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Chapter 1
Simulation of Arbitrary Mixed-Mode Crack Growth Using an Energy-Based Approach

B.R. Davis, P.A. Wawrzynek, and A.R. Ingraffea

Abstract A finite-element-based simulation technique is being developed to predict 3-D, arbitrary, non-planar evolution of mixed-mode crack growth. The approach combines a geometrically explicit crack front re-meshing scheme, and an energy-based growth formulation to predict extension magnitudes along the crack front. The technique also leverages a new 3-D mixed-mode energy release rate decomposition using the virtual crack extension (VCE) method. The energy-based crack growth formulation, previously implemented for planar crack growth, is extended to non-planar growth situations by employing a basis-function approach to describe crack front extensions. Rather than determining point-by-point extensions, calculating a governing function alleviates numerical influences on the crack growth predictions. The simulation technique seeks to mitigate computationally biased crack growth, as found in prescribed and mesh dependent methods, for example.

Keywords Crack growth • Energy release rate • Mixed-mode • Virtual crack extension • Finite-element analysis

1.1 Introduction

Many finite-element-based techniques have been developed to simulate 3-D arbitrary crack growth in structural components. Crack growth problems become inherently more difficult when complex loadings induce mixed-mode behavior. The resulting crack front evolution could become curved and tortuous. Utilizing a fracture mechanics approach to predict such complicated crack shape advances requires three main components: (1) crack representation, (2) fracture parameter calculations, and (3) crack extension application.

Currently, the prominent techniques for simulating 3-D, mixed-mode, arbitrary crack growth include the extended finite element method (XFEM) [1–3], cohesive zone elements [4, 5], and explicit crack front re-meshing schemes [6–9]. Each approach has been well developed throughout the literature, but as articulated in [10], suffer from some form of computational bias or require input on the expected crack evolution. For example, cohesive zone element approaches predict crack paths that are governed by the element locations, shapes, sizes and orientations. In some cases, finite element meshes are designed to adhere to the expected growth pattern, contradicting the notion of arbitrary crack evolution by linking a numerical artifact (the finite element mesh) with a physical phenomenon (crack growth). Alternatively, XFEM and re-meshing schemes often rely on user-prescribed growth increments to advance the front.

The unique crack growth simulation technique developed in [10] utilizes a new energy-based formulation that allows for the direct calculation of point-by-point crack front extensions. The formulation is derived from an energy expansion that depends on the first order variation of the energy release rate. The variation of energy release rate, made readily available via the virtual crack extension (VCE) method, acts as an influence function relating changes in energy from load increments to changes in geometry along the crack front. This term provides a mechanics-based approach to extract explicit crack extensions, avoiding the need to supply user-prescribed magnitudes of advance. The technique also employs a geometrically explicit crack representation that is continually updated through re-meshing schemes. As a result, mesh biased growth

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is reduced, allowing for the crack front to evolve arbitrarily. This form of crack representation is not a limitation of the technique, but deemed more practical for development purposes. The energy-based formulation can certainly be used with XFEM crack representations, for example.

The implementation of the simulation technique described in [10] is limited to mode I, planar crack growth conditions. It is the objective of the current work to extend the technique to simulate mixed-mode and non-planar, 3-D crack front evolution. To generalize the approach, an updated toolset is required. The commercial FRANC3D fracture analysis software [11] is used to represent and re-mesh the evolving non-planar crack front geometries. A new 3-D, mixed-mode VCE implementation (submitted and under review for publication) is developed to calculate the necessary energy release rates along the crack front. Finally, in conjunction with a trajectory criterion, a new basis-function approach to calculate extensions along an arbitrary front is constructed. The basis-function approach seeks to address sensitivity challenges associated with using a local, point-by-point approach for non-planar growth predictions. The focus of this proceeding will demonstrate the current status of the toolset development.

The next section will provide background and a description of the new 3-D, mixed-mode VCE implementation. Note that the full formulation and derivation of the mixed-mode VCE method will be available in an upcoming publication. Two examples are presented to verify the VCE mixed-mode energy release rate calculations. The following section will introduce the crack growth formulation, and its augmentation with the new basis-function methodology.

1.2 The Mixed-Mode Virtual Crack Extension Method

The VCE method for calculating energy release rates (also known as the stiffness derivative method) was first introduced by Dixon and Pook [12], and Watwood [13], and further developed by Hellen [14] and Parks [15]. Early VCE calculations utilized finite perturbations of meshes to approximate the required stiffness derivatives. This finite difference approach often introduced geometric approximation and numerical truncation errors. Using variational principle theories, a direct integral approach simplified the VCE method and improved accuracy and efficiency by eliminating the need for finite perturbations [16, 17]. The advent of the variational approach also allowed for the calculation of higher-order derivatives of energy release rate, the salient feature of the current method. Hwang et al. [18] generalized the variational formulations for planar 3-D cracks. A symmetric/anti-symmetric approach to decompose energy release rates, first demonstrated for a 2-D VCE method by Ishikawa [19] and later extended to 3-D equivalent domain J-integrals by [20–22], is used in the new 3-D VCE implementation.

1.2.1 Virtual Crack Extension Formulation

Following the formulation from [18], the VCE method is derived from the expression for the potential energy, Π, of a finite element system:

$$\Pi = \frac{1}{2} u^T K u - u^T f, \tag{1.1}$$

where u, K, and f are the displacement vector, the stiffness matrix, and the applied force vector, respectively.

The energy release rate, G, at crack-front position i is defined as the negative variation of the potential energy with respect to a virtual, incremental, crack-front extension, δa, in the normal direction of the front at that position:

$$G_i \equiv -\frac{\delta \Pi}{\delta A_i} = -\frac{\delta \Pi}{\delta a_i \ell_i}. \tag{1.2}$$

In the 3-D sense, the virtual extension has an associated area, δA_i, comprised of a virtual extension, δa_i, and an effective width, ℓ_i. Applying the variation to the finite element expression for potential energy leads to the following form of the energy release rate:

$$G_i = -\frac{1}{\ell_i} \left(\frac{1}{2} u^T \frac{\delta K}{\delta a_i} u - u^T \frac{\delta f}{\delta a_i} \right). \tag{1.3}$$

For simplicity in the current implementation, it is assumed that applied forces, f, are not influenced by the virtual incremental extension, and therefore the variational force term, $\frac{\delta f}{\delta a_i}$, is zero. The simplification reduces Eq. 1.3 to:

$$G_i = -\frac{1}{\ell_i}\left(\frac{1}{2}u^T\frac{\delta K}{\delta a_i}u\right). \tag{1.4}$$

Note, if the virtual extensions alter the nature of the applied load (e.g. with crack-face pressures, thermal, and body-force loadings), the effect must be accounted for with the variational force term, $\frac{\delta f}{\delta a_i}$, and included throughout the formulation.

The expression for the first-order variation of the energy release rate with respect to incremental crack extension, δa_j, follows from Eq. 1.4:

$$\frac{\delta G_i}{\delta a_j} = -\frac{1}{\ell_i}\left(u^T\frac{\delta K}{\delta a_i}\frac{\delta u}{\delta a_j} + \frac{1}{2}u^T\frac{\delta^2 K}{\delta a_i \delta a_j}u\right). \tag{1.5}$$

Note that Eqs. 1.4 and 1.5 represent the total global formulations of the VCE expressions for the energy release rate and rate of energy release rate. The integration and calculations are performed in a global coordinate system.

1.2.2 3-D Mixed-Mode Virtual Crack Extension Formulation

From Eq. 1.4, the energy release rate is composed of the stiffness derivative and nodal displacements. The stiffness derivative is fracture-mode invariant, leaving the displacements near the crack front to be decomposed into mode I, II, and III components. Using a local crack front coordinate system for each position along the front and the symmetric/anti-symmetric field decomposition found in [20], the local displacement modal contributions are determined such that:

$$u = u_I + u_{II} + u_{III}. \tag{1.6}$$

The decomposed displacements from Eq. 1.6 are substituted into Eq. 1.4, yielding the following expression for total energy release rate at position i:

$$G_i = -\frac{1}{\ell_i}\left[\frac{1}{2}(u_I + u_{II} + u_{III})^T\frac{\delta K}{\delta a_{i\,L}}(u_I + u_{II} + u_{III})\right]. \tag{1.7}$$

Note that the stiffness derivative is also calculated in the local crack-front coordinate system to maintain consistency, and is denoted by subscript L. Expanding Eq. 1.7 leads to the separation of the decomposed energy release rate modes:

$$G_i = -\frac{1}{\ell_i}\left[\begin{array}{l}\left(\frac{1}{2}u_I{}^T\frac{\delta K}{\delta a_{i\,L}}u_I\right) + \left(\frac{1}{2}u_{II}{}^T\frac{\delta K}{\delta a_{i\,L}}u_{II}\right) + \left(\frac{1}{2}u_{III}{}^T\frac{\delta K}{\delta a_{i\,L}}u_{III}\right) \\ + \left(u_I{}^T\frac{\delta K}{\delta a_{i\,L}}u_{II}\right) + \left(u_{II}{}^T\frac{\delta K}{\delta a_{i\,L}}u_{III}\right) + \left(\frac{1}{2}u_{III}{}^T\frac{\delta K}{\delta a_{i\,L}}u_I\right)\end{array}\right]. \tag{1.8}$$

Extending from Ishikawa's 2-D VCE mode decomposition [19], the mode I and II energy release rates are as follows:

$$G_{Ii} = -\frac{1}{\ell_i}\left(\frac{1}{2}u_I{}^T\frac{\delta K}{\delta a_{i\,L}}u_I\right), \tag{1.9}$$

$$G_{IIi} = -\frac{1}{\ell_i}\left(\frac{1}{2}u_{II}{}^T\frac{\delta K}{\delta a_{i\,L}}u_{II}\right). \tag{1.10}$$

The unique contribution of the current implementation is the inclusion of the mode III energy release rate distribution along the crack front. The remaining terms in Eq. 1.8, excluding Eqs. 1.9 and 1.10, comprise a pure mode III component, $\frac{1}{2}u_{III}{}^{T}\frac{\delta K}{\delta a_i}_{L}u_{III}$, and three modal-interaction coupling terms. Understanding the influence of the coupling-mode terms is critical in determining their respective contributions to the 3-D mixed-mode energy release rates. Consider the following coupling mode terms from Eq. 1.8:

$$G_{I/II_i} = -\frac{1}{\ell_i}\left(u_I{}^{T}\frac{\delta K}{\delta a_i}_{L}u_{II}\right), \tag{1.11}$$

$$G_{II/III_i} = -\frac{1}{\ell_i}\left(u_{II}{}^{T}\frac{\delta K}{\delta a_i}_{L}u_{III}\right), \tag{1.12}$$

$$G_{III/I_i} = -\frac{1}{\ell_i}\left(u_{III}{}^{T}\frac{\delta K}{\delta a_i}_{L}u_{I}\right). \tag{1.13}$$

Using symmetric and anti-symmetric arguments, it can be shown and verified numerically that calculating the coupling terms in Eqs. 1.11 and 1.13 about a symmetric domain lead to a cancellation effect. The result is $G_{III_i} = G_{III_i} = 0$. However, Eq. 1.12 has an additive effect that results in $G_{II/III_i} \neq 0$. The only contributing non-zero components to G_{II/III_i} are out-of-plane shear. Therefore, any addition to the total energy release rate from G_{II/III_i} must be a factor of the out-of-plane mode III energy release rate:

$$G_{III_i} = -\frac{1}{\ell_i}\left(\frac{1}{2}u_{III}{}^{T}\frac{\delta K}{\delta a_i}_{L}u_{III}\right) + G_{II/III_i}. \tag{1.14}$$

With the individual mixed-mode terms determined and the modal-interaction coupling terms accounted for, the VCE total energy release is successfully decomposed, satisfying the following summation:

$$G_i = G_{I_i} + G_{II_i} + G_{III_i}. \tag{1.15}$$

1.2.3 Numerical Examples

In this section two verification analyses are presented to demonstrate the accuracy of the new, 3-D, mixed-mode energy release rate implementation using the VCE method. The mixed-mode VCE results are compared with analytical and M-integral methods. Each model and crack front geometry is meshed using the FRANC3D software.

The first problem considered a 45°-inclined circular crack centrally embedded within a rectangular isotropic body, as shown in Fig. 1.1. The model geometry was appropriately sized to approximate crack behavior within an infinite body. Analytical expressions for the mixed-mode I/II/III stress intensity factors for an inclined penny crack under remote tension [23] are used as a reference solution.

The second numerical example is a half-penny-shaped surface crack in an isotropic cylindrical specimen [24, 25]. Figure 1.2 shows the global geometry and loading conditions that induce mode II/III behavior along the crack front. The local, in-plane geometry of the crack front is depicted in Fig. 1.2. The VCE energy release rate results are compared with M-integral calculations using the FRANC3D software.

Figures 1.3 and 1.4 display the mixed-mode and total energy release rate distributions calculated by the VCE implementation for each example. The results compare extremely well with the reference solutions. The average percent differences between the VCE and reference results are 0.11 % and 0.23 % for the inclined circular crack and the surface-cracked cylinder, respectively.

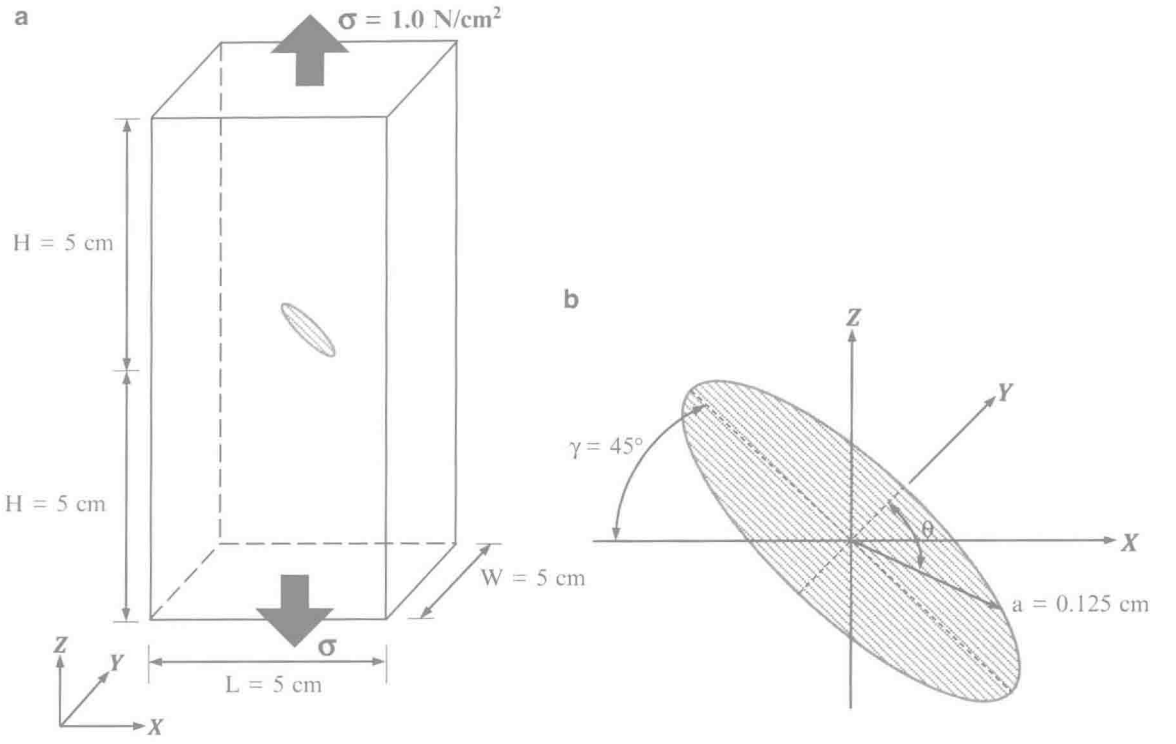

Fig. 1.1 Embedded inclined penny crack: (**a**) global geometry and loading conditions and (**b**) local crack-front geometry

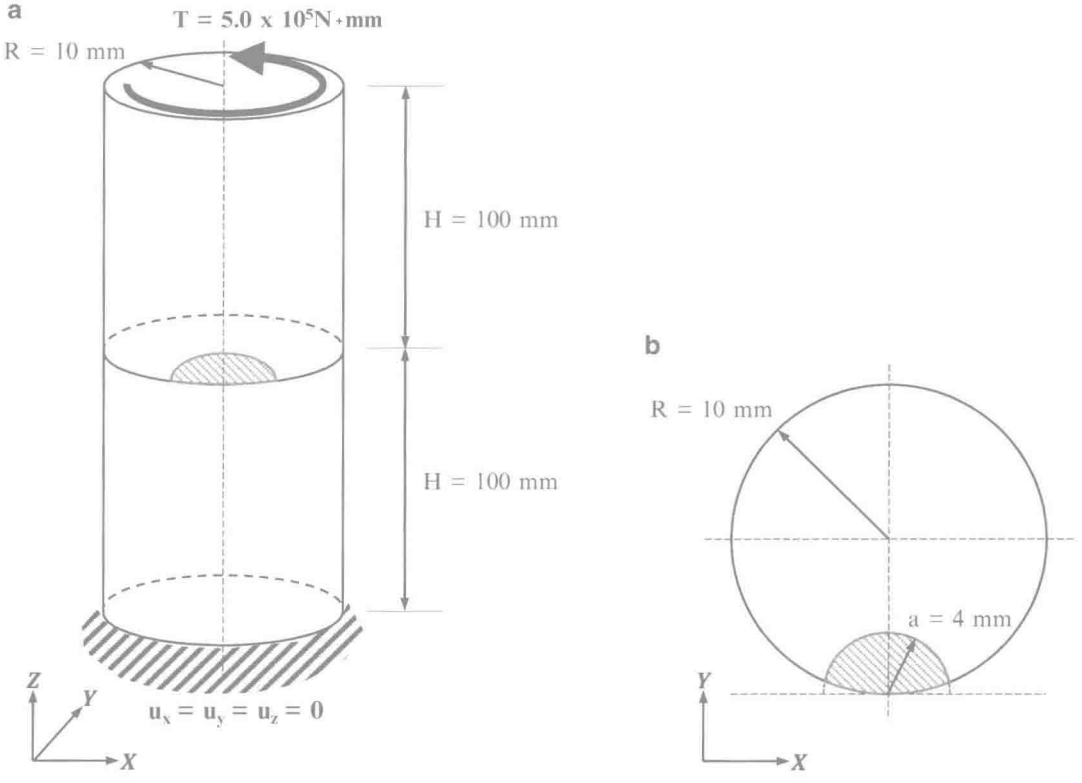

Fig. 1.2 Geometry and loading conditions for the penny-shaped surface crack in a cylindrical specimen: (**a**) global geometry and (**b**) local crack-plane geometry

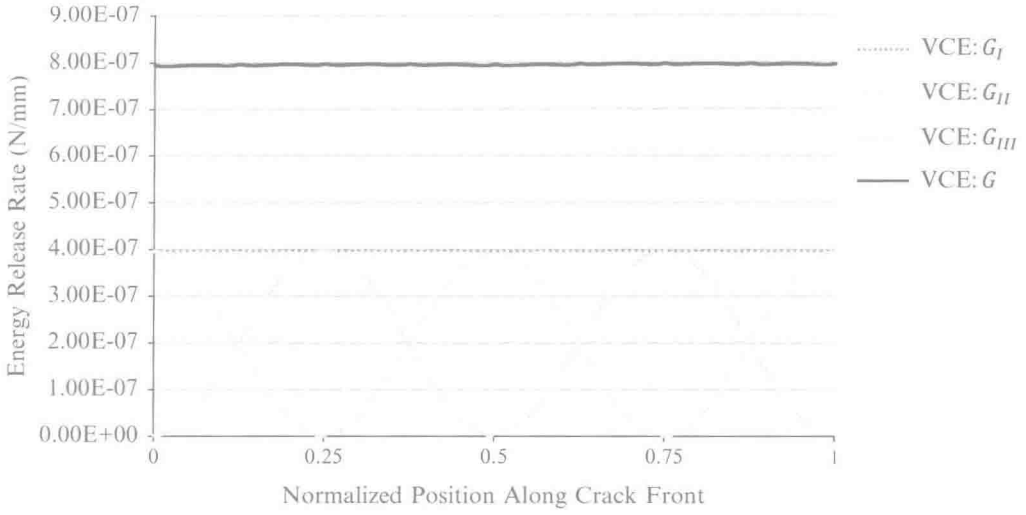

Fig. 1.3 Mixed-mode and total energy release rate distributions calculated by the VCE method for the inclined penny-crack specimen

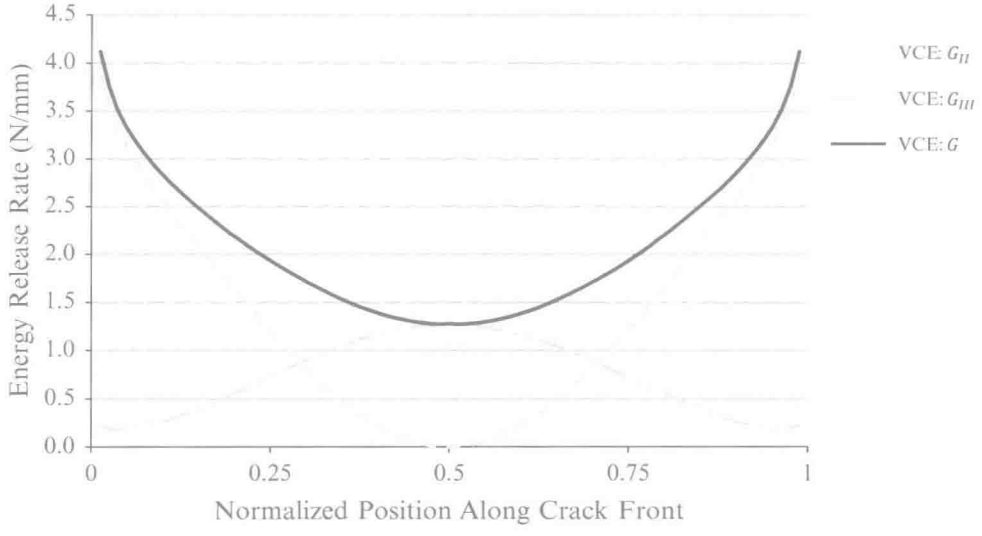

Fig. 1.4 Mixed-mode and total energy release rate distributions calculated by the VCE method for the surface-cracked cylindrical specimen

1.3 Crack Growth Formulation

The prediction of mixed-mode, non-planar crack front extensions requires two features. The first is a crack trajectory or kinking approach to determine the angle of advance for a position along the front. The second is a crack growth formulation to calculate the magnitudes of advances.

1.3.1 Crack Trajectory Criterion

For this work, a maximum critical energy release rate criterion is employed to obtain the extension angles [26]. For each position on the front, a series of virtual extensions that sweep radially along the local crack front normal are applied and energy release rates calculated. From this distribution the angle associated with the maximum energy release rate is chosen as the local position's extension angle. Certainly, other mixed-mode trajectory and kinking criteria can be used, such as maximum tensile stress and maximum shear stress.

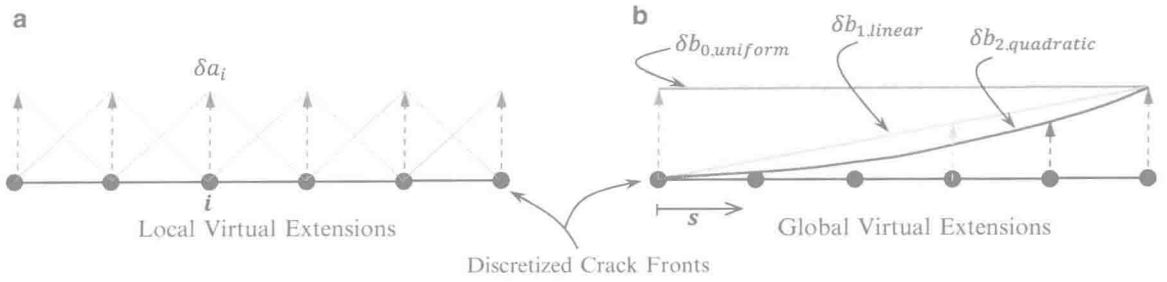

Fig. 1.5 Comparison between (**a**) local, point-by-point virtual crack extensions at position i, and (**b**) global basis-function virtual extensions along crack front length s

1.3.2 Basis-Function Growth Approach

The salient feature of this work is the modification of the crack growth formulation introduced in [10] to handle mixed-mode, non-planar growth situations. The planar growth formulation utilizes a balance equation extended from the first-order expansion of the energy release rate:

$$G_{ic} = G_i^0 + \frac{\delta G_i}{\delta P} \odot \Delta P_i + \frac{\delta G_i}{\delta a_j} \Delta a_j, \tag{1.16}$$

where G_{ic} is a critical energy release rate characterizing the material's resistance to crack growth; G_i^0 is the initial energy release rate prior to load increment ΔP_i; $\frac{\delta G_i}{\delta P} \odot \Delta P_i$ represents the energy contribution due to the applied load increment; $\frac{\delta G_i}{\delta a_j} \Delta a_j$ represents the energy contribution due to crack shape change. The $\frac{\delta G_i}{\delta a_j}$ term is the aforementioned rate of energy release rate parameter obtained through the VCE method that acts as an influence function relating energy changes to crack shape changes. By comparing energy contributions at an initial load level and after an applied load increment, Eq. 1.16 can be rearranged to directly obtain Δa_j, the point-by-point crack front extensions. Note that the critical energy release rate used for material toughness can be substituted with a mixed-mode fracture criterion or effective critical value and is not limited to a single toughness parameter.

Two major issues were encountered when attempting to apply the formulation for planar cracks described in [10] to non-planar growth situations. First, numerical sensitivities associated with the VCE calculations and the complicated front geometries were observed. These nominally insignificant undulations in energy release rate distributions along the front transpire through the growth formulation to create inconsistencies in the calculated crack front extension. These inconsistencies can accumulate and render the crack growth predictions nonsensical. Another concern is the accuracy of the VCE rate of energy release rate calculation. The suspect term within Eq. 1.5 is the variation of nodal displacements, $\frac{\delta u}{\delta a_j}$. The term, essentially, is obtained by applying a local virtual point perturbation at each position along the crack front and then obtaining a finite element solution. Similar to applying a nodal force in a standard finite element model, the local virtual point perturbation creates a region of high distortion near the perturbation.

To assuage both sources of error a new approach is applied to the VCE method. Rather than using local, point-by-point virtual extensions, δa_j, to calculate the rate of energy release rate, a basis-function approach is used. A set of linearly independent functions distributed along the crack front, δb_j, is used as virtual extensions. The expression for the rate of energy release rate now becomes:

$$\frac{\delta G_i}{\delta b_j} = -\frac{1}{\ell_i} \left(u^T \frac{\delta K}{\delta a_i} \frac{\delta u}{\delta b_j} + \frac{1}{2} u^T \frac{\delta^2 K}{\delta a_i \delta b_j} u \right). \tag{1.17}$$

The necessary calculations for each term are executed in a similar fashion to the standard local VCE approach. However, the second order virtual extensions are now globally applied across the crack front. Figure 1.5 shows a schematic demonstrating the difference between the local, point-by-point approach, and an example global, basis-function approach comprised of uniform, linear, and quadratic functions.

The augmented rate of energy release rate expression alters Eq. 1.16 so that the formulation calculates the coefficients of the basis functions that, when combined, represent a function for the predicted extensions along the crack front. The predicted extensions are then applied at the determined angle in the same manner described in the planar crack growth scheme.

The basis-function technique provides insulation against numerical noise by restricting the extension along the front to be smooth and continuous. The function characterizing the predicted growth mitigates the influence of the minor fluctuations in the energy release rate distributions on the individual front extensions. Additionally, by distributing the virtual extensions along the entire front, the point perturbation issue in the variation of nodal displacements is alleviated. This is analogous to applying a distributed force, i.e. a pressure, instead of nodal forces to avoid distortion in finite element results.

At this point no non-planar crack front evolution has been predicted using the basis-function approach. However, preliminary simulations have used the basis-function methodology to predict planar crack growth in a double cantilever beam specimen with applied displacement increments. The simulated uniform growth of the straight crack front compares well with analytical calculations and the local approach results from [10]. The predicted crack lengths have a relative difference of less than 1 %.

1.4 Conclusions

The presented work develops the toolset required for a new simulation technique to predict arbitrary mixed-mode crack growth. The technique relies on an explicit geometric representation of the crack through the meshing tools of FRANC3D. Fracture mechanics parameters of interest are calculated by a new 3-D mixed-mode VCE method. Finally, the energy-based crack growth formulation of [10] is modified through a novel basis-function approach to determine non-planar crack front extensions.

The development and implementation of the toolset is ongoing. Preliminary results show promise for mixed-mode and non-planar growth simulations. The VCE implementation is being extended to calculate energy release rates for anisotropic material systems, such as delaminations in composite laminates. The basis-function approach continues to be tested, and is being integrated within an incremental iterative crack growth simulation scheme for non-planar growth simulations.

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Chapter 2
Experimental and Predicted Crack Paths for Al-2024-T351 Under Mixed-Mode I/II Fatigue

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Abstract The aerospace industry has experience with a range of structural failures, oftentimes due to fatigue cracks in aircraft fuselage components that are exposed to relatively high stress levels during cyclic loading effects that lead to fatigue crack initiation at material defects and near stress concentrations. These aircraft components are under complex stress states. In this study, mixed mode I/II fatigue experiments and simulations are performed for an Arcan fixture and a 6.35 mm thick Al-2024-T351 specimen, a popular aerospace alloy. Experiments were performed for Arcan loading angles that gave rise to a range of Mode I/II crack tip conditions from $0 \leq \Delta KII/\Delta KI \leq \infty$. Measurements include the crack paths, loading cycles, and maximum and minimum loads for each loading angle. Simulations were performed using three-dimensional finite element analysis (3D-FEA) with 10-noded tetrahedral elements via the custom in-house FEA code, CRACK3D. While modeling the entire fixture-specimen geometry, a modified version of the virtual crack closure technique (VCCT) with automatic crack tip re-meshing and a maximum circumferential stress criterion was used to predict the direction of crack growth. Results indicate excellent agreement between experiments and simulations for the measured crack paths during the first several millimeters of crack extension.

Keywords Mixed mode • Fatigue • Arcan • Crack path • Modeling

2.1 Introduction

The aerospace industry has experience with a range of structural failures, oftentimes due to fatigue cracks in aircraft fuselage components that are exposed to relatively high stress levels during cyclic loading effects incurred during repeated take-off and landing events that lead to fatigue crack initiation at material defects and near stress concentrations. Numerous event over the last 30 years have been recorded where fatigue cracks in the fuselage have propagated to critical areas resulting the cabin opening up mid-flight and causing cabin depressurization along with numerous injuries and, in one case, death [1–3].

In fact, fatigue cracks are expected to form in the fuselage of modern airplanes due to repeated (a) pressurization and decompression of the cabin during every flight and (b) loading effects during take-off and landing. Thus, the propagation of cracks into critical joints continues to be an area of concern, especially since such propagation under complex stress states is not completely understood. Although procedures are currently in place to inspect and repair fatigue cracks, the ability to better predict how far a crack will propagate and in which direction it would grow when subjected to various loading conditions could save millions of dollars in premature inspection and repair, while also identifying the severity of an existing flaw in an aero-structure.

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