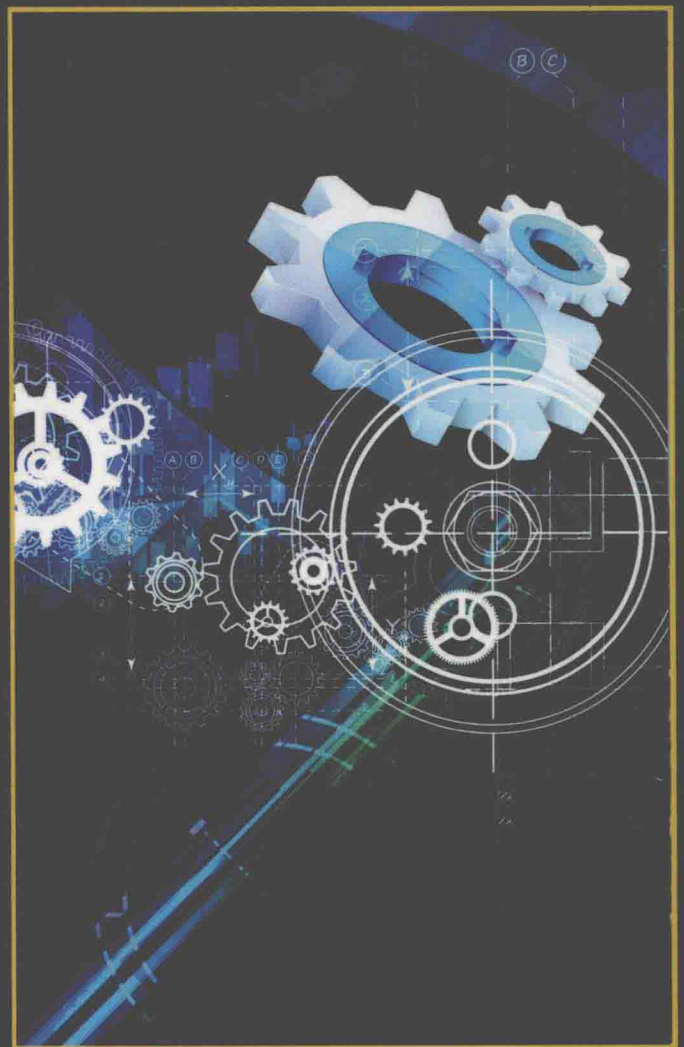


DYNAMICS IN ENGINEERING PRACTICE

ELEVENTH EDITION

Dara W. Childs
Andrew P. Conkey



CRC Press
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*This book is dedicated to the students who have taken my courses
in dynamics since 1968, particularly the hardworking
students at Texas A&M University in College Station, Texas.*

Preface

The following central changes have been made moving from the tenth to the eleventh edition:

1. A large number of problems have been added; specifically, 59 new problems have been added to the original problem sets of Chapters 2 through 5, an increase of 28%.
2. Chapter 6 has been added, covering the application of Lagrange's equations for deriving equations of motion. For one-degree-of-freedom (1DOF) problems, the tenth edition consistently provided a dual view of developing the equation of motion (EOM) using either Newtonian dynamics or work-energy. Hence, transitioning from the work-energy approach in Chapters 3 and 5 for 1DOF problems to multidegree-of-freedom (MDOF) problems in Chapter 6 via Lagrange's equations is straightforward. Thirty-eight new problems are provided in Chapter 6.

Dynamics concerns the motion of particles and rigid bodies and, as traditionally presented, has proven to be a "killer" course for both students who study engineering and professors who try to teach it. Students generally fail to grasp the pervasive nature of dynamics in engineering practice and view the subject matter as a collection of "tricks," rather than a unified body of knowledge based on an extremely limited number of basic principles. Professors grow tired of teaching courses that have a high level of failure and attrition, and professors in successor courses tend to complain that students "don't know how to derive equations of motion or do free-body diagrams."

This book's premise is that many of these difficulties arise because of the inappropriate nature of currently available dynamics texts. These books have a peculiar disconnect from either the requirements of subsequent course work or the practice of dynamics in an engineering career. The practice of engineering involves the development of (1) *general* kinematic equations, that is, equations that tell us the position, velocity, and acceleration of particles and/or rigid bodies without regard to the forces acting on the bodies, and (2) *general* EOMs that tell us how the motion of particles and rigid bodies evolves as a function of time under the action of forces. Dynamics describes the *continuous* evolution of motion. It is not a series of "snapshots" at a few discrete times; yet many current textbooks treat it as such.

The natural language of dynamics is differential equations, and most of today's dynamics students have

either studied differential equations or are concurrently enrolled. Nonetheless, many dynamics books carry on an extended conspiracy to avoid differential equations. Given that most current texts regularly use algebraic equations, for example, $f = ma$ and $T = I\alpha$, rather than the differential equation statements $\Sigma f = m\ddot{r}$ and $T = I\ddot{\theta}$, students regularly have trouble with follow-on courses that require the development and solution of governing differential EOM.

An additional shortcoming of most current texts is the treatment of computer skills. Most engineering freshmen have an introduction to at least one programming language and then frequently suffer at least 1 year of neglect before starting over with a computer-based numerical methods course. To be direct, modern dynamics cannot be fully appreciated without recourse to computer solutions. This book provides enough computer example problems and tools for students to grasp the subject.

In the past, many engineering curricula covered dynamics twice. The "survivors" of the algebra-based first course proceeded to a differential-equation-based second course. The *essential* modeling skills, involving the development of general kinematics equations and general EOMs, were covered in the second course. The continuing contraction of engineering curricula is forcing an end to this practice. Many curricula developers are also struggling to find space for coverage of traditional vibration topics. Engineering educators should now be prepared to consider the *real* requirements of dynamics in both curricula and engineering practice and develop courses that meet these requirements. The contents of this book were developed to teach students how to model general dynamic systems. The writer's experience has been that most students have an easier time learning general skills, where effectiveness can be demonstrated for a wide range of problems, than a collection of problem-specific tasks.

Many current dynamics books have potentially superb problems; however, the wrong questions are being asked. In my experience, companies hiring engineers will never ask them to work the problems in these books or even apply the tricks the students are taught to solve these problems. Engineers are employed to derive *general* kinematics equations and EOMs. These general equations can be used to solve for maximum accelerations, loads, velocities, etc., and answer real engineering design problems. Evidence that our students understand this situation is provided by the massive resale of such books on the secondhand book market.

Students regularly keep books on thermodynamics, fluid mechanics, strength of materials, controls, etc., but rush to dispose of their undergraduate dynamics texts.

Dynamics books that are written with a conscious avoidance of differential equations tend to either omit elements of mechanical vibrations or provide a separate (late) chapter covering the topic. Given the differential-equation orientation of this book, elements of mechanical vibrations are integrated, from the outset, in the particle and planar-dynamics material. “Real-world” systems of particles and rigid bodies inevitably include springs and energy-dissipation mechanisms, including viscous, Coulomb, and aerodynamic damping. One- and two-degree-of-freedom (2DOF) problems are introduced in the application of particle and planar rigid-body kinetics. Damped and undamped natural frequencies and linear damping factors are introduced and defined for 1DOF problems. 2DOF problems are used to define system natural frequencies (eigenvalues) and mode shapes (eigenvectors) and to demonstrate modal uncoupling of MDOF vibration problems. The 2DOF material can be skipped without affecting the readability and utility.

The idea of an equilibrium position or positions defined by a differential EOM is introduced, leading to linearization of nonlinear EOM for small motion about an equilibrium position. Nonlinear terms are linearized by using Taylor-series expansions about an equilibrium position. Stable and unstable static equilibrium positions are demonstrated.

In considering his campaign, President George H.W. Bush referred to “The Vision Thing” (Wikipedia). In fact, vision is the missing element in current dynamics books and courses. The “vision” of dynamics presented in this book is *active* students *actively* developing *general* kinematic equations and EOMs, *actively* analyzing the systems to the extent possible, and then *actively* setting up and solving these equations via current computer techniques.

This book combines subjects that would normally be covered in both an introductory and an intermediate or advanced dynamics course. It is intended to be of value to students through their first course and subsequently in engineering practice, and was also written for working engineers who are trying to analyze real dynamic systems.

Book Outline

The outline followed in achieving the book’s vision is traditional. Chapter 1 covers some fundamental requirements of dynamics, including units, force, and mass, and provides a brief history of the development

of dynamics. This material is intended to be a quick review of material covered earlier in physics.

Chapter 2 covers kinematics of a particle, including displacement, velocity, and acceleration in one and two dimensions. Cartesian, polar, and path coordinates are introduced, and coordinate transformations of the components of a vector in a plane are emphasized. Alternative kinematic statements of the same example using Cartesian, polar, and path coordinates are emphasized, using the coordinate transformations to move between the coordinate representations.

Chapter 3 deals with the kinetics of a particle. It begins with Newton’s laws for rectilinear motion. The rectilinear examples introduce physical modeling, covering forces due to gravity and spring forces, as well as linear, quadratic, and Coulomb damping. 1DOF vibration problems are covered, introducing the concepts of damped and undamped natural frequencies, damping factors, resonance, etc. Solutions are developed for free and forced motion and steady-state response due to harmonic excitation. The examples emphasize the development of the EOM from $\Sigma \mathbf{f} = m\ddot{\mathbf{r}}$ and the solution of the equation via either (1) direct time integration or (2) the energy-integral substitution, $\ddot{x} = d(\dot{x}^2/2)/dx$. Next, the notion of degrees of freedom and kinematic equations of constraint are introduced, using masses connected by pulleys as examples. These examples include several 1DOF vibration problems.

Planar motion in two dimensions is introduced next, with applications involving Cartesian, polar, and path coordinates. The Cartesian examples include trajectory motion with and without quadratic drag and motion on a plane, including Coulomb friction forces. Polar coordinate applications include the simple pendulum, and this example is used as the initial demonstration of linearization of an EOM. EOMs are developed for the pendulum, including viscous and quadratic damping. The path-coordinate applications tend to be traditional with beads sliding on wires or along a specified surface.

The fact that dynamics problems regularly have MDOF is obscured by many introductory dynamics texts that only present 1DOF examples. 2DOF problems are introduced in Section 3.5 with simple spring-mass systems. An approach is presented for developing EOMs for masses connected by linear springs and dampers. Matrix statements are presented for linear 2DOF vibration problems with coordinate coupling provided by the stiffness and damping matrices. A double pendulum is introduced and analyzed as a second 2DOF example. Nonlinear EOMs are developed for the two particles in the double pendulum from $\Sigma \mathbf{f} = m\ddot{\mathbf{r}}$. These equations are subsequently linearized to yield a matrix model with coordinate coupling in the inertia matrix entries.

Analysis procedures for 2DOF vibration problems are introduced by analyzing the free motion of a spring-mass system, leading to a quadratic characteristic equation to define the eigenvalues and natural frequencies. Solutions for the eigenvectors from calculated eigenvalues are presented next. Uncoupling of the coupled matrix vibration equations via the matrix of eigenvectors, leading to uncoupled modal differential equations, is also explained. Solutions are presented for free and forced motion, proceeding from solutions of the modal differential equations. Forced motion resulting from harmonic excitation is also presented. 2DOF vibration problems are provided to show students that such problems exist, and methods to derive and analyze the EOMs that model the systems are presented. This is done to help students understand how these systems behave, with multiple natural frequencies and resonance possibilities. The vibration coverage in Chapter 3 is a reasonable introduction to aspects of vibration but is not a substitute for a good engineering-vibrations text.

Work-energy applications for 1DOF examples are covered in Section 3.6. Kinetic and potential energies are introduced, and the general work-energy expression, $\text{Work}_{n,c} = \Delta(T + V)$, is developed. Work due to potential forces is not covered. Such forces are modeled by potential energy functions from the outset. The energy-integral substitution, $d(\dot{x}^2/2)dx = \ddot{x}$, is employed to show the direct relation between the work-energy equation and the original $\Sigma f = m\ddot{r}$ differential EOM. A considerable amount of material is presented on the derivation of EOM for 1DOF examples starting from the work-energy equation.

Chapter 3 concludes with the coverage of linear momentum and moment-of-momentum topics. These are largely algebraic in nature and are customarily covered adequately by prior physics courses. The coefficient of restitution for particle collisions is covered in Section 3.7.2.

Chapter 4 covers planar kinematics of rigid bodies. Many engineering dynamics problems involve planar dynamics, that is, motion of rigid bodies in a plane (such as the plane of this page) versus 3D vector kinematics that would be required to keep track of the position and orientation of an airplane or satellite. The kinematic relationships of Chapter 2 are adequate for the tasks presented in this chapter. The traditional approach for teaching the analysis of planar mechanisms involves vector velocity and acceleration equations for Cartesian motion that require a great deal of vector algebra with cross products. This traditional approach is presented in Chapter 4; however, an alternate approach using geometry is also presented for finding relationships between variables. (My students have found the geometric approach to be much easier and are able to develop general kinematic equations for mechanisms quickly.)

A considerable effort is expended at the start of this chapter to cover the rolling-without-slipping kinematic condition, given that students traditionally have trouble grasping this concept. Multiple representative examples including several commonly occurring mechanisms are analyzed using both traditional and geometric approaches.

Chapter 5 covers planar kinetics of rigid bodies, starting with inertia properties and including the mass moment of inertia, the radius of gyration, and the parallel-axis formula. For planar motion of a rigid body, governing force and moment equations are developed in Section 5.3, and the kinetic energy definition is developed in Section 5.4. In Chapter 3, Newtonian and energy approaches are considered sequentially. In this chapter, students are assumed to have experience and confidence in developing the EOM for 1DOF examples using either Newtonian (free-body) approaches or the energy. Hence, for most 1DOF planar-kinetics examples, the EOM is derived using both approaches.

Fixed-axis-rotation examples provide the initial application of the governing equations. In Section 5.6, EOMs are developed for various compound-pendulum geometries. EOMs are developed, pivot reactions are defined in terms of the pendulum's rotation angle, and stability is examined for small motion about equilibrium positions. Linear damping is also considered. A bar supported by linear spring and damper supports is investigated, with considerations of motion about equilibrium. The base acceleration of a compound pendulum's pivot-point is also investigated.

Systems in general planar motion that require both force and moment equations are introduced in Section 5.7.1, including rolling-without-slipping examples. Various 1DOF examples are modeled in Section 5.7.2. In Section 5.7.3, EOM developments are carried out using Newtonian and energy approaches for one-body examples that include external forces. Generalized forces are introduced and defined for the energy approach. In Section 5.7.3, multibody, 1DOF examples are introduced, demonstrating the advantages of the energy approach in deriving the EOM for this type of problem.

Several MDOF examples are introduced in Section 5.7.4, including a double compound pendulum. Torsional vibrations and systems for which beams are used as springs are also considered. Section 5.7.5 covers models for various planar mechanisms, including developing the EOM from Newtonian approaches as well as the kinematic-constraint equations that were developed in Chapter 4. Section 5.8 concludes the chapter with the development and application of moment-of-momentum equations for the planar motion of a rigid body.

Chapter 6 explains how to develop equations of motion for dynamics using Lagrange's equations. Particle and planar rigid-body examples are considered.

Following a derivation of Lagrange's equations from principles of virtual work in Section 6.2, worked examples are provided in Section 6.3 for systems involving generalized coordinates (coordinates for which no kinematic constraints exist). The example problems move steadily forward in explaining how to account for external and dissipative forces, using the same procedures outlined in Chapters 3 and 5, in particular Section 5.7.3, for 1DOF problems.

Examples are examined in Section 6.4 for which either linear momentum or moment of momentum is conserved, showing that either circumstance is "flagged" using Lagrangian dynamics because one or more generalized coordinates are missing from the Lagrangian function. The examples also demonstrate that the circumstance of conservation of momenta can be utilized to reduce model dimensionality.

The mechanisms covered in Chapter 5 present immediate examples of dynamic systems involving nonlinear kinematic constraints. Section 6.5 presents the modification to Lagrange's equations via Lagrange multipliers that are needed to account for

kinematic constraints. Four example problems are worked through to demonstrate the approach.

A major advantage of the Lagrangian approach is that forces of restraint are automatically eliminated from the equations of motion. Generally speaking, it is easier to determine missing reaction forces via free-body diagrams and Newtonian dynamics. However, examples are presented in Section 6.6 that show how these reaction forces can be calculated within the Lagrangian method by introducing dummy coordinates plus kinematic constraints that limit their behavior.

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