

CONTEMPORARY MATHEMATICS

697

Ordered Algebraic Structures and Related Topics

International Conference on
Ordered Algebraic Structures and Related Topics
October 12–16, 2015
Centre International de Rencontres Mathématiques (CIRM),
Luminy, France

Fabrizio Brogna
Françoise Delon
Max Dickmann
Danielle Gondard-Cozette
Victoria Ann Powers
Editors



The volume contains the proceedings of the international conference “Ordered Algebraic Structures and Related Topics” held from October 12–16, 2015, at CIRM, Luminy, Marseille, France.

Papers contained in this volume cover topics in real analytic geometry, real algebra, and real algebraic geometry including complexity issues, model theory of various algebraic and differential structures, Witt equivalence of fields, and the moment problem.

ISBN 978-1-4704-2966-9



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American Mathematical Society
Providence, Rhode Island

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2010 *Mathematics Subject Classification*. Primary 03Cxx, 06Fxx, 11Exx, 12-xx, 14Pxx, 14Qxx, 32Sxx, 44A60, 54C30, 58A07.

Library of Congress Cataloging-in-Publication Data

Names: International Conference on Ordered Algebraic Structures and Related Topics (2015 : Marseille, France) | Broglia, Fabrizio, 1948- editor.

Title: Ordered algebraic structures and related topics : International Conference on Ordered Algebraic Structures and Related Topics, October 12–16, 2015, Centre international de rencontres mathématiques (CIRM), Luminy, France / Fabrizio Broglia [and four others], editors.

Description: Providence, Rhode Island : American Mathematical Society, [2017] | Series: Contemporary mathematics ; volume 697 | Includes bibliographical references.

Identifiers: LCCN 2017015042 | ISBN 9781470429669 (alk. paper)

Subjects: LCSH: Ordered algebraic structures–Congresses. | Model theory–Congresses. | Geometry, Algebraic–Congresses. | Forms, Quadratic–Congresses. | Semigroups–Congresses. | AMS: Mathematical logic and foundations – Model theory – Model theory. msc | Order, lattices, ordered algebraic structures – Ordered structures – Ordered structures. msc | Number theory – Forms and linear algebraic groups – Forms and linear algebraic groups. msc | Field theory and polynomials. msc | Algebraic geometry – Real algebraic and real analytic geometry – Real algebraic and real analytic geometry. msc | Algebraic geometry – Computational aspects in algebraic geometry – Computational aspects in algebraic geometry. msc | Several complex variables and analytic spaces – Singularities – Singularities. msc | Integral transforms, operational calculus – Integral transforms, operational calculus – Moment problems. msc | General topology – Maps and general types of spaces defined by maps – Real-valued functions. msc | Global analysis, analysis on manifolds – General theory of differentiable manifolds – Real-analytic and Nash manifolds. msc

Classification: LCC QA172 .I5784 2017 | DDC 511.3/3–dc23

LC record available at <https://lcn.loc.gov/2017015042>

DOI: <http://dx.doi.org/10.1090/conm/697>

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10 9 8 7 6 5 4 3 2 1 22 21 20 19 18 17

Ordered Algebraic Structures and Related Topics

Preface

The present volume contains the proceedings of the meeting “Ordered Algebraic Structures and Related Topics”, held at the Centre International de Rencontres Mathématiques (CIRM), Luminy, Marseilles, October 12-16, 2015. The meeting, which had 78 participants, was convened to mark the 30th anniversary of the Paris seminar “Structures algébriques ordonnées”, organized by F. Delon, M. Dickmann and D. Gondard, whose activities are briefly reviewed in the following paragraphs.

The seminar began its activities in November 1984, motivated by the common scientific interests of the organizers: real algebraic geometry, real algebra, logical questions around power series fields, etc. The seminar has always been first and foremost interdisciplinary: for example, there have been many talks on the subjects “valuations and orders” and “model theory” which are transverse to most of the main themes of the seminar.

The initial interests of the seminar participants evolved over time, but the seminar’s format and even its familiar name, DDG, were nevertheless preserved. The early emphasis on real geometry and algebra, ordered groups, and other subjects in the 1980s, turned, twenty years later, towards the area of positive polynomials and relations with optimization and the moment problem, as well as to exponential-logarithmic series. This flexibility is a pillar of its longevity.

We have produced yearly proceedings leaving a written trace of many expositions; these records are not peer-reviewed¹ in order to speed the exchange of information among the interested researchers, in France and elsewhere. These notes, which are distributed to about 200 subscribers, are another pillar of the seminar’s longevity.

From the outset, the seminar was also intended to offer young researchers a platform for the exposition, discussion, and development of their work. Some of the subjects discussed in the seminar at an early stage of their development later grew into full-fledged research areas; examples are the theory of abstract real spectra and early work on the Birkhoff-Pierce conjecture.

These 30 years of uninterrupted activity resulted in some 440 talks and 284 written contributions² more than 60% of them by mathematicians from 25 countries other than France. Many international collaborations grew out of this interaction.

A trimester on Real Geometry was held at the Emile Borel Center (Institut Henri Poincaré, Paris) in 2005 that coincided with the seminar’s 20th anniversary. For the 30th anniversary, we gathered in the CIRM meeting a large number of

¹ But are nevertheless published with the editors’ agreement. A refereed selection of the contributions of the period 1984-1987 was published in the series “Publications Mathématiques de l’Université Paris 7”.

² To July 2013.

researchers whose work has contributed to the advancement of the main fields covered by the seminar:

- (1) Ordered groups, rings and fields, real algebra and valuations.
- (2) Model theory and algorithmic aspects.
- (3) Positive polynomials and optimization.
- (4) Real algebraic and analytic geometry; o-minimality.
- (5) Quadratic forms, abstract spaces of orderings, real semigroups.

The future of the seminar after the 2015 meeting had been a concern for the organizers: should we continue, or was 2015 the right moment to stop? The question was eventually resolved in favor of continuing the seminar for two reasons. First, by the encouragement of colleagues and friends familiar with the seminar's past work – many of them present at Luminy – who by a large majority favored continuing the seminar. Second, by the appointment in July 2015 of Tamara Servi as a Lecturer at the University Paris Diderot (Paris 7), whose scientific work is close to some of the seminar's themes, and who gladly joined us, providing a welcome influx of young blood. Since October 2015, the seminar's nickname has been DDGS.

The October 2015 meeting was marked by the unexpected death of Murray A. Marshall, on May 1st, 2015, aged 75, Professor Emeritus at the University of Saskatchewan, Saskatoon, Canada. He had been scheduled to deliver a one-hour address at the meeting. For many years Murray was a recurrent speaker at the DDG seminar, as well as co-author of some of the DDG seminar and Luminy meeting organizers. Ten of his approximately thirty co-authors attended the Luminy meeting and a special session at the meeting was dedicated to him.

Most readers of the current volume are familiar with Murray's brilliant, extensive, and influential scientific career that covered the following areas of research, of some of which he initiated:

- Algebraic Theory of Quadratic Forms
- Abstract Spaces of Orderings
- Abstract Real Spectra
- Positive Polynomials
- The Moment Problem

Several papers in the current volume bear witness, explicitly and implicitly, to Murray's mathematical talent, initiative, and influence.

We dedicate this volume to the memory of Murray Marshall.

On the permanent website of the meeting

<http://scientific-events.weebly.com/1155.html>

the reader can find a report of the homage to Murray Marshall, together with pictures of him. The website also contains abstracts, notes or slides, and some videos of the talks, as well as details about an evening session on "Women in Mathematics".

With great sadness we also learned of the death, on April 25, 2016, of François Lucas, a long time friend and member of the Paris Logic Group. François was a regular expositor at the seminar and co-authored several papers with two of its organizers. After receiving his PhD at the University Paris 7, François took up a position as a Lecturer at the University of Angers. In the last years of his life his research focused on the Birkhoff-Pierce conjecture.

We gratefully acknowledge the material support for the 2015 meeting from the following institutions:

- Centre International de Rencontres Mathématiques (CIRM)
- National Science Foundation (USA)
- Institut de Mathématiques de Jussieu – Paris Rive Gauche (CNRS, Université Paris 6, Université Paris 7).
- Institut Universitaire de France (IUF)
- Groupement de recherche “Structuration de la théorie des nombres” (CNRS, GDR-STN)

Max Dickmann and Danielle Gondard
Paris, January 18, 2017

Program of the conference

Ordered Algebraic Structures and Related Topics CIRM; Luminy, France. October 12-16, 2015

Monday, October 12 – **Ordered Groups, Real Algebra and Valuations.**

- 9 - 9:10. Opening
- 9:15 - 10. A. Pfister (Mainz, Germany), An elementary proof of Hilbert's theorem on ternary quartics.
- 10:30 - 11. M. Spivakovsky (Toulouse), Generalized Puiseux expansion of a valuation and local uniformization in arbitrary characteristic.
- 11:10 - 11:40. F.-V. Kuhlmann (Katowice, Poland), Symmetrically complete ordered fields.
- 11:50 - 12:20. B. Teissier (Paris 7), Valued noetherian local rings versus valued fields.
- 15:45 - 16:45. J. Königsmann (Oxford, UK), Survey. How Galois theory knows arithmetic via valuations.
- 17:15 - 17:45. L. Bröcker (Münster, Germany) On p-adic semialgebraic sets.
- 17:55 - 18:25. N. Schwartz (Passau, Germany), The sums of squares in a real field.
- 18:35 - 19:05. E. Becker (Dortmund, Germany), Sums of Powers in Algebraic Function Fields of One Variable

Tuesday, October 13 – **Model Theory and Algorithm Topics.**

- 9 - 10. S. Basu (Purdue Univ., USA), A survey of recent advances in quantitative and algorithmic real algebraic geometry.
- 10:30 - 11. D. Perrucci (Buenos Aires, Argentina), Elementary recursive degree bounds for the Positivstellensatz, Hilbert's 17th problem, and the Real Nullstellensatz.
- 11:10 - 11:40. C. Riener (Aalto Univ., Finland), Topological complexity of symmetric semi-algebraic sets.
- 11:50 - 12:20. H. Lombardi (Besançon), On the constructive algebra of real numbers.
- 15:45 - 16:45. L. van den Dries (Urbana, USA), Survey: The ordered differential field of transseries.
- 17:15 - 17:45. M. Tressl (Manchester, UK), Model Theory of Real Closed Rings.
- 17:55 - 18:25. S. Kuhlmann (Konstanz, Germany), Real Closed Fields and Models of Peano Arithmetic.
- 18:35 - 19:05. F. Jahnke (Münster, Germany) Dp-minimal ordered fields.

- 20:30 - 22:30. **Young researchers evening session.**

- A. Blaszcok (Univ. of Silesia, Poland).
- Q. Brouette (Univ. Mons, Belgium).
- C. Goel (Univ. Konstanz, Germany).
- M. Infusino (Univ. Konstanz, Germany).
- M. Thomas (Univ. Konstanz, Germany).
- G. Violet (Univ. Konstanz, Germany).

Wednesday, October 14 – **Quadratic forms; Abstract Spaces of Orderings; Real Semigroups.**

- 9 - 9:30. Homage to Murray Marshall.
- 9:30 - 10:20. F. Miraglia (S. Paulo, Brazil) Survey: Abstract theory of quadratic forms and its applications in ring theory.
- 10:45 - 11:15. P. Gładki (Katowice, Poland), Witt equivalence of various classes of fields.
- 11:20 - 11:50. Katarzyna Kuhlmann (Katowice, Poland), Orderings and $\mathbb{R}$ -places of function fields.
- 11:55 - 12:25. V. Astier (Dublin, Ireland), Signatures of hermitian forms and orderings on some algebras with involution.
- 18:00. **Poster session opening.**
- 20:45 – 22:15. **Women in Mathematics session.**
 - Presentation of the recently launched International Mathematical Union Committee for Women in Mathematics. Marie-Françoise Roy (chair of the committee).
 - Roundtable discussion on the gender gap in mathematics.
 - Panel: Francesca Acquistapace (Italy), Anna Blaszcok (Poland), Charu Goel (Germany), Noa Lavi (Israel), Vicky Powers (USA), Marie-Françoise Roy (France), Carol Wood (USA).

Thursday, October 15 – **Real Algebraic and Analytic Geometry; o-minimality.**

- 9 - 10. J.-P. Rolin (Dijon), Survey: Logarithmic-exponential series and fractal analysis.
- 10:30 - 11. Tobias Kaiser (Passau, Germany), Lebesgue motivic invariants.
- 11:10 - 11:40. T. Servi (Pisa, Italy), Lebesgue integration of oscillating and subanalytic functions.
- 11:50 - 12:20. I. Itenberg (Paris 6), Real Hurwitz numbers.
- 15:45 - 16:45. J. Fernando (Madrid, Spain), Survey: On polynomial, regular and Nash images of Euclidean spaces.
- 17:15 - 17:45. H. Schoutens (CUNY, NY USA) O-minimalism: first order properties of o-minimal theories.
- 17:55 - 18:25. Z. Izhakian (Bar Ilan Univ., Israel) Supertropical algebra and representations.
- 18:35 - 19:05. F. Acquistapace (Pisa, Italy), A class of globally defined semianalytic sets. (Joint work with F. Broglia and J. Fernando)

Friday, October 16 – **Positive Polynomials and Optimisation.**

- 9 - 10. D. Plaumann (Konstanz, Germany), Survey: Positive polynomials according to Murray Marshall.

- 10:30 - 11. G. Blekherman (Georgia Tech., USA), Degree Bounds in Rational Sums of Squares Representations on Varieties.
- 11:10 - 11:40. T. Netzer (Leipzig, Germany), Proving Kazhdan's Property (T) with sums of squares techniques.
- 11:50 - 12:20. B. Reznick (Urbana, USA), Ternary forms with lots of zeros.
- 14:30 - 15:30. K. Schmüdgen (Leipzig, Germany), Survey: The truncated moment problem and positive polynomials.
- 16 - 16:30. R. Sinn (Georgia Tech., USA) Gram Spectrahedra (Joint work with D. Plaumann and C. Vinzant).
- 16:40 - 17:10. M. Schweighofer (Konstanz, Germany), Inclusion of spectrahedra, free spectrahedra and coin tossing (Joint work with Bill Helton, Igor Klep and Scott McCullough).
- 17:20 - 17:50. C. Scheiderer (Konstanz, Germany), Extremal nonnegative forms.

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