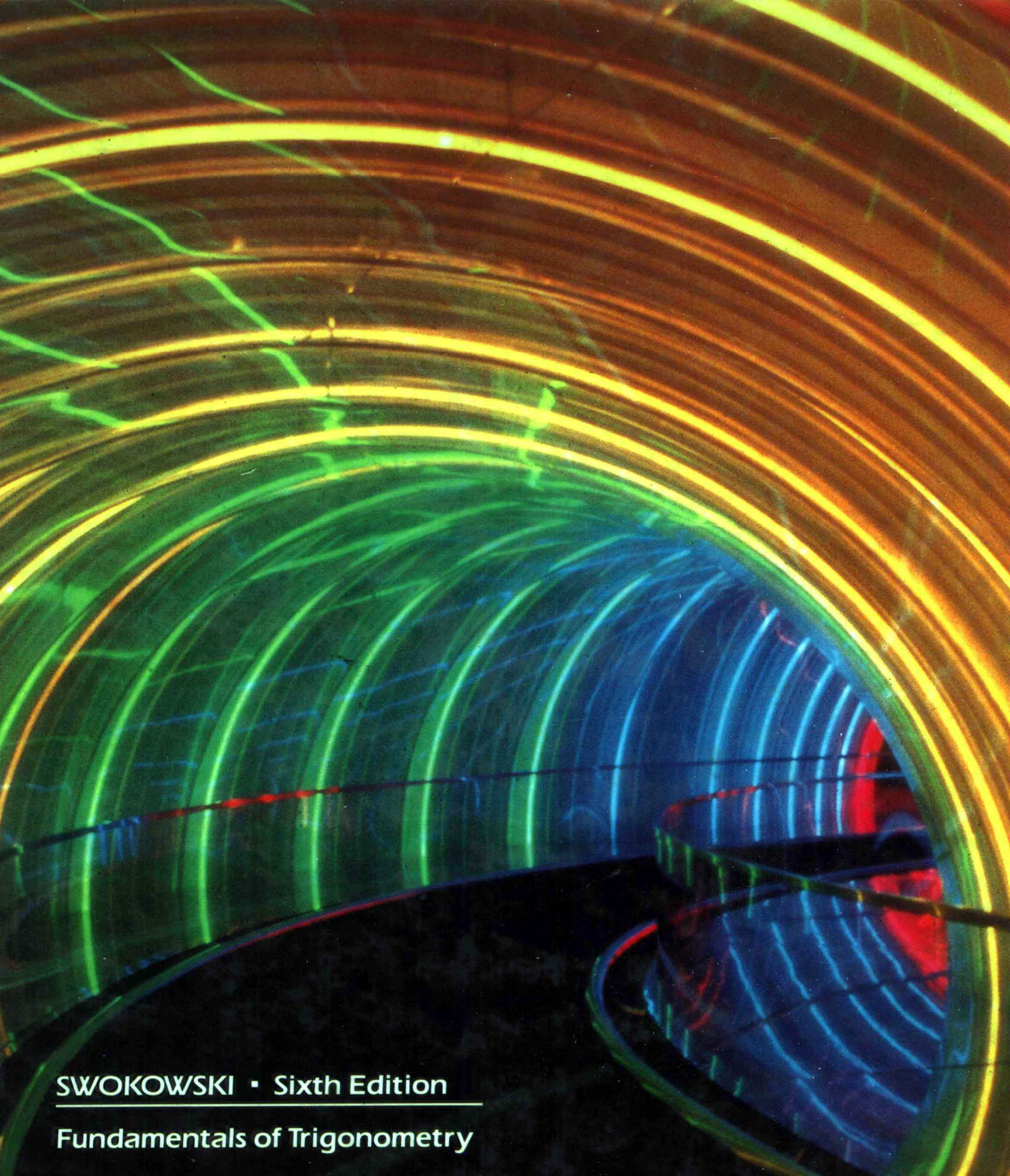

SWOKOWSKI • Sixth Edition

Fundamentals of Trigonometry

Fundamentals of Trigonometry

Sixth Edition

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Preface

This book is a major revision of the previous five editions of *Fundamentals of Trigonometry*. One of my goals was to maintain the mathematical soundness of earlier editions, while making discussions somewhat less formal by rewriting, placing more emphasis on graphs, and by adding new examples and figures. Another objective was to stress the usefulness of the subject matter through a great variety of applied problems from many different disciplines. Finally, suggestions for improvements from users of previous editions led me to change the order in which certain topics are presented. The comments that follow highlight some of the changes and features of this edition.

CHANGES FOR THE SIXTH EDITION

- The review material in Chapter 1 has been reorganized, with more emphasis given to graphical interpretations for the domain and range of a function.
- Chapter 2 begins with a section on angles, then proceeds to the unit circle definition of the trigonometric functions, which is followed by a right triangle approach.
- Trigonometric tables have been deemphasized, since calculators are much more efficient and accurate. Teachers who feel that students should be instructed on the use of tables and the technique of linear interpolation will find suitable material in Appendix I.
- Section 3.6 on the inverse trigonometric functions has been rewritten, with better motivation being provided for domains and graphs.
- Applications of oblique triangles and vectors are given greater emphasis in Chapter 4.
- The approach to complex numbers in Section 5.1 has been simplified.
- Chapter 6 has been completely rewritten, with much more attention given to the natural exponential and logarithmic functions and their applications.

FEATURES

Applications Previous editions contained applied problems, but most of them were in the fields of engineering, physics, chemistry, or biology. In this revision other subjects are also considered, such as physiology, medicine, sociology, ecology, oceanography, marine biology, business, and economics.

Examples Each section contains carefully chosen examples to help students understand and assimilate new concepts. Whenever feasible, applications are included to demonstrate the usefulness of the subject matter.

Exercises Exercise sets begin with routine drill problems and gradually progress to more difficult types. As a rule, applied problems appear near the end of the set, to allow students to gain confidence in manipulations and new ideas before attempting questions that require an analysis of practical situations.

There is a review section at the end of each chapter, consisting of a list of important topics and pertinent exercises.

Answers to the odd-numbered exercises are given at the end of the text. Instructors may obtain an answer booklet for the even-numbered exercises from the publisher.

Calculators Calculators are given much more emphasis in this edition. It is possible to work most of the exercises without the aid of a calculator; however, instructors may wish to encourage their use to shorten numerical computations. Some sections contain problems labeled *Calculator Exercises,* for which a calculator should definitely be employed.

Text Design A change in page size has made it possible to place most figures in margins, as close as possible to where they are first mentioned in the text. A new use of a second color for figures and statements of important facts should make it easier to follow discussions and remember major ideas. Graphs are usually labeled and color-coded to clarify complex figures. Many figures have been added to exercise sets to help visualize important aspects of applied problems.

Flexibility Hundreds of syllabi from schools that used previous editions attest to the flexibility of the text. Sections and chapters can be rearranged in many different ways, depending on the objectives and the length of the course.

SUPPLEMENTS

Instructors may obtain a manual containing worked-out solutions for approximately one-third of the exercises, authored by Stephen Rodi of Austin Community College. Test banks that can be used for quizzes and examinations are also available from the publisher. Students who need additional help may purchase, from their bookstore, *A Programmed Study Guide* by Roy Dobyns of Carson-Newman College. This guide is designed to assist self-study by reinforcing the mathematics presented in the lectures and the text.

ACKNOWLEDGMENTS

I wish to thank Michael Cullen of Loyola Marymount University for supplying all the new exercises dealing with applications. This large assortment of problems provides strong motivation for the mathematical concepts introduced in the text. Because of his significant input on exercise sets, Michael should be considered as a coauthor of this edition.

This revision has benefited from the comments of users of previous editions. I wish to thank the following individuals, who reviewed the manuscript and offered many helpful suggestions:

James Arnold, University of Wisconsin–Milwaukee
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Michael D. Taylor, University of Central Florida

I also wish to single out the following mathematics educators, who met with me and representatives from my publisher for several days during the summer of 1984: Michael Eames of Keene State College, Carol Edwards of St. Louis Community College, John Hornsby of the University of New Orleans, and William Robinson of Ventura College. Their general comments about pedagogy and their specific recommendations about the content of mathematics courses were extremely helpful in improving the book.

I am thankful to John Spellmann of Southwest Texas State University, who checked the answers to the exercises.

I am grateful for the excellent cooperation of the staff of Prindle, Weber & Schmidt. Two people in the company deserve special mention. They are Senior Editor David Pallai and my production editor Kathi Townes. The present form of the book was greatly influenced by their efforts, and I owe them both a debt of gratitude. Moreover, their personal friendship has often been a source of comfort during the years we have worked together.

In addition to all of the persons named here, I express my sincere appreciation to the many unnamed students and teachers who have helped shape my views on mathematics education.

EARL W. SWOKOWSKI

Contents

CHAPTER 1

Prerequisites for Trigonometry

This chapter contains topics necessary for the study of trigonometry. ■ After a review of real numbers, coordinate systems, and graphs in two dimensions, we turn our attention to one of the most important concepts in mathematics—the notion of function.

1.1 Real Numbers

Real numbers are employed in all phases of mathematics, and you are undoubtedly well acquainted with symbols used to represent them, such as

$$73, \quad -5, \quad \tfrac{49}{12}, \quad \sqrt{2}, \quad \sqrt[3]{-85}, \quad -8.674, \quad 0.33333\ldots, \quad 596.25.$$

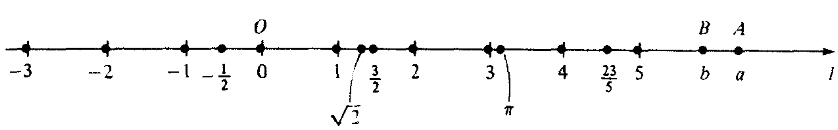
We shall assume familiarity with the fundamental properties of addition, subtraction, multiplication, division, exponents, and radicals. Throughout this chapter, unless otherwise specified, lowercase letters $a, b, c, \ldots$ will denote real numbers.

The **positive integers** 1, 2, 3, 4, ... may be obtained by adding the real number 1 successively to itself. The **integers** consist of all positive and negative integers together with the real number 0. A **rational number** is a real number that can be expressed as a quotient a/b, where a and b are integers and $b \neq 0$. Real numbers that are not rational are called **irrational.** The ratio of the circumference of a circle to its diameter is irrational. This real number is denoted by π and the notation $\pi \approx 3.1416$ is used to indicate that π is *approximately equal* to 3.1416. Another familiar example of an irrational number is $\sqrt{2}$.

Real numbers are often expressed in terms of decimals. For rational numbers the decimals are either terminating or repeating, such as $\frac{5}{4} = 1.25$ or $\frac{177}{55} = 3.2181818\ldots$, where the digits 1 and 8 repeat indefinitely. Decimal representations for irrational numbers are always nonterminating and nonrepeating.

Real numbers may be represented geometrically by points on a line l in such a way that for each real number a there corresponds one and only one point, and conversely, to each point P on l there corresponds precisely one real number. Such an association is referred to as a **one-to-one correspondence.** We first choose an arbitrary point O, called the **origin,** and associate with it the real number 0. Points associated with the integers are then determined by laying off successive line segments of equal length on either side of O as illustrated in Figure 1.1. The points corresponding to rational numbers such as $\frac{23}{5}$ and $-\frac{1}{2}$ are obtained by subdividing the

FIGURE 1.1



equal line segments. Points associated with irrational numbers such as π can be approximated to within any degree of accuracy by locating successively the points corresponding to 3, 3.1, 3.14, 3.141, 3.1415, 3.14159, and so on.

The number a that is associated with a point A on l is called the **coordinate** of A. An assignment of coordinates to points on l is called a **coordinate system** for l, and l is called a **coordinate line,** or a **real line.** A direction can be assigned to l by taking the **positive direction** along l to the right and the **negative direction** to the left. The positive direction is noted by placing an arrowhead on l as shown in Figure 1.1. Numbers that correspond to points to the right of O in Figure 1.1 are called **positive real numbers,** whereas those that correspond to points to the left of O are **negative real numbers.** The real number 0 is neither positive nor negative.

At times it is convenient to use the notation and terminology of sets. A **set** may be thought of as a collection of objects of some type. The objects are called **elements** of the set. Throughout our work $\mathbb{R}$ will denote the set of real numbers and $\mathbb{Z}$ the set of integers. If every element of a set S is also an element of a set T, then S is called a **subset** of T. For example, $\mathbb{Z}$ is a subset of $\mathbb{R}$. Two sets S and T are said to be **equal,** written $S = T$, if S and T contain precisely the same elements. The notation $S \neq T$ means that S and T are not equal.

We frequently use symbols to represent arbitrary elements of a set. For example, we may use x to denote a real number, although no *particular* real number is specified. A letter that is used to represent any element of a given set is sometimes called a **variable.** A symbol that represents a *specific* element is called a **constant.** In most of our work, letters near the end of the alphabet, such as x, y, and z, will be used for variables, whereas letters such as a, b, and c will denote constants. Throughout this text, unless otherwise specified, variables represent real numbers. The **domain of a variable** is the set of real numbers represented by the variable. To illustrate, $\sqrt{x}$ is a real number if and only if $x \geq 0$, and hence the domain of x is the set of nonnegative real numbers. Similarly, given the expression $1/(x - 2)$, we must exclude $x = 2$ in order to avoid division by zero; consequently, in this case the domain is the set of all real numbers different from 2.

If the elements of a set S have a certain property, we sometimes write $S = \{x: \quad \}$, where the property describing the variable x is stated in the space after the colon. For example, $\{x: x \text{ is an even integer}\}$ denotes the set of all even integers. Finite sets are sometimes identified by listing all the elements within braces. Thus, if the set T consists of the first five positive integers, we may write $T = \{1, 2, 3, 4, 5\}$.

If a and b are real numbers and $a - b$ is positive, we say that ***a* is greater than *b*** and write $a > b$. An equivalent statement is ***b* is less than *a*,** written $b < a$. The symbols $>$ and $<$ are called **inequality signs,** and expressions such as $a > b$ or $b < a$ are called **inequalities.** Referring to Figure 1.1, we see that if A and B are points on l with coordinates a and b, respectively, then $a > b$ (or $b < a$) *if and only if A lies to the right of B.* As illustrations,

$$5 > 3, \qquad -6 < -2, \qquad -\sqrt{2} < 1, \qquad 2 > 0, \qquad -5 < 0.$$

Note in general that $a > 0$ if and only if a is positive, and $a < 0$ if and only if a is negative. We sometimes refer to the **sign** of a real number as being positive or negative if the number is positive or negative, respectively.

The notation $a \geq b$, which is read **a is greater than or equal to b,** means that either $a > b$ or $a = b$ (but not both). The symbol $a \leq b$ is read **a is less than or equal to b** and means that either $a < b$ or $a = b$. The expression $a < b < c$ means that both $a < b$ and $b < c$, in which case we say that **b is between a and c.** We may also write $c > b > a$. For instance,

$$1 < 5 < \tfrac{11}{2}, \qquad -4 < \tfrac{2}{3} < \sqrt{2}, \qquad 3 > -6 > -10.$$

Other variations of the inequality notation are used. For example, $a < b \leq c$ means both $a < b$ and $b \leq c$. Similarly, $a \leq b < c$ means both $a \leq b$ and $b < c$. Finally, $a \leq b \leq c$ means both $a \leq b$ and $b \leq c$.

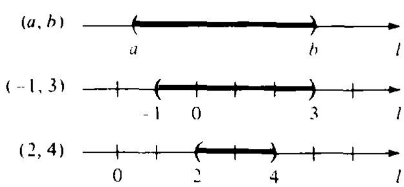
If $a < b$, the symbol (a, b) is sometimes used to denote all real numbers between a and b. This set is called an **open interval.**

DEFINITION OF OPEN INTERVAL

$$(a, b) = \{x : a < x < b\}$$

FIGURE 1.2

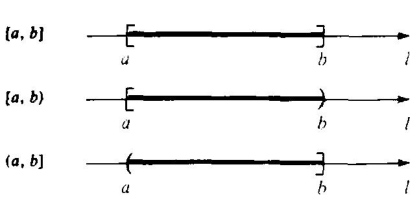
Open intervals (a, b), $(-1, 3)$, and $(2, 4)$.



The numbers a and b are called the **endpoints** of the interval. The **graph** of the open interval (a, b) consists of all points on a coordinate line that lie between the points corresponding to a and b, as illustrated by the black part of l in Figure 1.2. The parentheses in the figure indicate that the endpoints of the interval are not included in the graph. For convenience, we use the terms *open interval* and *graph of an open interval* interchangeably.

If we wish to include an endpoint of an interval, a bracket is used instead of a parenthesis. If $a < b$, then **closed intervals,** denoted by $[a, b]$, and **half-open intervals,** denoted by $[a, b)$ or $(a, b]$, are defined as follows.

FIGURE 1.3



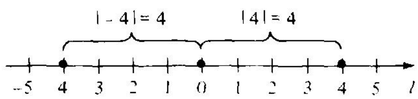
$$[a, b] = \{x : a \leq x \leq b\}$$

$$[a, b) = \{x : a \leq x < b\}$$

$$(a, b] = \{x : a < x \leq b\}$$

Typical graphs are sketched in Figure 1.3. A bracket indicates that the corresponding endpoint is part of the graph.

FIGURE 1.4



If a is a real number, then it is the coordinate of some point A on a coordinate line l, and the symbol $|a|$ is used to denote the number of units (or distance) between A and the origin, without regard to direction. The nonnegative number $|a|$ is called the ***absolute value*** of a. Referring to Figure 1.4, we see that for the point with coordinate -4, we have $|-4| = 4$. Similarly, $|4| = 4$. In general, if a is negative we change its sign to find $|a|$, whereas if a is nonnegative, then $|a| = a$. The next definition summarizes this discussion.

DEFINITION

If a is a real number, then the **absolute value** of a, denoted by $|a|$, is

$$|a| = \begin{cases} a & \text{if } a \geq 0. \\ -a & \text{if } a < 0. \end{cases}$$

The use of this definition is illustrated in the following example.

EXAMPLE 1 Find $|3|$, $|-3|$, $|0|$, $|\sqrt{2} - 2|$, and $|2 - \sqrt{2}|$.

SOLUTION Since 3, $2 - \sqrt{2}$, and 0 are nonnegative,

$$|3| = 3, \quad |2 - \sqrt{2}| = 2 - \sqrt{2}, \quad \text{and} \quad |0| = 0.$$

Since -3 and $\sqrt{2} - 2$ are negative, we use the formula $|a| = -a$ to obtain

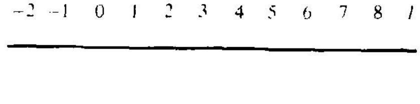
$$|-3| = -(-3) = 3 \quad \text{and} \quad |\sqrt{2} - 2| = -(\sqrt{2} - 2) = 2 - \sqrt{2}. \quad \blacksquare$$

Note that in Example 1, $|-3| = |3|$ and $|2 - \sqrt{2}| = |\sqrt{2} - 2|$. It can be shown that

$$|a| = |-a| \quad \text{for every real number } a.$$

We shall use the concept of absolute value to define the distance between any two points on a coordinate line. Let us begin by noting that the distance between the points with coordinates 2 and 7, shown in Figure 1.5, equals 5 units on l. This distance is the difference, $7 - 2$, obtained by subtracting the smaller coordinate from the larger. If we employ absolute values, then since $|7 - 2| = |2 - 7|$, it is unnecessary to be concerned about the order of subtraction. We shall use this as our motivation for the next definition.

FIGURE 1.5



DEFINITION

Let a and b be the coordinates of two points A and B, respectively, on a coordinate line l. The **distance between A and B**, denoted by $d(A, B)$, is

$$d(A, B) = |b - a|.$$

The number $d(A, B)$ is also called the **length of the line segment AB.** Observe that since $d(B, A) = |a - b|$ and $|b - a| = |a - b|$,

$$d(A, B) = d(B, A).$$

Also note that the distance between the origin O and the point A is

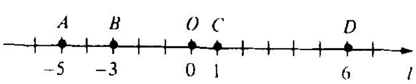
$$d(O, A) = |a - 0| = |a|.$$

EXAMPLE 2 Let A, B, C, and D have coordinates -5, -3, 1, and 6, respectively, on a coordinate line l (see Figure 1.6). Find $d(A, B)$, $d(C, B)$, $d(O, A)$, and $d(C, D)$.

SOLUTION Using the definition of the distance between points,

$$d(A, B) = |-3 - (-5)| = |-3 + 5| = |2| = 2$$
$$d(C, B) = |-3 - 1| = |-4| = 4$$
$$d(O, A) = |-5 - 0| = |-5| = 5$$
$$d(C, D) = |6 - 1| = |5| = 5$$

FIGURE 1.6



These answers can be checked by referring to Figure 1.6. ■

The concept of absolute value has uses other than that of finding distances between points. Generally, it is employed whenever we are interested in the "magnitude" or "numerical value" of a real number without regard to its sign.

If x is a variable, then expressions of the form

$$x + 3 = 0, \quad x^2 - 5 = 4x, \quad \text{or} \quad (x^2 - 9)\sqrt[3]{x + 1} = 0$$

are called **equations** in x. A number a is a **solution** or **root of an equation** if a true statement is obtained when a is substituted for x. We also say that a **satisfies** the equation. For example, 5 is a solution of the equation $x^2 - 5 = 4x$ since substitution gives us $(5)^2 - 5 = 4(5)$, or $20 = 20$, which is a true statement. To **solve** an equation means to find all the solutions.

An equation is called an **identity** if every number in the domain of the variable is a solution. An example of an identity is

$$\frac{1}{x^2 - 4} = \frac{1}{(x + 2)(x - 2)}$$

since this equation is true for every number in the domain of x. An equation is called a **conditional equation** if there are real numbers in the domain of the variable that are *not* solutions.

Two equations are **equivalent** if they have exactly the same solutions. For example, the equations

$$x = 3, \quad x - 1 = 2, \quad 5x = 15, \quad \text{and} \quad 2x + 1 = 7$$

are all equivalent.

We shall assume that the reader has had experience in finding solutions of equations in one variable. In particular, recall that the solutions of a **quadratic equation** $ax^2 + bx + c = 0$, for $a \neq 0$, may be obtained as follows.

QUADRATIC FORMULA

The solutions of the equation $ax^2 + bx + c = 0$, for $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The solutions are real if $b^2 - 4ac$ is nonnegative. The case $b^2 - 4ac < 0$ is discussed in Chapter 5.

EXAMPLE 3 Find the solutions of $2x^2 + 7x - 15 = 0$.

SOLUTION Using the Quadratic Formula with $a = 2$, $b = 7$, and $c = -15$ gives us

$$x = \frac{-7 \pm \sqrt{49 + 120}}{4} = \frac{-7 \pm \sqrt{169}}{4} = \frac{-7 \pm 13}{4}.$$

Hence, $$x = \frac{-7 + 13}{4} = \frac{3}{2} \quad \text{or} \quad x = \frac{-7 - 13}{4} = -5.$$

Consequently, the solutions are $\frac{3}{2}$ and -5.

The given equation can also be solved by factoring. We begin by writing

$$2x^2 + 7x - 15 = (2x - 3)(x + 5) = 0.$$

Since a product can equal zero only if one of the factors is zero, we obtain

$$2x - 3 = 0 \quad \text{or} \quad x + 5 = 0.$$

This leads to the same solutions, $\frac{3}{2}$ and -5. ■

Exercises 1.1

In Exercises 1–4 replace the symbol □ with either <, >, or =.

1 (a) $-7 \square -4$
(b) $3 \square -1$
(c) $1 + 3 \square 6 - 2$

2 (a) $-3 \square -5$
(b) $-6 \square 2$
(c) $\frac{1}{4} \square 0.25$

3 (a) $\frac{1}{3} \square 0.33$
(b) $\frac{125}{57} \square 2.193$
(c) $\frac{22}{7} \square \pi$

4 (a) $\frac{1}{7} \square 0.143$
(b) $\frac{3}{4} + \frac{2}{3} \square \frac{19}{12}$
(c) $\sqrt{2} \square 1.4$

Express the statements in Exercises 5–16 in terms of inequalities.

5 -8 is less than -5.

6 2 is greater than 1.9.

7 0 is greater than -1.

8 $\sqrt{2}$ is less than π.

9 x is negative.

10 y is positive.

11 a is between 5 and 3.

12 b is between $\frac{1}{10}$ and $\frac{1}{3}$.

13 b is greater than or equal to 2.

14 x is less than or equal to -5.

15 c is not greater than 1.

16 d is nonnegative.

Express the intervals in Exercises 17–22 in terms of inequalities.

17 $(-2, 1)$

18 $[2, 6)$

19 $(3, 5]$

20 $[-3, -2]$

21 $[0, 2\pi]$

22 $(-\pi/2, \pi/2)$

Rewrite the numbers in Exercises 23–26 without using symbols for absolute value.

23 (a) $|4 - 9|$
(b) $|-4| - |-9|$
(c) $|4| + |-9|$

24 (a) $|3 - 6|$
(b) $|0.2 - \frac{1}{5}|$
(c) $|-3| - |-4|$

25 (a) $3 - |-3|$
(b) $|\pi - 4|$
(c) $(-3)/|-3|$

26 (a) $|8 - 5|$
(b) $-5 + |-7|$
(c) $(-2)|-2|$

In Exercises 27–30 the given numbers are coordinates of three points A, B, and C (in that order) on a coordinate line l. For each, find (a) $d(A, B)$; (b) $d(B, C)$; (c) $d(C, B)$; and (d) $d(A, C)$.

27 $-6, -2, 4$

28 $3, 7, -5$

29 $8, -4, -1$

30 $-9, 1, 10$

Use the Quadratic Formula to solve the equations in Exercises 31–40.

31 $2x^2 - x - 3 = 0$

32 $u^2 + 2u - 6 = 0$

33 $3x^2 - 2x - 8 = 0$

34 $v^2 + 3v - 5 = 0$

35 $2x^2 - 4x - 5 = 0$

36 $3x^2 - 6x + 2 = 0$

37 $4y^2 - 20y + 25 = 0$

38 $9t^2 + 6t + 1 = 0$

39 $4x^4 - 37x^2 + 9 = 0$

40 $2x^4 - 9x^2 + 4 = 0$

1.2 Coordinate Systems in Two Dimensions

In Section 1.1 we discussed a method of assigning coordinates to points on a line. Coordinate systems can also be introduced in planes by means of *ordered pairs*. The term **ordered pair** refers to two real numbers, where one is designated as the "first" number and the other as the "second." The symbol (a, b) is used to denote the ordered pair consisting of the real numbers a and b, where a is first and b is second. There are many uses for ordered pairs. We used them in Section 1.1 to denote open intervals. In this section they will represent points in a plane. Although ordered pairs are employed in different situations, there is little chance that we will confuse them, since it should always be clear from our discussion whether the symbol (a, b) represents an interval, a point, or some other mathematical object. We consider two ordered pairs (a, b) and (c, d) equal, and write

$$(a, b) = (c, d) \quad \text{if and only if} \quad a = c \quad \text{and} \quad b = d.$$

This implies, in particular, that $(a, b) \neq (b, a)$ if $a \neq b$.

A **rectangular,** or **Cartesian* coordinate system** may be introduced in a plane by considering two perpendicular coordinate lines in the plane that intersect in the origin O on each line. Unless specified otherwise, the same unit of length is chosen on each line. Usually one of the lines is horizontal with positive direction to the right, and the other line is vertical with positive direction upward, as indicated by the arrowheads in Figure 1.7(i). The two lines are called **coordinate axes,** and the point O is called the **origin.** The horizontal line is usually referred to as the ***x*-axis** and the vertical line as the ***y*-axis,** and they are labeled x and y, respectively. The plane is then called a **coordinate plane,** or an ***xy*-plane.** In certain applications different labels such as d or t are used for the coordinate lines. The coordinate axes divide the plane into four parts called the **first, second, third,** and **fourth quadrants** and labeled I, II, III, and IV, respectively. (See Figure 1.7(i).)

Each point P in an xy-plane may be assigned a unique ordered pair (a, b) as shown in Figure 1.7(ii). The number a is called the ***x*-coordinate** (or **abscissa**) of P, and b is called the ***y*-coordinate** (or **ordinate**). We say that *P has coordinates (a, b)*. Conversely, every ordered pair (a, b) determines a point P in the xy-plane with coordinates a and b. We often refer

* The term "Cartesian" is used in honor of the French mathematician and philosopher René Descartes (1596–1650), who was one of the first to employ such coordinate systems.