The Classical Theory of Fields

Fourth Edition

Course of Theoretical Physics
Volume 2

L. D. Landau and E. M. Lifshitz

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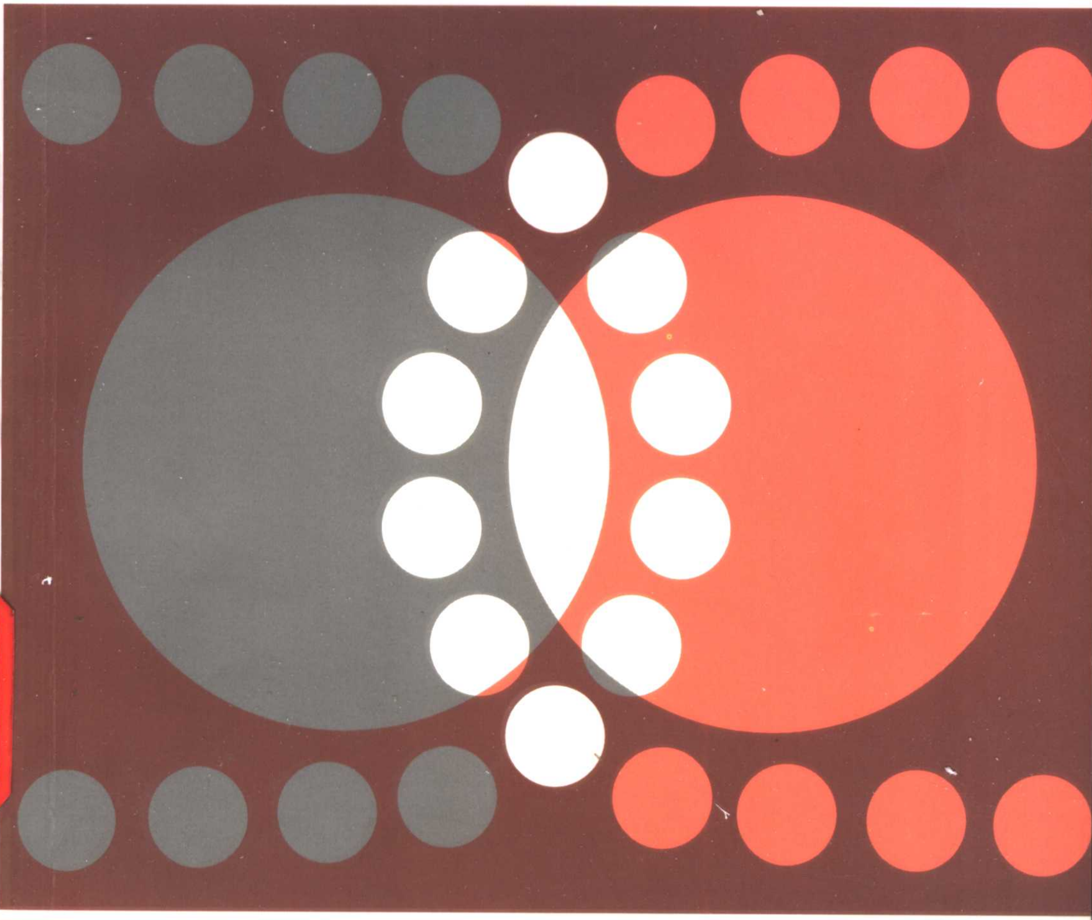

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EXCERPTS FROM THE PREFACES TO THE FIRST AND SECOND EDITIONS

THIS book is devoted to the presentation of the theory of the electromagnetic and gravitational fields. i.e. electrodynamics and general relativity. A complete, logically connected theory of the electromagnetic field includes the special theory of relativity, so the latter has been taken as the basis of the presentation. As the starting point of the derivation of the fundamental relations we take the variational principles, which make possible the attainment of maximum generality, unity and simplicity of presentation.

In accordance with the overall plan of our Course of Theoretical Physics (of which this book is a part), we have not considered questions concerning the electrodynamics of continuous media, but restricted the discussion to "microscopic electrodynamics"—the electrodynamics of point charges *in vacuo*.

The reader is assumed to be familiar with electromagnetic phenomena as discussed in general physics courses. A knowledge of vector analysis is also necessary. The reader is not assumed to have any previous knowledge of tensor analysis, which is presented in parallel with the development of the theory of gravitational fields.

Moscow, December 1939
Moscow, June 1947

L. LANDAU, E. LIFSHITZ

PREFACE TO THE FOURTH ENGLISH EDITION

THE first edition of this book appeared more than thirty years ago. In the course of reissues over these decades the book has been revised and expanded; its volume has almost doubled since the first edition. But at no time has there been any need to change the method proposed by Landau for developing the theory, or his style of presentation, whose main feature was a striving for clarity and simplicity. I have made every effort to preserve this style in the revisions that I have had to make on my own.

As compared with the preceding edition, the first nine chapters, devoted to electrodynamics, have remained almost without changes. The chapters concerning the theory of the gravitational field have been revised and expanded. The material in these chapters has increased from edition to edition, and it was finally necessary to redistribute and rearrange it.

I should like to express here my deep gratitude to all of my helpers in this work—too many to be enumerated—who, by their comments and advice, helped me to eliminate errors and introduce improvements. Without their advice, without the willingness to help which has met all my requests, the work to continue the editions of this course would have been much more difficult. A special debt of gratitude is due to L. P. Pitaevskii, with whom I have constantly discussed all the vexing questions.

The English translation of the book was done from the last Russian edition, which appeared in 1973. No further changes in the book have been made. The 1994 corrected reprint includes the changes made by E. M. Lifshitz in the Seventh Russian Edition published in 1987.

I should also like to use this occasion to sincerely thank Prof. Hamermesh, who has translated this book in all its editions, starting with the first English edition in 1951. The success of this book among English-speaking readers is to a large extent the result of his labor and careful attention.

E. M. LIFSHITZ

PUBLISHER'S NOTE

As with the other volumes in the *Course of Theoretical Physics*, the authors do not, as a rule, give references to original papers, but simply name their authors (with dates). Full bibliographic references are only given to works which contain matters not fully expounded in the text.

EDITOR'S PREFACE TO THE SEVENTH RUSSIAN EDITION

E. M. Lifshitz began to prepare a new edition of *Teoria Polia* in 1985 and continued his work on it even in hospital during the period of his last illness. The changes that he proposed are made in the present edition. Of these we should mention some revision of the proof of the law of conservation of angular momentum in relativistic mechanics, and also a more detailed discussion of the question of symmetry of the Christoffel symbols in the theory of gravitation. The sign has been changed in the definition of the electromagnetic field stress tensor. (In the present edition this tensor was defined differently than in the other volumes of the Course.)

June 1987 L. P. Pitaevskii

NOTATION

Three-dimensional quantities

Three-dimensional tensor indices are denoted by Greek letters
Element of volume, area and length: dV, $d\mathbf{f}$, $d\mathbf{l}$
Momentum and energy of a particle: $\mathbf{p}$ and $\mathscr{E}$
Hamiltonian function: $\mathscr{H}$
Scalar and vector potentials of the electromagnetic field: ϕ and $\mathbf{A}$
Electric and magnetic field intensities: $\mathbf{E}$ and $\mathbf{H}$
Charge and current density: ρ and $\mathbf{j}$
Electric dipole moment: $\mathbf{d}$
Magnetic dipole moment: $\mathbf{m}$

Four-dimensional quantities

Four-dimensional tensor indices are denoted by Latin letters i, k, l, ... and take on the values 0, 1, 2, 3
We use the metric with signature $(+ - - -)$
Rule for raising and lowering indices—see p. 14
Components of four-vectors are enumerated in the form $A^i = (A^0, \mathbf{A})$
Antisymmetric unit tensor of rank four is e^{iklm}, where $e^{0123} = 1$ (for the definition, see p. 17)
Element of four-volume $d\Omega = dx^0 dx^1 dx^2 dx^3$
Element of hypersurface dS^i (defined on pp. 19–20)
Radius four-vector: $x^i = (ct, \mathbf{r})$
Velocity four-vector: $u^i = dx^i/ds$
Momentum four-vector: $p = (\mathscr{E}/c, \mathbf{p})$
Current four-vector: $j^i = (c\rho, \rho\mathbf{v})$
Four-potential of the electromagnetic field: $A^i = (\phi, \mathbf{A})$
Electromagnetic field four-tensor $F_{ik} = \frac{\partial A_k}{\partial x^i} - \frac{\partial A_i}{\partial x^k}$ (for the relation of the components of F_{ik} to the components of $\mathbf{E}$ and $\mathbf{H}$, see p. 61)
Energy-momentum four-tensor T^{ik} (for the definition of its components, see p. 78)

CONTENTS

CHAPTER 1

THE PRINCIPLE OF RELATIVITY

§ 1. Velocity of propagation of interaction

For the description of processes taking place in nature, one must have a *system of reference*. By a system of reference we understand a system of coordinates serving to indicate the position of a particle in space, as well as clocks fixed in this system serving to indicate the time.

There exist systems of reference in which a freely moving body, i.e. a moving body which is not acted upon by external forces, proceeds with constant velocity. Such reference systems are said to be *inertial*.

If two reference systems move uniformly relative to each other, and if one of them is an inertial system, then clearly the other is also inertial (in this system too every free motion will be linear and uniform). In this way one can obtain arbitrarily many inertial systems of reference, moving uniformly relative to one another.

Experiment shows that the so-called *principle of relativity* is valid. According to this principle all the laws of nature are identical in all inertial systems of reference. In other words, the equations expressing the laws of nature are invariant with respect to transformations of coordinates and time from one inertial system to another. This means that the equation describing any law of nature, when written in terms of coordinates and time in different inertial reference systems, has one and the same form.

The interaction of material particles is described in ordinary mechanics by means of a potential energy of interaction, which appears as a function of the coordinates of the interacting particles. It is easy to see that this manner of describing interactions contains the assumption of instantaneous propagation of interactions. For the forces exerted on each of the particles by the other particles at a particular instant of time depend, according to this description, only on the positions of the particles at this one instant. A change in the position of any of the interacting particles influences the other particles immediately.

However, experiment shows that instantaneous interactions do not exist in nature. Thus a mechanics based on the assumption of instantaneous propagation of interactions contains within itself a certain inaccuracy. In actuality, if any change takes place in one of the interacting bodies, it will influence the other bodies only after the lapse of a certain interval of time. It is only after this time interval that processes caused by the initial change begin to take place in the second body. Dividing the distance between the two bodies by this time interval, we obtain the *velocity of propagation of the interaction*.

We note that this velocity should, strictly speaking, be called the *maximum* velocity of propagation of interaction. It determines only that interval of time after which a change occurring in one body *begins* to manifest itself in another. It is clear that the existence of a maximum velocity of propagation of interactions implies, at the same time, that motions of bodies with greater velocity than this are in general impossible in nature. For if such a motion could occur, then by means of it one could realize an interaction with a velocity exceeding the maximum possible velocity of propagation of interactions.

Interactions propagating from one particle to another are frequently called "signals", sent out from the first particle and "informing" the second particle of changes which the first has experienced. The velocity of propagation of interaction is then referred to as the *signal velocity*.

From the principle of relativity it follows in particular that the velocity of propagation of interactions is the *same* in *all* inertial systems of reference. Thus the velocity of propagation of interactions is a universal constant. This constant velocity (as we shall show later) is also the velocity of light in empty space. The velocity of light is usually designated by the letter c, and its numerical value is

$$c = 2.998 \times 10^{10} \text{ cm/sec}. \tag{1.1}$$

The large value of this velocity explains the fact that in practice classical mechanics appears to be sufficiently accurate in most cases. The velocities with which we have occasion to deal are usually so small compared with the velocity of light that the assumption that the latter is infinite does not materially affect the accuracy of the results.

The combination of the principle of relativity with the finiteness of the velocity of propagation of interactions is called the *principle of relativity of Einstein* (it was formulated by Einstein in 1905) in contrast to the principle of relativity of Galileo, which was based on an infinite velocity of propagation of interactions.

The mechanics based on the Einsteinian principle of relativity (we shall usually refer to it simply as the principle of relativity) is called *relativistic*. In the limiting case when the velocities of the moving bodies are small compared with the velocity of light we can neglect the effect on the motion of the finiteness of the velocity of propagation. Then relativistic mechanics goes over into the usual mechanics, based on the assumption of instantaneous propagation of interactions; this mechanics is called *Newtonian* or *classical*. The limiting transition from relativistic to classical mechanics can be produced formally by the transition to the limit $c \to \infty$ in the formulas of relativistic mechanics.

In classical mechanics distance is already relative, i.e. the spatial relations between different events depend on the system of reference in which they are described. The statement that two nonsimultaneous events occur at one and the same point in space or, in general, at a definite distance from each other, acquires a meaning only when we indicate the system of reference which is used.

On the other hand, time is absolute in classical mechanics; in other words, the properties of time are assumed to be independent of the system of reference; there is one time for all reference frames. This means that if any two phenomena occur simultaneously for any one observer, then they occur simultaneously also for all others. In general, the interval of time between two given events must be identical for all systems of reference.

It is easy to show, however, that the idea of an absolute time is in complete contradiction to the Einstein principle of relativity. For this it is sufficient to recall that in classical mechanics, based on the concept of an absolute time, a general law of combination of velocities is valid, according to which the velocity of a composite motion is simply equal to the (vector) sum of the velocities which constitute this motion. This law, being universal, should also be applicable to the propagation of interactions. From this it would follow that the velocity of propagation must be different in different inertial systems of reference, in contradiction to the principle of relativity. In this matter experiment completely confirms the principle of relativity. Measurements first performed by Michelson (1881) showed complete lack of dependence of the velocity of light on its direction of propagation; whereas according to classical mechanics the velocity of light should be smaller in the direction of the earth's motion than in the opposite direction.

Thus the principle of relativity leads to the result that time is not absolute. Time elapses differently in different systems of reference. Consequently the statement that a definite time interval has elapsed between two given events acquires meaning only when the reference frame to which this statement applies is indicated. In particular, events which are simultaneous in one reference frame will not be simultaneous in other frames.

To clarify this, it is instructive to consider the following simple example.

Let us look at two inertial reference systems K and K' with coordinate axes XYZ and $X'Y'Z'$ respectively, where the system K' moves relative to K along the $X(X')$ axis (Fig. 1).

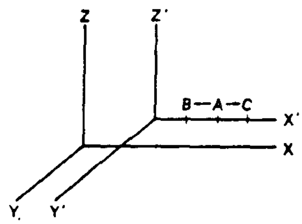

FIG. 1.

Suppose signals start out from some point A on the X' axis in two opposite directions. Since the velocity of propagation of a signal in the K' system, as in all inertial systems, is equal (for both directions) to c, the signals will reach points B and C, equidistant from A, at one and the same time (in the K' system).

But it is easy to see that the same two events (arrival of the signal at B and C) can by no means be simultaneous for an observer in the K system. In fact, the velocity of a signal relative to the K system has, according to the principle of relativity, the same value c, and since the point B moves (relative to the K system) toward the source of its signal, while the point C moves in the direction away from the signal (sent from A to C), in the K system the signal will reach point B earlier than point C.

Thus the principle of relativity of Einstein introduces very drastic and fundamental changes in basic physical concepts. The notions of space and time derived by us from our daily experiences are only approximations linked to the fact that in daily life we happen to deal only with velocities which are very small compared with the velocity of light.

§ 2. Intervals

In what follows we shall frequently use the concept of an *event*. An event is described by the place where it occurred and the time when it occurred. Thus an event occurring in a certain material particle is defined by the three coordinates of that particle and the time when the event occurs.

It is frequently useful for reasons of presentation to use a fictitious four-dimensional space, on the axes of which are marked three space coordinates and the time. In this space events are represented by points, called *world points*. In this fictitious four-dimensional space there corresponds to each particle a certain line, called a *world line*. The points of this line determine the coordinates of the particle at all moments of time. It is easy to show that to a particle in uniform rectilinear motion there corresponds a straight world line.

We now express the principle of the invariance of the velocity of light in mathematical form. For this purpose we consider two reference systems K and K' moving relative to each other with constant velocity. We choose the coordinate axes so that the axes X and X' coincide, while the Y and Z axes are parallel to Y' and Z'; we designate the time in the systems K and K' by t and t'.

Let the first event consist of sending out a signal, propagating with light velocity, from a point having coordinates $x_1 y_1 z_1$ in the K system, at time t_1 in this system. We observe the propagation of this signal in the K system. Let the second event consist of the arrival of the signal at point $x_2 y_2 z_2$ at the moment of time t_2. The signal propagates with velocity c; the distance covered by it is therefore $c(t_1 - t_2)$. On the other hand, this same distance equals $[(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2]^{\frac{1}{2}}$. Thus we can write the following relation between the coordinates of the two events in the K system:

$$(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2-c^2(t_2-t_1)^2 = 0. \tag{2.1}$$

The same two events, i.e. the propagation of the signal, can be observed from the K' system:

Let the coordinates of the first event in the K' system be $x'_1 y'_1 z'_1 t'_1$, and of the second: $x'_2 y'_2 z'_2 t'_2$. Since the velocity of light is the same in the K and K' systems, we have, similarly to (2.1):

$$(x'_2-x'_1)^2+(y'_2-y'_1)^2+(z'_2-z'_1)^2-c^2(t'_2-t'_1)^2 = 0. \tag{2.2}$$

If $x_1 y_1 z_1 t_1$ and $x_2 y_2 z_2 t_2$ are the coordinates of *any* two events, then the quantity

$$s_{12} = [c^2(t_2-t_1)^2-(x_2-x_1)^2-(y_2-y_1)^2-(z_2-z_1)^2]^{\frac{1}{2}} \tag{2.3}$$

is called the *interval* between these two events.

Thus it follows from the principle of invariance of the velocity of light that if the interval between two events is zero in one coordinate system, then it is equal to zero in all other systems.

If two events are infinitely close to each other, then the interval ds between them is

$$ds^2 = c^2dt^2-dx^2-dy^2-dz^2. \tag{2.4}$$

The form of expressions (2.3) and (2.4) permits us to regard the interval, from the formal point of view, as the distance between two points in a fictitious four-dimensional space (whose axes are labelled by x, y, z, and the product ct). But there is a basic difference between the rule for forming this quantity and the rule in ordinary geometry: in forming the square of the interval, the squares of the coordinate differences along the different axes are summed, not with the same sign, but rather with varying signs.†

As already shown, if $ds = 0$ in one inertial system, then $ds' = 0$ in any other system. On the other hand, ds and ds' are infinitesimals of the same order. From these two conditions it follows that ds^2 and ds'^2 must be proportional to each other:

$$ds^2 = a\,ds'^2$$

where the coefficient a can depend only on the absolute value of the relative velocity of the

† The four-dimensional geometry described by the quadratic form (2.4) was introduced by H. Minkowski, in connection with the theory of relativity. This geometry is called *pseudo-euclidean*, in contrast to ordinary euclidean geometry.

two inertial systems. It cannot depend on the coordinates or the time, since then different points in space and different moments in time would not be equivalent, which would be in contradiction to the homogeneity of space and time. Similarly, it cannot depend on the direction of the relative velocity, since that would contradict the isotropy of space.

Let us consider three reference systems K, K_1, K_2, and let V_1 and V_2 be the velocities of systems K_1 and K_2 relative to K. We then have:

$$ds^2 = a(V_1)ds_1^2, \qquad ds^2 = a(V_2)ds_2^2.$$

Similarly we can write

$$ds_1^2 = a(V_{12})ds_2^2,$$

where V_{12} is the absolute value of the velocity of K_2 relative to K_1. Comparing these relations with one another, we find that we must have

$$\frac{a(V_2)}{a(V_1)} = a(V_{12}). \tag{2.5}$$

But V_{12} depends not only on the absolute values of the vectors $\mathbf{V}_1$ and $\mathbf{V}_2$, but also on the angle between them. However, this angle does not appear on the left side of formula (2.5). It is therefore clear that this formula can be correct only if the function $a(V)$ reduces to a constant, which is equal to unity according to this same formula.

Thus,

$$ds^2 = ds'^2, \tag{2.6}$$

and from the equality of the infinitesimal intervals there follows the equality of finite intervals: $s = s'$.

Thus we arrive at a very important result: the interval between two events is the same in all inertial systems of reference, i.e. it is invariant under transformation from one inertial system to any other. This invariance is the mathematical expression of the constancy of the velocity of light.

Again let $x_1 y_1 z_1 t_1$ and $x_2 y_2 z_2 t_2$ be the coordinates of two events in a certain reference system K. Does there exist a coordinate system K', in which these two events occur at one and the same point in space?

We introduce the notation

$$t_2 - t_1 = t_{12}, \qquad (x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2 = l_{12}^2.$$

Then the interval between events in the K system is:

$$s_{12}^2 = c^2t_{12}^2 - l_{12}^2$$

and in the K' system

$$s_{12}'^2 = c^2t_{12}'^2 - l_{12}'^2,$$

whereupon, because of the invariance of intervals,

$$c^2t_{12}^2 - l_{12}^2 = c^2t_{12}'^2 - l_{12}'^2.$$

We want the two events to occur at the same point in the K' system, that is, we require $l'_{12} = 0$. Then

$$s_{12}^2 = c^2t_{12}^2 - l_{12}^2 = c^2t_{12}'^2 > 0.$$

Consequently a system of reference with the required property exists if $s_{12}^2 > 0$, that is, if the interval between the two events is a real number. Real intervals are said to be *timelike*.

Thus, if the interval between two events is timelike, then there exists a system of reference in which the two events occur at one and the same place. The time which elapses between the two events in this system is

$$t'_{12} = \frac{1}{c}\sqrt{c^2t_{12}^2 - l_{12}^2} = \frac{s_{12}}{c}. \tag{2.7}$$

If two events occur in one and the same body, then the interval between them is always timelike, for the distance which the body moves between the two events cannot be greater than ct_{12}, since the velocity of the body cannot exceed c. So we have always

$$l_{12} < ct_{12}.$$

Let us now ask whether or not we can find a system of reference in which the two events occur at one and the same time. As before, we have for the K and K' systems $c^2t_{12}^2 - l_{12}^2 = c^2t'^2_{12} - l'^2_{12}$. We want to have $t'_{12} = 0$, so that

$$s_{12}^2 = -l'^2_{12} < 0.$$

Consequently the required system can be found only for the case when the interval s_{12} between the two events is an imaginary number. Imaginary intervals are said to be *spacelike*.

Thus if the interval between two events is spacelike, there exists a reference system in which the two events occur simultaneously. The distance between the points where the events occur in this system is

$$l'_{12} = \sqrt{l_{12}^2 - c^2t_{12}^2} = is_{12}. \tag{2.8}$$

The division of intervals into space- and timelike intervals is, because of their invariance, an absolute concept. This means that the timelike or spacelike character of an interval is independent of the reference system.

Let us take some event O as our origin of time and space coordinates. In other words, in the four-dimensional system of coordinates, the axes of which are marked x, y, z, t, the world point of the event O is the origin of coordinates. Let us now consider what relation other events bear to the given event O. For visualization, we shall consider only one space dimension and the time, marking them on two axes (Fig. 2). Uniform rectilinear motion of a particle, passing through $x = 0$ at $t = 0$, is represented by a straight line going through O and inclined to the t axis at an angle whose tangent is the velocity of the particle. Since the maximum possible velocity is c, there is a maximum angle which this line can subtend with the t axis. In Fig. 2 are shown the two lines representing the propagation of two signals

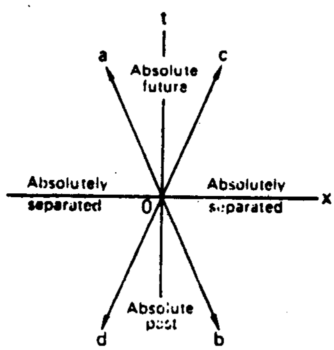

FIG. 2

(with the velocity of light) in opposite directions passing through the event O (i.e. going through $x = 0$ at $t = 0$). All lines representing the motion of particles can lie only in the regions *aOc* and *dOb*. On the lines *ab* and *cd*, $x = \pm ct$. First consider events whose world points lie within the region *aOc*. It is easy to show that for all the points of this region $c^2t^2 - x^2 > 0$. In other words, the interval between any event in this region and the event O is timelike. In this region $t > 0$, i.e. all the events in this region occur "after" the event O. But two events which are separated by a timelike interval cannot occur simultaneously in any reference system. Consequently it is impossible to find a reference system in which any of the events in region *aOc* occurred "before" the event O, i.e. at time $t < 0$. Thus all the events in region *aOc* are future events relative to O in *all* reference systems. Therefore this region can be called the *absolute future* relative to O.

In exactly the same way, all events in the region *bOd* are in the *absolute past* relative to O; i.e. events in this region occur before the event O in all systems of reference.

Next consider regions *dOa* and *cOb*. The interval between any event in this region and the event O is spacelike. These events occur at different points in space in every reference system. Therefore these regions can be said to be *absolutely remote* relative to O. However, the concepts "simultaneous", "earlier", and "later" are relative for these regions. For any event in these regions there exist systems of reference in which it occurs after the event O, systems in which it occurs earlier than O, and finally one reference system in which it occurs simultaneously with O.

Note that if we consider all three space coordinates instead of just one, then instead of the two intersecting lines of Fig. 2 we would have a "cone" $x^2+y^2+z^2-c^2t^2 = 0$ in the four-dimensional coordinate system x, y, z, t, the axis of the cone coinciding with the t axis. (This cone is called the *light cone*.) The regions of absolute future and absolute past are then represented by the two interior portions of this cone.

Two events can be related causally to each other only if the interval between them is timelike; this follows immediately from the fact that no interaction can propagate with a velocity greater than the velocity of light. As we have just seen, it is precisely for these events that the concepts "earlier" and "later" have an absolute significance, which is a necessary condition for the concepts of cause and effect to have meaning.

§ 3. Proper time

Suppose that in a certain inertial reference system we observe clocks which are moving relative to us in an arbitrary manner. At each different moment of time this motion can be considered as uniform. Thus at each moment of time we can introduce a coordinate system rigidly linked to the moving clocks, which with the clocks constitutes an inertial reference system.

In the course of an infinitesimal time interval dt (as read by a clock in our rest frame) the moving clocks go a distance $\sqrt{dx^2+dy^2+dz^2}$. Let us ask what time interval dt' is indicated for this period by the moving clocks. In a system of coordinates linked to the moving clocks, the latter are at rest, i.e., $dx' = dy' = dz' = 0$. Because of the invariance of intervals

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2 = c^2dt'^2,$$

from which

$$dt' = dt\sqrt{1 - \frac{dx^2+dy^2+dz^2}{c^2dt^2}}.$$

But

$$\frac{dx^2+dy^2+dz^2}{dt^2}=v^2,$$

where v is the velocity of the moving clocks; therefore

$$dt'=\frac{ds}{c}=dt\sqrt{1-\frac{v^2}{c^2}}. \tag{3.1}$$

Integrating this expression, we can obtain the time interval indicated by the moving clocks when the elapsed time according to a clock at rest is t_2-t_1:

$$t_2'-t_1'=\int_{t_1}^{t_2} dt\sqrt{1-\frac{v^2}{c^2}}. \tag{3.2}$$

The time read by a clock moving with a given object is called the *proper time* for this object. Formulas (3.1) and (3.2) express the proper time in terms of the time for a system of reference from which the motion is observed.

As we see from (3.1) or (3.2), the proper time of a moving object is always less than the corresponding interval in the rest system. In other words, moving clocks go more slowly than those at rest.

Suppose some clocks are moving in uniform rectilinear motion relative to an inertial system K. A reference frame K' linked to the latter is also inertial. Then from the point of view of an observer in the K system the clocks in the K' system fall behind. And conversely, from the point of view of the K' system, the clocks in K lag. To convince ourselves that there is no contradiction, let us note the following. In order to establish that the clocks in the K' system lag behind those in the K system, we must proceed in the following fashion. Suppose that at a certain moment the clock in K' passes by the clock in K, and at that moment the readings of the two clocks coincide. To compare the rates of the two clocks in K and K' we must once more compare the readings of the same moving clock in K' with the clocks in K. But now we compare this clock with *different* clocks in K—with those past which the clock in K' goes at this new time. Then we find that the clock in K' lags behind the clocks in K with which it is being compared. We see that to compare the rates of clocks in two reference frames we require several clocks in one frame and one in the other, and that therefore this process is not symmetric with respect to the two systems. The clock that appears to lag is always the one which is being compared with different clocks in the other system.

If we have two clocks, one of which describes a closed path returning to the starting point (the position of the clock which remained at rest), then clearly the moving clock appears to lag relative to the one at rest. The converse reasoning, in which the moving clock would be considered to be at rest (and vice versa) is now impossible, since the clock describing a closed trajectory does not carry out a uniform rectilinear motion, so that a coordinate system linked to it will not be inertial.

Since the laws of nature are the same only for inertial reference frames, the frames linked to the clock at rest (inertial frame) and to the moving clock (non-inertial) have different properties, and the argument which leads to the result that the clock at rest must lag is not valid.