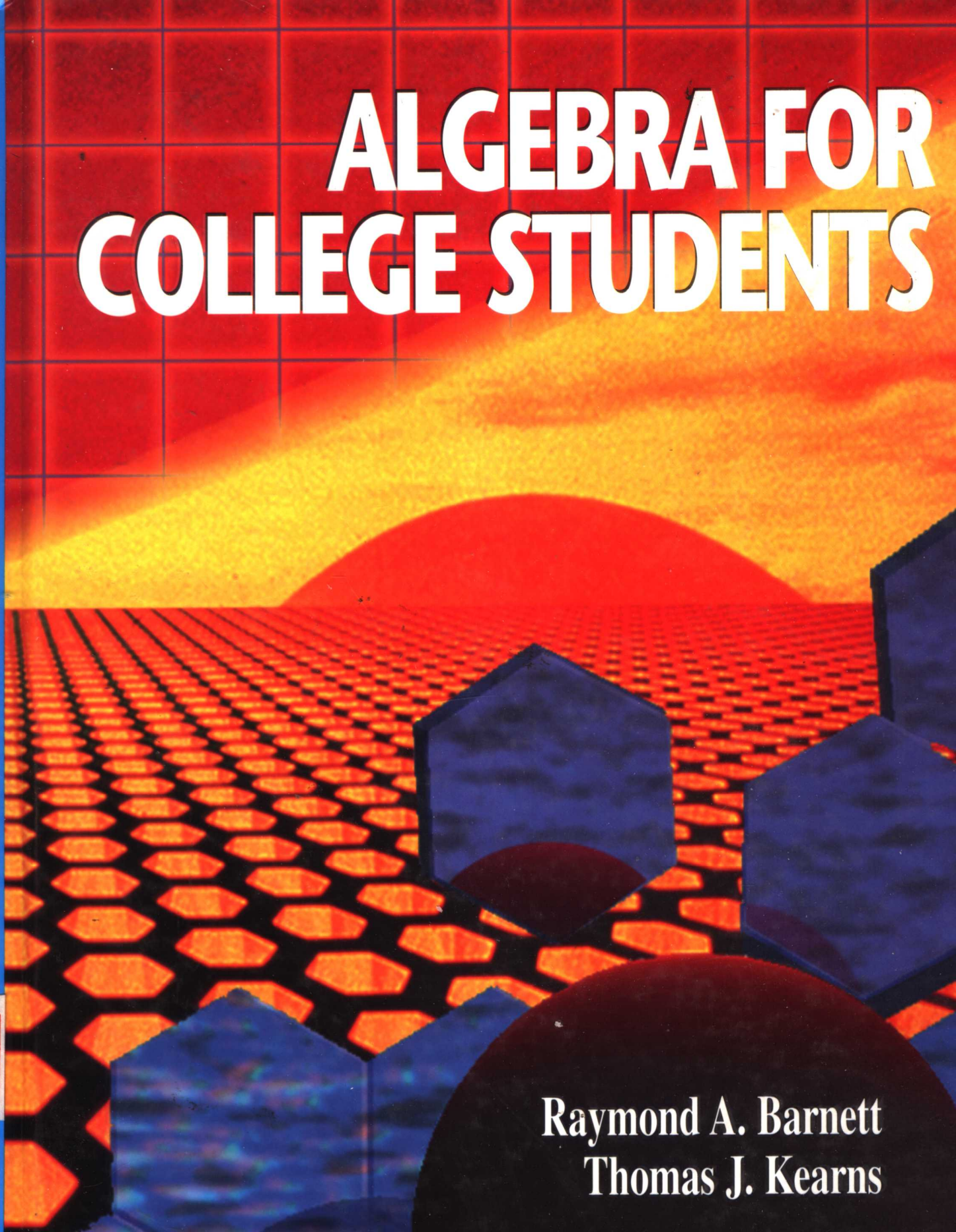
ALGEBRA FOR COLLEGE STUDENTS



Raymond A. Barnett
Thomas J. Kearns

ALGEBRA FOR COLLEGE STUDENTS

Raymond A. Barnett
Merritt College

Thomas J. Kearns
Northern Kentucky University

WCB McGraw-Hill

Boston, Massachusetts Burr Ridge, Illinois Dubuque, Iowa
Madison, Wisconsin New York, New York San Francisco, California St. Louis, Missouri

WCB/McGraw-Hill

A Division of The McGraw-Hill Companies

Algebra for College Students

This book is printed on acid-free paper.

5 6 7 8 9 0 VNH VNH 9 0 9

ISBN 0-07-005001-5

This book was set in Times Roman by York Graphic Services, Inc.
The editors were Michael Johnson and David A. Damstra;
the production supervisor was Leroy A. Young.
The cover was designed by Initial Graphic Systems, Inc.
Von Hoffmann Press, Inc., was printer and binder.

Library of Congress Cataloging-in-Publication Data

Barnett, Raymond A.
Algebra for college students / Raymond A. Barnett, Thomas J. Kearns.
p. cm.
Includes index.
ISBN 0-07-005001-5
1. Algebra. I. Kearns, Thomas J. II. Title.
QA152.2.B3685 1995
512.9—dc20 94-18497

About the Authors

Raymond A. Barnett is an experienced teacher and author. He received his B.A. in mathematical statistics from the University of California at Berkeley and his M.A. in mathematics from the University of Southern California. He then went on to become a member of the Department of Mathematics at Merritt College and head of that department for 4 years. He is a member of the Mathematical Association of America (MAA), the National Council of Teachers of Mathematics (NCTM), and the American Association for the Advancement of Science. He is the author or coauthor of eighteen books in mathematics that are still in print—all with a reputation for extremely readable prose and high-quality mathematics.

Thomas J. Kearns received his B.S. from Santa Clara University and his M.S. and Ph.D. from the University of Illinois at Urbana-Champaign. After several years of teaching at the University of Delaware, he was appointed to the faculty at Northern Kentucky University, where he served as chairman of the Department of Mathematical Sciences for ten years. He is a member of the MAA, the NCTM, the American Mathematical Society, and the Operations Research Society of America. He has coauthored five texts with Raymond A. Barnett, as well as texts in college algebra and elementary statistics.

Preface

This book contains a blend of intermediate and college algebra topics written at an intermediate algebra level. The goal of the text is to provide a sound transition between elementary algebra and more advanced courses in mathematics. It is written for students with differing backgrounds and abilities; an average or below average student will experience success and an above average student will be challenged.

♦ KEY FEATURES

Compressed Review

The review of basic topics from elementary algebra is covered in Chapter 1 and can be omitted by fully prepared classes. Topics that remain troublesome to most students, however, are still thoroughly reviewed in context at appropriate places in the text.

Early and Continuous Emphasis on Graphing

Early and continuous emphasis on graphing allows for visual interpretation of many mathematical concepts and provides another dimension of mathematical understanding. For example, the relationship between the graph of an equation

$$y = (\text{an expression in } x)$$

and the solution of the related equation

$$(\text{an expression in } x) = 0$$

is emphasized. This enables students to "see" the number of real roots to a given equation and to estimate their approximate values.

Early Functions

Functions and function notation are introduced early in the text (Section 3-8) in the simple setting of linear equations and functions. Function notation is used in the next two chapters to reinforce ideas before the topic is expanded in a richer setting in Chapter 6. This is an example of the "spiral approach" the authors use on some difficult topics: the topic is first introduced in a simple setting and then is returned to in successively richer and more complex settings.

Use of Technology

Exploratory problems requiring the use of technology are included in many exercise sets. These problems are clearly identified with the icon  and may be omitted without loss of continuity. Graphing calculator output is also used in the text to

further illustrate information that can be obtained from graphs or to illustrate graphs that require advanced techniques or extensive point plotting. A graphing calculator supplement is available for instructors who want calculator-specific examples for the TI-81, TI-82, TI-85, and the Casio *fx*-7700G.

Additional Topics

In addition to the standard intermediate algebra topics, the text includes a number of college algebra topics. These additional topics include: **elementary linear programming** (Section 3-7); expanded treatment of **roots of a polynomial equation** (Section 5-9); **matrix operations and methods** (Sections 9-3, 9-4, 9-5); a more extensive discussion of **conic sections** (Chapter 10), and **more emphasis on graphing** throughout.

♦ ADDITIONAL IMPORTANT FEATURES

Learning System

The text is **written for student comprehension.** Each concept or technique is illustrated with an **example,** followed by a **matched problem.** Answers to the matched problems are provided at the end of each section so that students can immediately check their understanding. This example–matched problem structure encourages active rather than passive reading of the text.

We've included **numerous illustrative examples,** including examples for the more challenging exercises and applications. Completely worked-out solutions include dashed **Think Boxes** that present the steps which will be performed mentally as the student becomes more confident. Cautions 

are included to point out common errors. Annotations are included as needed to help clarify key steps and reinforce concepts.

An **informal style,** proceeding **from the concrete to the abstract,** is used for exposition. Definitions are illustrated with simple examples. Formal statements of theorems are not emphasized.

Graded Exercise Sets

Graded exercise sets are divided into A, B, and C groupings. The A problems are straightforward and representative of the easier examples in the section. The B problems represent the more challenging examples in the section, and aid in student comprehension and critical thinking. The C-level problems provide a mixture of harder mechanics, theory, and extension of the material. The C exercises are more conceptual in nature and include challenging problems that do not match worked examples in the section. In short, the exercises are designed so that an average or below average student will be able to experience some success and more capable students will still be challenged.

Applications

The subject matter is related to the real world through numerous realistic applications from the physical sciences, business and economics, the life sciences, and the

social sciences. Applications noted by either one ★ or two stars ★★ provide the opportunities for *collaborative learning* and may be used to encourage *group discussion* and/or as a basis for *written reports.*

Major Topic Developments

The text continues to use a spiral technique for major topics wherever possible. That is, a topic will be introduced in a relatively simple framework and then returned to, reinforced, and developed further in later sections. For example, consider these topics:

Solving equations: Sections 1-7, 2-1, 2-4, 3-3, 4-2, 4-3, 5-5, 7-6, 8-5, 9-1, 9-5, 10-5

Word problems: 2-2, 2-4, 3-1, 3-4, 3-5, 3-6, 3-7, 4-4, 5-6, 5-7, 6-5, 8-5, 9-1, 11-5, 11-6, 11-7

Graphing: Sections 1-7, 2-1, 2-4, 2-5, 2-6, 3-1, 3-5, 3-6, 4-5, 4-6, 4-7, 6-3, 6-6, 8-1, 8-2, Chapter 10

Inequalities: Sections 1-1, 2-5, 2-6, 3-5, 3-6, 4-7

Functions: Sections 3-8, 4-5, 5-8, 5-9, Chapters 6 and 8, Section 11-1

The use of this spiral technique continues from the companion text *Elementary Algebra: Structure and Use* (Sixth Edition).

Systematic Review

Chapter summary sections include a review of the chapter with all important terms and symbols. Also included are a **comprehensive review exercise** set for the chapter and a short **chapter test. Cumulative review exercises** are included after Chapters 3, 5, 7, and 10.

History

Historical comments add interesting notes and perspectives.

Answers

Answers to all chapter review exercises, chapter tests, cumulative review exercises, and all odd-numbered problems from the section exercises are provided in the back of the book. Answers to the exercises in the chapter summaries (review exercises, chapter tests, and cumulative review exercises) are keyed, by numbers in italics, to corresponding sections in the text.

Design

The design and art program make the book visually appealing and easy to read, and reinforce mathematical concepts.

The **type style** for exponents and fractions is designed for clarity.

Captions identify all examples and application problems.

Boxed material is used for emphasis, and **schematics** for clarity.

Common student errors are identified by a special **caution** symbol at places where they naturally occur.

Think boxes (dashed boxes) are used to enclose steps that are usually done mentally (see Sections 1-2, 1-3, 1-4, 1-7, 2-1, 2-2).

Annotation of examples and development is found throughout the text to help students through critical steps (see any section in Chapter 1.)

Functional use of color guides students through critical steps (see Sections 1-5, 1-6, 1-7, 2-4).

Formula Summaries

Summaries of algebraic formulas, symbols, and real-number properties, all keyed to the sections in which they are introduced, are included inside the front cover of the book for convenient reference. Summaries of geometric and other common formulas and of the metric system are provided inside the back cover of the book.

♦ STUDENT SUPPLEMENTS

Student's Solutions Manual

A STUDENT'S SOLUTIONS MANUAL is available at a nominal cost through your bookstore. The manual includes key ideas and formulas, solutions to odd-numbered end-of-section exercises, all solutions to the end-of-chapter exercises and chapter tests, and an appendix on setting up word problems.

MATHWORKS is a self-paced interactive tutorial specifically linked to the text. It reinforces selected topics and provides unlimited opportunities to review concepts and to practice problem solving. It requires virtually *no* computer training and is available for IBM, IBM-compatible, and Macintosh computers.

Course VIDEOTAPES are available.

Graphing Calculator Supplement

THE GRAPHING CALCULATOR ENHANCEMENT MANUAL presents an integrated approach that utilizes calculator-based graphing to enhance understanding and development. It includes calculator exercises and examples as well as appendixes on how to use the most popular calculators.

Activities Manual

The ACTIVITIES MANUAL contains individual and group activities that help illustrate and reinforce algebra topics. These range from creative and motivational games to critical thinking and writing exercises.

♦ INSTRUCTOR SUPPLEMENTS

Resource Manual

An INSTRUCTOR'S RESOURCE MANUAL provides sample tests, transparency masters, an applications index, and additional teaching suggestions and assistance.

Solutions Manual

An INSTRUCTOR'S SOLUTIONS MANUAL contains detailed solutions to even end-of-section exercises, and all cumulative review exercises, as well as the answers to all problems.

Computerized Test Generator

The PROFESSOR'S ASSISTANT is a unique computerized test generator available to instructors. This system allows the instructor to create tests using algorithmically generated test questions and those from a standard test bank. This testing system enables the instructor to choose questions either manually or randomly by section, question type, difficulty level, and other criteria. This system is available for IBM, IBM-compatible, and Macintosh computers.

Printed Test Bank A printed and bound TEST BANK is also available. This is a hard-copy listing of the questions found in the standard test bank.

For further information, please contact your local McGraw-Hill sales representative.

♦ ERROR CHECK

This text has been carefully and independently checked and proofread by a number of people. Because of this, the authors and publisher believe it to be substantially error-free. However, if errors remain, the authors and publisher would be grateful to be notified and receive corrections. Corrections should be sent to:

Mathematics Editor
College Division, 43rd Floor
McGraw-Hill Book Company
1221 Avenue of the Americas
New York, NY 10020

♦ ACKNOWLEDGMENTS

In addition to the authors, many others are involved in the publication of a book and the authors have benefited from a great deal of help in preparing this text. We are grateful to the many users of the text *Intermediate Algebra: Structure and Use* for their kind remarks and helpful suggestions, many of which have been incorporated into this text. We thank the following, in particular, for their reviews regarding this text:

Jeffrey O. Bauer, Northeast Community College

James Brown, West Essex Community College

Brian Buhrman, Iowa Western Community College

Gary Carlson, Utah Valley Community College

Margaret D. Donlan, University of Delaware

Kendall Griggs, Hutchinson Community College

Martin Johnson, Montgomery Community College

Herbert F. Kramer, Longview Community College

Grace Malaney, Donnelly College

Ashley M. Martin, Fairmont State College

Peggy Miller, Kearney State College

Murray Peterson, College of Marin

Analy Scorsone, Lexington Community College

Mark Serebransky, Camden County College

Jane E. Sieberth, Franklin University

Richard Werner, Santa Rosa Junior College

Deborah H. White, College of the Redwoods

Brad Wind, Marshall University

Kelly Wyatt, Umpqua Community College

We especially thank Steven Blasberg and Alexander Majzoub for careful checking of examples, exercises, and answers; Fred Safier of City College of San Francisco for his accurate checking of exercise sets and his skillful preparation of the solutions manual; Karen Minette, Associate Mathematics Editor, for guiding the preparation of the supplements; Anita Wagner for her expert manuscript editing; David Damstra, Senior Editing Supervisor, for his conscientious supervision of production; Nancy Evans, Marketing Manager, for her tireless efforts in promoting the book; and Michael Johnson, Mathematics Editor, for the services he provided the authors and his guidance of the entire project from beginning to end.

Raymond A. Barnett
Thomas J. Kearns

To the Student

Mastery of the material in this text is important. It will be critical for your success in all college mathematics courses and quantitative courses in other disciplines. There are several things you can do that will help you reach the goal of mastering this material.

First, approach the subject with a **positive attitude.** This text has been written to help you to understand intermediate college algebra and to use it to solve problems. Over the past 25 years tens of thousands of students have succeeded in algebra with the help of books related to this edition. **You, too, can succeed!** But you must want to and believe that you can.

Second, **do your work on a regular basis.** Mathematics is not a spectator sport. You cannot learn to swim, or draw, or speak a foreign language simply by watching someone else. Similarly, you cannot learn mathematics by just reading worked examples or watching your instructor work problems. You must **work problems.** This takes time and effort. Moreover, mathematical learning is cumulative—as you progress through a subject like algebra, you continually need what you have already learned. Thus, it is very important that you keep up with assigned work. **Don't fall behind.**

Third, try the following **study process.** It will help you use this text effectively:

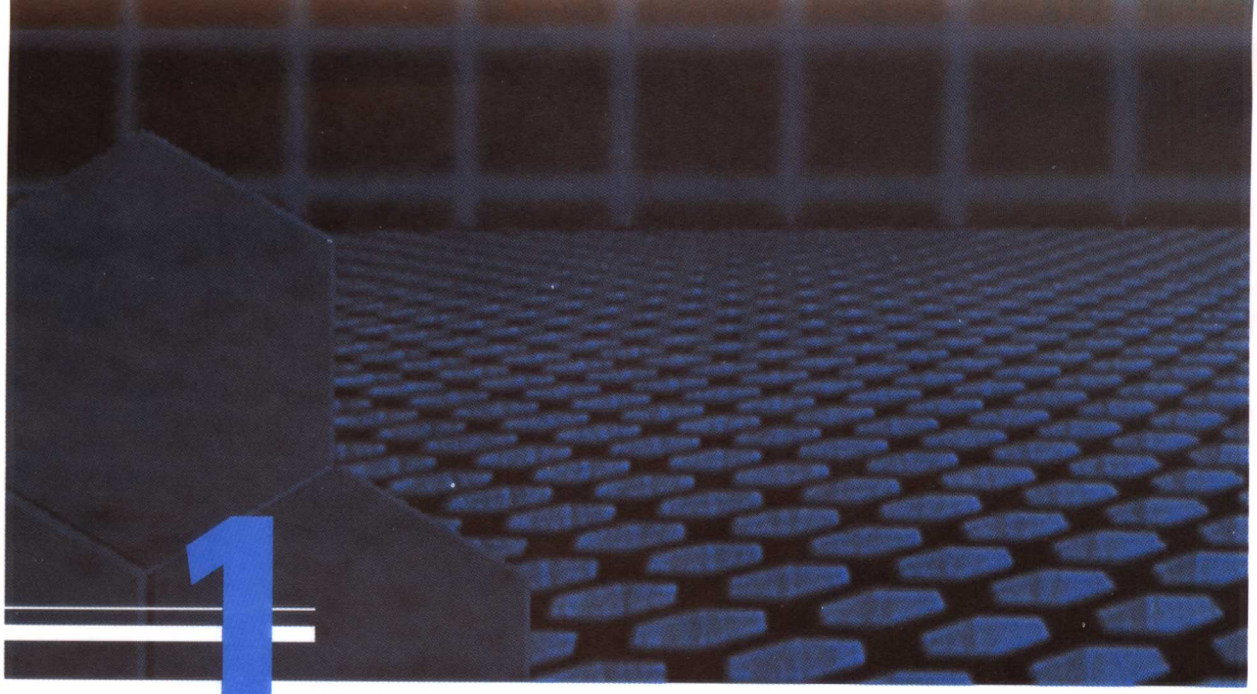
1. Read the mathematical development.	Keep pencil and paper at hand while you read. Make notes. Check any details that aren't provided. Try examples of your own.
2. Work through the illustrative example.	Try to understand each step. There will be a similar problem, called a "Matched Problem," after the example.
3. Try the matched problem following the example.	The answer to the Matched Problem can be found at the end of the section.
4. Review the main ideas and any new terminology in the section.	Pay particular attention to boxed material and any terms in bold type.
5. Work the assigned exercises at the end of the section.	This is the most important part of the learning process!

There are more than enough problems in this text for you to work. Use your assignments as a guide. However, if you are having trouble, you may have to do more of the A problems to get started. If you continue to have trouble with the problems, see your instructor. If you find the assignments too easy, try more of the C problems and check with your instructor—you may be ready for the next course in your curriculum.

Good luck!

Raymond A. Barnett
Thomas J. Kearns

Contents



1 Elementary Algebraic Concepts

Algebra is often referred to as "generalized arithmetic." In arithmetic we deal with basic arithmetic operations: addition, subtraction, multiplication, and division on specific numbers. In algebra we continue to use all that we know in arithmetic, but we also reason and work with *variables,* symbols that represent (or are placeholders) for one or more numbers. The rules for manipulating and reasoning with these symbols depend on certain properties of numbers, since the symbols represent numbers. In this chapter we review important number systems and some of the basic properties that determine how we can manipulate algebraic expressions. We also recall the operations of addition, subtraction, and multiplication on polynomials. We then look at the process of factoring a polynomial, that is, writing the polynomial as a product. Finally, we review two key ideas from elementary algebra: solving and graphing equations. We recall the cartesian coordinate system, how to graph algebraic relationships between two variables, and how graphs are related to solutions of equations.

1-1 Basic Concepts

- Sets
- Variables and Algebraic Expressions
- The Set of Real Numbers and the Real Number Line
- Equality and Inequality Relations

In this section, we recall the familiar number systems of arithmetic and beginning algebra, all contained in the set of real numbers identified with points on the number line. We begin with a brief introduction to the ideas and terminology of *sets,* and also review the concepts of *variables*, *algebraic expressions*, *equality*, and *inequality*.

SETS

Our use of the word "set" will not differ appreciably from the way it is used in everyday language. Words such as "set," "collection," "bunch," and "flock" all convey the same idea. Thus, we think of a **set** as a collection of objects with the important property that given any object we can tell whether it is or is not in the set. Capital letters, such as A, B, and C, are often used to designate particular sets. For example,

$$A = \{3, 5, 7\} \qquad B = \{4, 5, 6\}$$

specify sets A and B consisting of the numbers enclosed within the braces { }.

A set is usually described in one of two ways:

1. By **listing** the elements between braces:

$$\{3, 5, 7\}$$

where the order in which the elements are listed does not matter. Note that a given element is not listed more than once.

2. By enclosing a **rule** within braces that determines the elements in the set:

$\{x \mid x^2 = 81\}$ Read "the set of all x such that $x^2 = 81$."

Rule
Such that
All x
The set of

If each element of set A is also an element of set B, we say that A is a **subset** of set B. For example, the set of all women in a class is a subset of the whole class. The definition of a subset allows a set to be a subset of itself. If two sets have exactly the same elements, the sets are said to be **equal.** Symbolically:

Subsets and Equality

$A \subset B$	means	"A is a subset of B"	$\{3, 5\} \subset \{3, 5, 7\}$
$A = B$	means	"A is equal to B"	$\{4, 6\} = \{6, 4\}$

Some texts use a variation of this notation where $A \subseteq B$ denotes that A is a subset of B and may possibly be equal to B, while $A \subset B$ denotes that A is a subset of B that is not equal to B. We will not need to make this distinction.

A more complete treatment of sets may be found in Appendix A.

♦ VARIABLES AND ALGEBRAIC EXPRESSIONS

The letter x used in equations such as $x^2 = 25$ is a variable. In general, a **variable** is a symbol used to represent unspecified elements from a set with two or more elements. This set is called the **replacement set** for the variable. A **constant,** on the other hand, is a symbol that names exactly one object. The symbol "8" is a constant, since it always names the number eight.

An **algebraic expression** is a meaningful symbolic form involving constants, variables, mathematical operations, and grouping symbols. For example,

$$2 + 8 \qquad 4 \cdot 3 - 7 \qquad 16 - 3(7 - 4)$$
$$5x - 3y \qquad 7(x + 2y) \qquad 4\{u - 3[u - 2(u + 1)]\}$$

are all algebraic expressions.

Two or more algebraic expressions, each taken as a single entity and joined by plus or minus signs, are called **terms.** For reasons that will become clear later in this chapter, a term includes the sign that precedes it. Two or more algebraic expressions joined by multiplication are called **factors.** For example,

$$\underbrace{3(x - y)}_{\text{Term}} + \underbrace{(x + y)(x - y)}_{\text{Term}}$$

(Factors: 3 and $(x - y)$; Factors: $(x + y)$ and $(x - y)$)

has two terms, $3(x - y)$ and $(x + y)(x - y)$, and each of these terms has two factors. The first term has factors 3 and $(x - y)$, and the second term has factors $(x + y)$ and $(x - y)$. A term may contain several factors, and a factor may contain several terms.

Example 1
Terms and Factors

Identify the terms and factors in the following algebraic expressions:

(A) $3xy$ **(B)** $x + y - z - 3$ **(C)** $x(x - 1) + 2x$

Solution

(A) The expression has three factors: 3, x, and y.
(B) The expression has four terms: x, y, $-z$, and -3.
(C) The expression consists of two terms: $x(x - 1)$ and $2x$. The first term has two factors, x and $x - 1$, and the second of these factors has two terms, namely x and -1. The second term, $2x$, has two factors, 2 and x.

Matched Problem 1

Identify the terms and factors in the following algebraic expressions:

(A) $a + b - 1$ **(B)** $23xyz$ **(C)** $x(x - 1)(x + y + 2)$

To **evaluate** an algebraic expression for particular values of the variables means to replace each variable by a given value and then calculate the resulting arithmetic value. Thus, for example, to evaluate $3xy$ when $x = 5$ and $y = 7$, we calculate

$$3xy = 3(5)(7) = 105$$