



Mechanics of engineering materials

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Preface

The S.I. edition of *Mechanics of Solids and Structures* by P. P. Benham and F. V. Warnock was first published in 1973. It appears to have been very well received over the ensuing period. This preface is therefore written both for those who are familiar with the past text and also those who are approaching this subject for the first time. Although the subject matter is still basically the same today as it has been for decades, there are a few developing topics which have been introduced into undergraduate courses such as finite element analysis, fracture mechanics and fibre composite materials. In addition, style of presentation and illustrations in engineering texts have changed for the better and certain limitations of the previous edition, e.g. the number of problems and worked examples, needed to be rectified. Professor Warnock died in 1976 after a period of happy retirement and so it was left to the other author and the publisher to take the initiative to construct a new textbook.

In order to provide fresh thinking and reduce the time of rewriting Dr Roy Crawford, Reader in Mechanical Engineering at the Queen's University of Belfast, was invited, and kindly agreed, to join the project as a co-author. Although the present text might be regarded as a further edition of the original book the new authorship team preferred to make a completely fresh start. This is reflected in the change of title which is widely used as an alternative to *Mechanics of Solids*. The dropping of the reference to structures does not imply any reduction in that topic as will be seen in the contents.

In order that the book should not become any larger with the proposed expansion of material in some chapters, it was decided that the previous three chapters on experimental stress analysis should be omitted as there are several excellent texts in this field. The retention of that part of the book dealing with mechanical properties of materials for design was regarded as important even though there are also specialized texts in this area.

The main part (Ch. 1–18) of this new book of course still deals with the basic subject of Solid Mechanics, or *Mechanics of Materials*, whichever

title one may prefer, being the study of equilibrium and displacement systems in engineering components and structures to enable designs to be effected in terms of stress and strain and the selection of materials. These eighteen chapters cover virtually all that is required in the three-year syllabus of a university or polytechnic degree course in engineering, or the examinations of the engineering Council, C.N.A.A. etc.

Although there is a fairly natural ordering of the material there is some scope for variation and lecturers will have their own particular detailed preferences.

As in the previous text, the first eleven chapters are concerned with forces and displacements in statically-determinate and indeterminate components and structures, and the analysis of uniaxial stress and strain due to various forms of loading such as bending, torsion, pressure and temperature change. The basic concepts of strain energy (Ch. 9) and elastic stability (Ch. 11) are also introduced. In Chapter 12 a study is made of two-dimensional states of stress and strain with special emphasis on principal stresses and the analysis of strain measurements using strain gauges. Chapter 13 combines two chapters of the previous book and brings together the topics of yield prediction and stress concentration which are of such importance in design.

Also included in these two chapters is an elementary introduction to the stress analysis and failure of fibre composite materials. These relatively new advanced structural materials are becoming increasingly used, particularly in the aerospace industry, and it is essential for engineers to receive a basic introduction to them. These thirteen chapters constitute the bulk of the syllabuses covered in first and second-year courses.

Four of the next five chapters appeared in the previous text and deal with more advanced or specialized topics such as thick-walled pressure vessels, rotors, thin plates and shells and post-yield or plastic behaviour, which will probably occupy part of final-year courses.

One essential new addition is an introductory chapter on finite element analysis. It may seem presumptuous even to attempt an introduction to such a broad subject in one chapter, but it is an attempt to provide initial encouragement and confidence to proceed to the complete texts on finite elements.

Chapters 19 to 22 cover much the same ground as in the previous text, but have been brought up to date particularly in relation to fracture mechanics. Since these chapters have such importance in relation to design, a number of worked examples have been introduced, together with problems at the end of each chapter. Bibliographies have still been included for further reading as required.

The first Appendix covers the essential material on properties of areas. The second deals with the simple principles of matrix algebra. A useful table of mechanical properties is provided in the third Appendix.

One of the recommendations of the Finniston Report to higher education was that theory should be backed up by more practical industrial applications. In this context the authors have attempted to incorporate into the worked examples and problems at the end of each

chapter realistic engineering situations apart from the conventional examination-type applications of theory.

There had been a number of enquiries for a solutions manual for the previous text and this can be very helpful to both lecturer and student. Consequently this text is accompanied by another volume which contains worked solutions to nearly 300 problems. The manual should be used alongside the main text, so that steps in each solution can be referred back to the appropriate development in the relevant chapter. It is most important not to approach solutions on the basis of plucking the "appropriate formula" out of the text, inserting the numbers, and manipulating a calculator!

Every effort has been made by the authors to ensure accuracy of text and solutions, but lengthy experience demonstrates human fallibility in this respect. When errors subsequently come to light they will be corrected at the next reprinting and readers' patience and comments will be appreciated!

Some use has been made of data and diagrams from other published literature and, in addition to the individual references, the authors wish to make grateful acknowledgement to all persons and organizations concerned.

P. P. Benham
R. J. Crawford

1987

Notation

α	angle, coefficient of thermal expansion
β	angle
γ	shear strain
δ	deflection, displacement
ϵ	direct strain
η	efficiency, viscosity
θ	angle, angle of twist, co-ordinate
λ	lack of fit
ν	Poisson's ratio
ρ	radius of curvature, density
σ	direct stress
τ	shear stress
ϕ	angle, co-ordinate, stress function
ω	angular velocity
A	area
C	complementary energy
D	diameter
E	Young's modulus of elasticity
F	force
G	shear or rigidity modulus of elasticity, strain energy release rate
H	force
I	second moment of area, product moment of area
J	polar second moment of area
K	bulk modulus of elasticity, fatigue strength factor, stress concentration factor, stress intensity factor
L	length
M	bending moment
N	number of stress cycles, speed of rotation
P	force
Q	shear force
R	force, radius of curvature, stress ratio

S	cyclic stress
T	temperature, torque
U	strain energy
V	volume
W	weight, load
X	body force
Y	body force
Z	body force, section modulus
a	area, distance
b	breadth, distance
c	distance
d	depth, diameter
e	eccentricity, base of Napierian logarithms
g	gravitational constant
h	distance
j	number of joints
k	diameter ratio of cylinder
l	length
m	mass, modular ratio, number of members
n	number
p	pressure
q	shear flow
r	co-ordinate, radius, radius of gyration
s	length
t	thickness, time
u	displacement in the x - or r -direction
v	deflection, displacement in the y - or θ -direction, velocity
w	displacement in the z -direction, load intensity
x	co-ordinate, distance
y	co-ordinate, distance
z	co-ordinate, distance

It should be noted that a number of these symbols have also been used to denote constants in various equations.

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