

Marek Musiela Marek Rutkowski

Martingale Methods
in Financial Modelling

金融建模中的鞅方法



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Marek Musiela Marek Rutkowski

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Stochastic Control

**Applications of
Mathematics**
*Stochastic Modelling
and Applied Probability*

36

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Preface

The origin of this book can be traced to courses on financial mathematics taught by us at the University of New South Wales in Sydney, Technical University of Warsaw (Politechnika Warszawska) and Institut National Polytechnique de Grenoble. Our initial aim was to write a short text around the material used in two one-semester graduate courses attended by students with diverse disciplinary backgrounds (mathematics, physics, computer science, engineering, economics and commerce). The anticipated diversity of potential readers explains the somewhat unusual way in which the book is written. It starts at a very elementary mathematical level and does not assume any prior knowledge of financial markets. Later, it develops into a text which requires some familiarity with concepts of stochastic calculus (the basic relevant notions and results are collected in the appendix). Over time, what was meant to be a short text acquired a life of its own and started to grow. The final version can be used as a textbook for three one-semester courses – one at undergraduate level, the other two as graduate courses.

The first part of the book deals with the more classical concepts and results of arbitrage pricing theory, developed over the last thirty years and currently widely applied in financial markets. The second part, devoted to interest rate modelling is more subjective and thus less standard. A concise survey of short-term interest rate models is presented. However, the special emphasis is put on models built upon market interest rates.

We are grateful to the Australian Research Council for providing partial financial support throughout the development of this book. We thank Alan Brace, Ben Goldys, Dieter Sondermann, Erik Schlögl, Lutz Schlögl, Alexander Mürmann, and Alexander Zilberman, who offered useful comments on the first draft, and Barry Gordon, who helped with editing. Special thanks to Catriona Byrne at Springer for her continual and invaluable assistance.

Our hope is that this book will help to bring the mathematical and financial communities closer together, by introducing mathematicians to some important problems arising in the theory and practice of financial markets, and by providing finance professionals with a set of useful mathematical tools in a comprehensive and self-contained manner.

Sydney, March 1997

Marek Musiela
Marek Rutkowski

Note on the Second Printing

During the 6 months that elapsed between the first and the second printing, we found a number of minor typographical errors in the text and formulae: these have been corrected. We have also adjusted or completed a number of bibliographical references, with the aim of keeping it, as far as possible, up to date.

M.M., M.R.
February 1998

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Part I

Spot and Futures Markets

