolya had a very modest but also a very strong and determined personality. He always talked in a low voice and avoided, if possible, crowded and loud meetings. He often managed to find his own trails (not recommended in a conference bulletin), which always were less populated, more quiet, and therefore more suitable for thinking and enjoying nature. The same happened if there was quiet hotel nearby (not recommended).

Travel to remote places and hiking was Kolya's passion. Together with his hobby of fixing old huge American cars and vans, it made him one of the most knowledgeable around on the beauty of (often remote and hardly accessible) US landscapes. Again, everything that Kolya did he did very fundamentally. No wonder that the mechanics in Birmingham had the utmost respect for him.

It is always the case that we read and write in reference letters that somebody is an expert in the field. With Kolya's passing away the Mathematical Community lost the Expert in billiards. He was the only one whom we could completely trust to judge that some very sophisticated proof on billiards dynamics is, in fact, correct.

The memories of Prof. Chernov as an outstanding mathematician and a wonderful person will always be with those were lucky enough to know Kolya. His scientific contributions will have a long lasting life.

We lost a wonderful colleague and a dear friend.

Leonid Bunimovich

