C. Henry Edwards
David E. Penney

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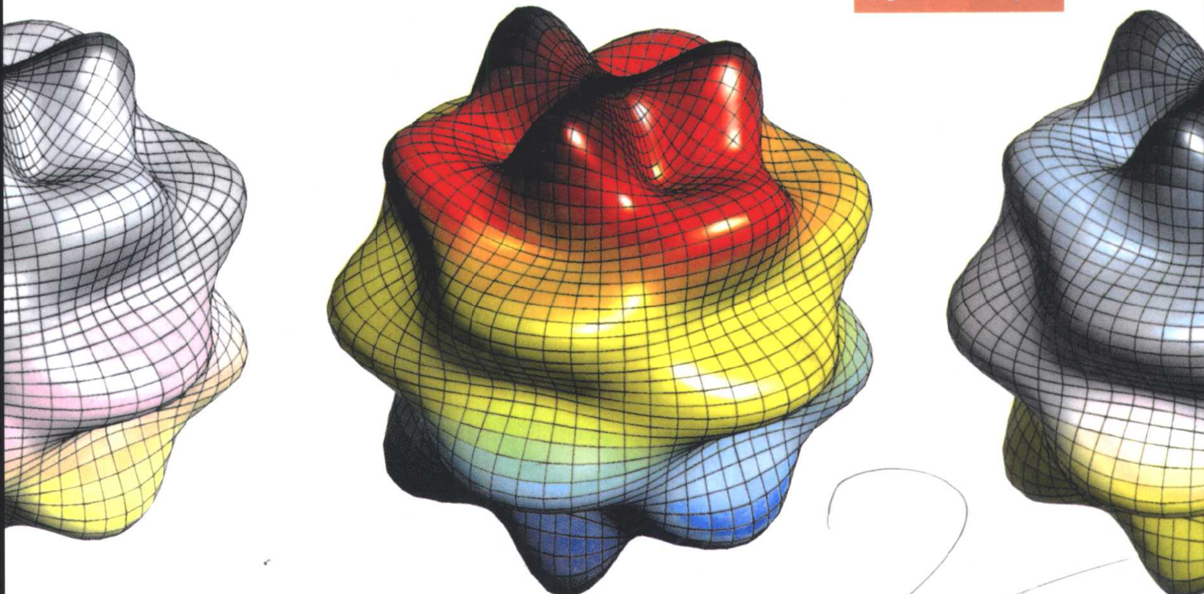
微分方程及边值问题

计算与模型

第 3 版



Differential Equations and Boundary Value Problems

Computing and Modeling, 3E

清华大学出版社

Pearson Education

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APPLICATION MODULES

The modules listed here follow the indicated sections in the text. Most provide computing projects that illustrate the content of the corresponding text sections. *Maple*, *Mathematica*, and MATLAB versions of these investigations are included in the Applications Manual that accompanies this text.

PREFACE

Many introductory differential equations courses in the recent past have emphasized the formal solution of standard types of differential equations using a (seeming) grab-bag of systematic solution techniques. Many students have concentrated on learning to match memorized methods with memorized equations. The evolution of the present text is based on experience teaching a course with a greater emphasis on conceptual ideas and the use of applications and computing projects to involve students in more intense and sustained problem-solving experiences.

The availability of technical computing environments like *Maple*, *Mathematica*, and MATLAB is reshaping the role and applications of differential equations in science and engineering and has shaped our approach in this text. New technology motivates a shift in emphasis from traditional manual methods to both qualitative and computer-based methods that

- render accessible a wider range of more realistic applications;
- permit the use of both numerical computation and graphical visualization to develop greater conceptual understanding; and
- encourage empirical investigations that involve deeper thought and analysis than standard textbook problems.

Major Features

The following features of this text are intended to support a contemporary differential equations course that augments traditional core skills with conceptual perspectives that students will need for the effective use of differential equations in their subsequent work and study:

- Coverage of seldom-used topics has been trimmed and new topics added to place a greater emphasis on core techniques as well as qualitative aspects of the subject associated with direction fields, solution curves, phase plane portraits, and dynamical systems. We combine symbolic, graphic, and numeric solution methods wherever it seems advantageous. A fresh computational flavor should be evident in figures, examples, problems, and applications throughout the text. About 15% of the examples in the text are new or newly revised for this edition.
- The organization of the book places an increased emphasis on linear systems of differential equations, which are covered in Chapters 4 and 5 (together with the necessary linear algebra), followed by a substantial treatment in Chapter 6 of nonlinear systems and phenomena (including chaos in dynamical systems).
- This book begins and ends with discussions and examples of the mathematical modeling of real-world phenomena. Students learn through mathematical

modeling and empirical investigation to balance the questions of what equation to formulate, how to solve it, and whether a solution will yield useful information.

- The first course in differential equations should also be a window on the world of mathematics. While it is neither feasible nor desirable to include proofs of the fundamental existence and uniqueness theorems along the way in an elementary course, students need to see precise and clear-cut statements of these theorems and to understand their role in the subject. We include appropriate existence and uniqueness proofs in the Appendix and occasionally refer to them in the main body of the text.
- While our approach reflects the widespread use of new computer methods for the solution of differential equations, certain elementary analytical methods of solution (as in Chapters 1 and 3) are important for students to learn. Effective and reliable use of numerical methods often requires preliminary analysis using standard elementary techniques; the construction of a realistic numerical model often is based on the study of a simpler analytical model. We therefore continue to stress the mastery of traditional solution techniques (especially through the inclusion of extensive problem sets).

Computing Features

The following features highlight the flavor of computing technology that distinguishes much of our exposition.

- Almost 700 *computer-generated figures*—over half of them new for this edition and most constructed using Mathematica or MATLAB—show students vivid pictures of direction fields, solution curves, and phase plane portraits that bring symbolic solutions of differential equations to life. For instance, the cover graphic shows an eigenfunction of the three-dimensional wave equation that illustrates surface waves on a spherical planet and was constructed using associated Legendre functions (see Section 10.5).
- About 45 *application modules* follow key sections throughout the text. Most of these applications outline "technology neutral" investigations illustrating the use of technical computing systems and seek to actively engage students in the application of new technology.
- A fresh *numerical emphasis* that is afforded by the early introduction of numerical solution techniques in Chapter 2 (on mathematical models and numerical methods). Here and in Chapter 4, where numerical techniques for systems are treated, a concrete and tangible flavor is achieved by the inclusion of numerical algorithms presented in parallel fashion for systems ranging from graphing calculators to MATLAB.
- A *conceptual perspective* shaped by the availability of computational aids, which permits a leaner and more streamlined coverage of certain traditional manual topics (like exact equations and variation of parameters) in Chapters 1, 3, and 5.

Modeling Features

Mathematical modeling is a goal and constant motivation for the study of differential equations. To sample the range of applications in this text, take a look at the following questions:

- What explains the commonly observed time lag between indoor and outdoor daily temperature oscillations? (Section 1.5)
- What makes the difference between doomsday and extinction in alligator populations? (Section 2.1)
- How do a unicycle and a two-axle car react differently to road bumps? (Sections 3.7 and 5.3)
- How can you predict the time of next perihelion passage of a newly observed comet? (Section 4.3)
- Why might an earthquake demolish one building and leave standing the one next door? (Section 5.3)
- What determines whether two species will live harmoniously together, or whether competition will result in the extinction of one of them and the survival of the other? (Section 6.3)
- Why and when does nonlinearity lead to chaos in biological and mechanical systems? (Section 6.5)
- If a mass on a spring is periodically struck with a hammer, how does the behavior of the mass depend on the frequency of the hammer blows? (Section 7.6)
- Why are flagpoles hollow instead of solid? (Section 8.6)
- What explains the difference in the sounds of a guitar, a xylophone, and drum? (Sections 9.6, 10.2, and 10.4)

Organization and Content

We have reshaped the usual approach and sequence of topics to accommodate new technology and new perspectives. For instance,

- After a precis of first-order equations in Chapter 1 (though with the coverage of certain traditional symbolic methods streamlined a bit), Chapter 2 offers an early introduction to mathematical modeling, stability and qualitative properties of differential equations, and numerical methods—a combination of topics that frequently are dispersed later in an introductory course.
- Chapters 4 and 5 provide a flexible treatment of linear systems. Motivated by current trends in science and engineering education and practice, Chapter 4 offers an early, intuitive introduction to first-order systems, models, and numerical approximation techniques. Chapter 5 begins with a self-contained treatment of the linear algebra that is needed and then presents the eigenvalue approach to linear systems. It includes a wide range of applications (ranging from railway cars to earthquakes) of all the various cases of the eigenvalue method. Section 5.5 includes a fairly extensive treatment of matrix exponentials, which are exploited in Section 5.6 on nonhomogeneous linear systems.
- Chapter 6 on nonlinear systems and phenomena ranges from phase plane anal-

ysis to ecological and mechanical systems to a concluding section on chaos and bifurcation in dynamical systems. Section 6.5 presents an elementary introduction to such contemporary topics as period doubling in biological and mechanical systems, the pitchfork diagram, and the Lorenz strange attractor (all illustrated with vivid computer graphics).

- Laplace transform methods (Chapter 7) and power series methods (Chapter 8) follow the material on linear and nonlinear systems but can be covered at any earlier point (after Chapter 3) the instructor desires.
- Chapters 9 and 10 treat the applications of Fourier series, separation of variables, and Sturm-Liouville theory to partial differential equations and boundary value problems. After the introduction of Fourier series, the three classical equations—the wave and heat equations and Laplace's equation—are discussed in the last three sections of Chapter 9. The Sturm-Liouville methods of Chapter 10 are developed sufficiently to include some rather significant and realistic applications.

This book includes enough material appropriately arranged for different courses varying in length from one quarter to two semesters. The briefer version, *Differential Equations: Computing and Modeling*, ends with Chapter 7 on Laplace transform methods (and thus omits the material on power series methods, Fourier series, separation of variables and partial differential equations).

Problems, Applications, and Solutions Manuals

Almost 20% of the text's over 1900 problems are new for this edition or are newly revised to include graphic or qualitative content. Accordingly, the answer section now includes almost 300 new computer-generated figures illustrating those which students are expected to construct.

The answer section for this revision has been expanded considerably to increase its value as a learning aid. It now includes the answers to most odd-numbered problems plus a good many even-numbered ones. The 625-page *Instructor's Solutions Manual* (0-13-047578-5) accompanying this book provides worked-out solutions for most of the problems in the book, and the 375-page *Student Solutions Manual* (0-13-047579-3) contains solutions for most of the odd-numbered problems.

The approximately 45 application modules in the text contain additional problem and project material designed largely to engage students in the exploration and application of computational technology. These investigations are expanded considerably in the 325-page *Applications Manual* (0-13-047577-7) that accompanies the text and supplements it with additional and sometimes more challenging investigations. Each section in this manual has parallel subsections "Using Maple," "Using Mathematica," and "Using MATLAB" that detail the applicable methods and techniques of each system and will afford student users an opportunity to compare the merits and styles of different computational systems.

Technology Manuals and Website

The author-written solutions and applications manuals described previously, as well as the additional technology manuals listed next, are available shrink-wrapped free with the textbook upon order using the indicated ISBN numbers:

- Text with *Student Solutions Manual* (0-13-114492-8)
- Text with *Applications Manual* (0-13-114491-X)
- Text with David Calvis, *Mathematica for Differential Equations: Projects, Insights, Syntax, and Animations* (0-13-114489-8)
- Text with Selwyn Hollis, *A Mathematica Companion for Differential Equations* (0-13-178327-0)
- Text with Robert Gilbert & George Hsiao, *Maple Projects for Differential Equations* (0-13-178326-2)
- Text with John Polking & David Arnold, *Ordinary Differential Equations Using* MATLAB, 2nd edition (0-13-075668-7)

Notebooks and worksheets supporting these manuals—plus additional software including a package of Maple worksheets keyed to this text by John Maloney—are available for downloading at the website **www.prenhall.com/edwards**. Many of the figures in this text were computer generated using Polking's MATLAB programs **dfield** and **pplane** that are linked at the site. Another MATLAB-based ODE package that has impressive graphical capabilities and is referenced in the text is **Iode** (see **www.math.uiuc.edu/iode**).

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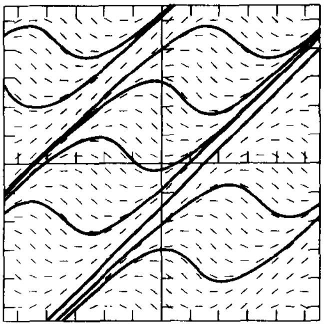

CHAPTER 1

First-Order Differential Equations

1.1 Differential Equations and Mathematical Models

The laws of the universe are written in the language of mathematics. Algebra is sufficient to solve many static problems, but the most interesting natural phenomena involve change and are described by equations that relate changing quantities.

Because the derivative $dx/dt = f'(t)$ of the function f is the rate at which the quantity $x = f(t)$ is changing with respect to the independent variable t, it is natural that equations involving derivatives are frequently used to describe the changing universe. An equation relating an unknown function and one or more of its derivatives is called a **differential equation**.

Example 1 The differential equation

$$\frac{dx}{dt} = x^2 + t^2$$

involves both the unknown function $x(t)$ and its first derivative $x'(t) = dx/dt$. The differential equation

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 7y = 0$$

involves the unknown function y of the independent variable x and the first two derivatives y' and y'' of y. ■

The study of differential equations has three principal goals:

1. To discover the differential equation that describes a specified physical situation.
2. To find—either exactly or approximately—the appropriate solution of that equation.
3. To interpret the solution that is found.

In algebra, we typically seek the unknown *numbers* that satisfy an equation such as $x^3 + 7x^2 - 11x + 41 = 0$. By contrast, in solving a differential equation, we are challenged to find the unknown *functions* $y = y(x)$ for which an identity such as $y'(x) = 2xy(x)$—that is, the differential equation

$$\frac{dy}{dx} = 2xy$$

—holds on some interval of real numbers. Ordinarily, we will want to find *all* solutions of the differential equation, if possible.

Example 2 If C is a constant and

$$y(x) = Ce^{x^2}, \tag{1}$$

then

$$\frac{dy}{dx} = C\left(2xe^{x^2}\right) = (2x)\left(Ce^{x^2}\right) = 2xy.$$

Thus every function $y(x)$ of the form in Eq. (1) *satisfies*—and thus is a solution of—the differential equation

$$\frac{dy}{dx} = 2xy \tag{2}$$

for all x. In particular, Eq. (1) defines an *infinite* family of different solutions of this differential equation, one for each choice of the arbitrary constant C. By the method of separation of variables (Section 1.4) it can be shown that every solution of the differential equation in (2) is of the form in Eq. (1). ■

Differential Equations and Mathematical Models

The following three examples illustrate the process of translating scientific laws and principles into differential equations. In each of these examples the independent variable is time t, but we will see numerous examples in which some quantity other than time is the independent variable.

Example 3 Newton's law of cooling may be stated in this way: The *time rate of change* (the rate of change with respect to time t) of the temperature $T(t)$ of a body is proportional to the difference between T and the temperature A of the surrounding medium (Fig. 1.1.1). That is,

$$\frac{dT}{dt} = -k(T - A), \tag{3}$$

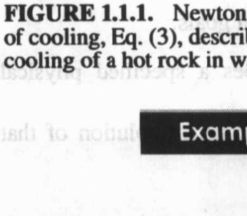

FIGURE 1.1.1. Newton's law of cooling, Eq. (3), describes the cooling of a hot rock in water.

where k is a positive constant. Observe that if $T > A$, then $dT/dt < 0$, so the temperature is a decreasing function of t and the body is cooling. But if $T < A$, then $dT/dt > 0$, so that T is increasing.

Thus the physical law is translated into a differential equation. If we are given the values of k and A, we should be able to find an explicit formula for $T(t)$, and then—with the aid of this formula—we can predict the future temperature of the body. ■

Example 4 Torricelli's law implies that the *time rate of change* of the volume V of water in a draining tank (Fig. 1.1.2) is proportional to the square root of the depth y of water in the tank:

$$\frac{dV}{dt} = -k\sqrt{y}, \tag{4}$$

where k is a constant. If the tank is a cylinder with vertical sides and cross-sectional area A, then $V = Ay$, so $dV/dt = A \cdot (dy/dt)$. In this case Eq. (4) takes the form

$$\frac{dy}{dt} = -h\sqrt{y}, \tag{5}$$

where $h = k/A$ is a constant. ■

Example 5 The *time rate of change* of a population $P(t)$ with constant birth and death rates is, in many simple cases, proportional to the size of the population. That is,

$$\frac{dP}{dt} = kP, \tag{6}$$

where k is the constant of proportionality. ■

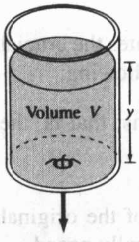

FIGURE 1.1.2. Torricelli's law of draining, Eq. (4), describes the draining of a water tank.

Let us discuss Example 5 further. Note first that each function of the form

$$P(t) = Ce^{kt} \tag{7}$$

is a solution of the differential equation

$$\frac{dP}{dt} = kP$$

in (6). We verify this assertion as follows:

$$P'(t) = Cke^{kt} = k\left(Ce^{kt}\right) = kP(t)$$

for all real numbers t. Because substitution of each function of the form given in (7) into Eq. (6) produces an identity, all such functions are solutions of Eq. (6).

Thus, even if the value of the constant k is known, the differential equation $dP/dt = kP$ has *infinitely many* different solutions of the form $P(t) = Ce^{kt}$, one for each choice of the "arbitrary" constant C. This is typical of differential equations. It is also fortunate, because it may allow us to use additional information to select from among all these solutions a particular one that fits the situation under study.

Example 6 the population at time $t = 0$ (hours, h) was 1000, and that the population doubled after 1 h. This additional information about $P(t)$ yields the following equations:

$$1000 = P(0) = Ce^0 = C,$$

$$2000 = P(1) = Ce^k.$$

It follows that $C = 1000$ and that $e^k = 2$, so $k = \ln 2 \approx 0.693147$. With this value of k the differential equation in (6) is

$$\frac{dP}{dt} = (\ln 2)P \approx (0.693147)P.$$

Substitution of $k = \ln 2$ and $C = 1000$ in Eq. (7) yields the particular solution

$$P(t) = 1000e^{(\ln 2)t} = 1000(e^{\ln 2})^t = 1000 \cdot 2^t \qquad \text{(because } e^{\ln 2} = 2\text{)}$$

that satisfies the given conditions. We can use this particular solution to predict future populations of the bacteria colony. For instance, the predicted number of bacteria in the population after one and a half hours (when $t = 1.5$) is

$$P(1.5) = 1000 \cdot 2^{3/2} \approx 2828.$$

■

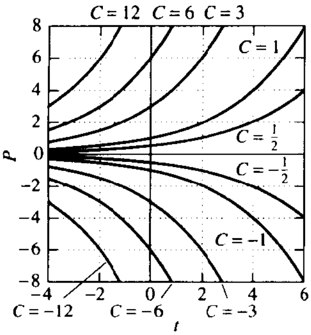

FIGURE 1.1.3. Graphs of $P(t) = Ce^{kt}$ with $k = \ln 2$.

The condition $P(0) = 1000$ in Example 6 is called an **initial condition** because we frequently write differential equations for which $t = 0$ is the "starting time." Figure 1.1.3 shows several different graphs of the form $P(t) = Ce^{kt}$ with $k = \ln 2$. The graphs of all the infinitely many solutions of $dP/dt = kP$ in fact fill the entire two-dimensional plane, and no two intersect. Moreover, the selection of any one point P_0 on the P-axis amounts to a determination of $P(0)$. Because exactly one solution passes through each such point, we see in this case that an initial condition $P(0) = P_0$ determines a unique solution agreeing with the given data.

Mathematical Models

Our brief discussion of population growth in Examples 5 and 6 illustrates the crucial process of *mathematical modeling* (Fig. 1.1.4), which involves the following:

1. The formulation of a real-world problem in mathematical terms; that is, the construction of a mathematical model.
2. The analysis or solution of the resulting mathematical problem.
3. The interpretation of the mathematical results in the context of the original real-world situation; for example, answering the question originally posed.

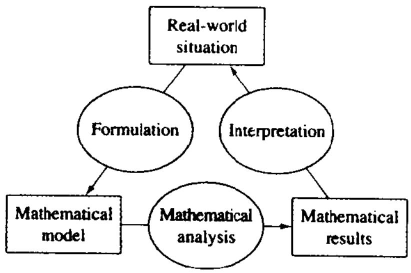

FIGURE 1.1.4. The process of mathematical modeling.

In the population example, the real-world problem is that of determining the population at some future time. A **mathematical model** consists of a list of variables (P and t) that describe the given situation, together with one or more equations relating these variables ($dP/dt = kP$, $P(0) = P_0$) that are known or are assumed to hold. The mathematical analysis consists of solving these equations (here, for P as a function of t). Finally, we apply these mathematical results to attempt to answer the original real-world question.

Nevertheless, it is quite possible that no one solution of the differential equation fits *all* the known information. In such a case we must suspect that the differential equation may not adequately describe the real world. For instance, the solutions of Eq. (6) are of the form $P(t) = Ce^{kt}$, where C is a positive constant, but for *no* choice of the constants k and C does $P(t)$ accurately describe the actual growth of the human population of the world over the past few centuries. We must therefore write a perhaps more complicated differential equation, one that takes into account the effects of population pressure on the birth rate, the declining food supply, and other factors. This should not be regarded as a failure of the model in Example 5, but as an insight into what additional factors must be considered in studying the growth of populations. Indeed, Eq. (6) is quite accurate under certain circumstances—for example, the growth of a bacterial population under conditions of unlimited food and space.