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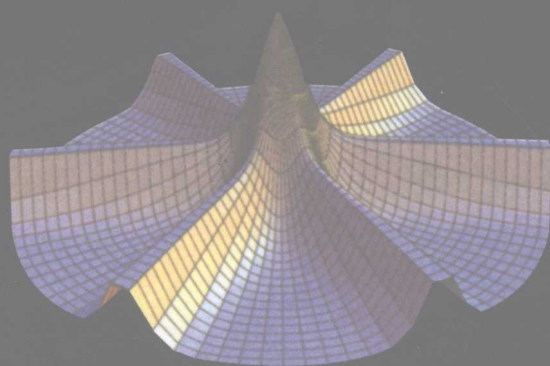


BEAUTIFUL MODELS
70 YEARS OF EXACTLY SOLVED
QUANTUM MANY-BODY PROBLEMS

模型之美

——量子多体问题严格解70年

Bill Sutherland



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内 容 简 介

本书全面介绍了可以严格求解的量子多体系统, 这是一个吸引人的优美学科。该学科开始于 70 多年前量子力学建立后不久对于一维海森伯磁体的著名的贝脱解。从此, 可以严格求解的量子多体系统的多样性和范围持续稳步增长。

本书体系完整统一, 可以作为研究生及大学物理专业本科生进一步学习的教科书, 也可以让非物理专业的学者了解这一领域的一些典型模型。本书表述详尽, 可以满足读者的求知欲和阅读兴趣; 内容全面, 提供了必要的背景材料以便读者深入该学科领域并查阅研究文献。

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Preface

This book is conceived as an advanced textbook – suitable for graduate students, interested non-experts, and even ambitious undergraduates – to provide a broad introduction to the subject of exactly solved quantum many-body systems. These systems are necessarily one-dimensional, but should that change in the future, I will happily include higher-dimensional examples in a new edition of this book! As the subtitle indicates, the topic began more than 70 years ago – soon after the invention of quantum mechanics – and has been steadily growing since. My only ambition for the book is to provide a largely self-contained presentation – as broad and coherent as feasible – that will interest the interested reader, and provide the background necessary should that reader wish to go further and enter the vast and intimidating research literature. Oh yes, and with a minimum of mistakes.

Before beginning, I would like to take this opportunity to thank a few of the people who have been essential in my career, and thus, directly or indirectly, made this book possible.

First, I thank C.N.Yang, with whom I did my thesis work back in 1965, on exactly solved problems, naturally. Frank Yang has always been extremely generous: with his time and patience, with his friends and colleagues, and especially with his deep understanding of the beauty of physics and the physicist's craft. He has helped shape my personal standards of craftsmanship, and as always, I write hoping that he will find my work interesting; that is also true of this book.

Second, I thank Sriram Shastry – the B.S.Shastry of the bibliography – my colleague, co-author and friend of 20+ years. Looking back, we wrote our first paper together in 1981, and have collaborated continuously every since, often over a 12 hour time difference. I am very proud of the work we have done together, but more importantly, working with

Sriram has been extremely enjoyable, which is most important for the long-haul. I invited Sriram to co-author this book, but he wisely declined. I do hope Sriram feels this book does the subject justice.

Third, I wish to thank T.A.Pond for looking after me in my student days. I first met Alec Pond as a sophomore at Washington University, when he gave the best physics class I have ever taken. He also hired me to work part-time with his graduate students, among whom was my future brother-in-law. I then followed him to Stony Brook for graduate school. Although by that time Alec was largely an administrator, he still took care of me with baby-sitting jobs and shirts, and especially by building up a first-class Physics Department and helping to bring C.N.Yang to Stony Brook.

I would like to thank Norio Kawakami and Yoshio Kuramoto for bringing me to Japan, and the Yukawa Institute for Theoretical Physics, its faculty and staff, and especially Akira Furasaki, for a lovely autumn visit when much of this book was written. Also, the University of Utah has supported the writing of this book with a sabbatical year, and the National Science Foundation has given support off-and-on over the years – starting first as a high school student in the summer of 1959.

To numerous students – Rudo Römer, Kazumoto Iguchi, Joel Campbell in particular – colleagues and friends – thanks. Also, I thank Maria Cranor for reading the manuscript.

Finally, I thank Dr. K.K.Phua of World Scientific for suggesting and supporting the publication of this book, and for introducing me to Ms. Lakshmi Narayanan – ‘my editor’ – who has made a hard job as painless as possible.

*Bill Sutherland
Salt Lake City, Utah
March 31, 2004*

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Chapter 1

Overview

This book is a text on a subject – exactly solved quantum many-body problems – that is usually considered to be ‘difficult’. This subject belongs within the realm of mathematical physics – too mathematical to be ‘respectable’ physics, yet not rigorous enough to be ‘real’ mathematics. However, over a period of more than seventy years, there has been much success in understanding the detailed ‘how and why’ of such models. The results are quite interesting for their own sake, and so there are perennial attempts to translate this body of work into either respectable physics or real mathematics; this is not that sort of book. Instead, this book attempts to discuss the models and the solutions in their own ‘intrinsic’ language. But before we begin this program, in this chapter I would first like to try and sketch the motivation and strategy with a minimum of equations. So this first chapter is offered to encourage a general audience to read on; it can certainly be skipped by the experts.

1.1 Orientation

Let us begin with an informal description of the type of physical problems we will be trying to capture with our exactly solved models. The proper language to discuss these problems is the language of statistical mechanics; let us agree on terms. We start with a *closed*, insulated container filled with a fixed amount of chemically pure fluid – say water, for example. We might see something like the top picture of Fig. 1.1. We wait until nothing appears to be changing within the container – the fluid is in *equilibrium*. We know from experience that the fluid will then be uniform and have a unique temperature and pressure – it is in a *thermodynamic state*. (Actually, we show the fluid with a slight density and pressure gradient due to an external gravitational field. Thus, it is really temperature and chemical potential that are uniform,