Edward ott

Chaos in Dynamical Systems

Second Edition

动力系统中的混沌 第2版

Cambridge

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CAMBRIDGE
UNIVERSITY PRESS

世界图书出版公司

书　　名：Chaos in Dynamical Systems 2nd ed.

作　　者：E. Ott

中 译 名：动力系统中的混沌 第2版

出 版 者：世界图书出版公司北京公司

印 刷 者：北京世图印刷厂

发　　行：世界图书出版公司北京公司 (北京朝内大街137号　100010)

联系电话：010-64015659, 64038347

电子信箱：kjsk@vip.sina.com

开　　本：16　开　　印　张：30.75

出版年代：2005年6月第1版　2006年2月第2次印刷

书　　号：7-5062-7271-7 / O・538

版权登记：图字: 01-2005-2491

定　　价：99.00元

Chaos in Dynamical Systems

In the new edition of this classic textbook Ed Ott has added much new material and has significantly increased the number of homework problems. The most important change is the addition of a completely new chapter on control and synchronization of chaos. Other changes include new material on riddled basins of attraction, phase locking of globally coupled oscillators, fractal aspects of fluid advection by Lagrangian chaotic flows, magnetic dynamos and strange nonchaotic attractors.

Over the past few decades scientists, mathematicians and engineers have come to understand that a large variety of systems exhibit complicated evolution with time. This complicated behavior, known as chaos, occurs so frequently that it has become important for workers in many disciplines to have a good grasp of the fundamentals and basic tools of the science of chaotic dynamics.

Topics in the book include: attractors; basins of attraction; one-dimensional maps; fractals; Hausdorff dimensions; symbolic dynamics; stable and unstable manifolds; Lyapunov exponents; metric and topological entropy; chaotic transients; fractal basin boundaries; chaotic scattering; quasiperiodicity; Hamiltonian systems; KAM tori; period doubling cascades; the intermittency transition to chaos; crises; bifurcations to chaos in scattering problems and in fractal basin boundaries; the characterization of dynamics by unstable periodic orbits; control and synchronization of chaos; and quantum chaos in time-dependent bounded systems, as well as in temporarily kicked and scattering problems. Homework problems are included throughout the book.

This new edition will be of interest to advanced undergraduates and graduate students in science, engineering and mathematics taking courses in chaotic dynamics, as well as to researchers in the subject.

EDWARD OTT is currently on the faculty of the University of Maryland where he holds the title of Distinguished University Professor of Physics and of Electrical and Computer Engineering. Before coming to Maryland in 1979, he was a Professor of Electrical Engineering at Cornell University (1968–1979). Prof. Ott's early research was on plasma physics and charged particle beams, including research on space plasmas, fusion plasmas, intense ion beams and electromagnetic wave generation by electron beams. Since the early 1980s, Prof. Ott's main research interests have been in nonlinear dynamics and its applications to problems in science and engineering. Some of this work includes contributions in the areas of bifurcations of chaotic sets, the fractal dimension of strange attractors, the structure of basin boundaries, applications of chaotic dynamics to problems in fluids and plasmas, and the control and synchronization of chaos. Prof. Ott has also been active in the education of students in nonlinear dynamics. He is an author of over 300 research articles in scientific journals.

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS
The Edinburgh Building, Cambridge CB2 2RU, UK
40 West 20th Street, New York, NY 10011-4211, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
Ruiz de Alarcón 13, 28014 Madrid, Spain
Dock House, The Waterfront, Cape Town 8001, South Africa

http://www.cambridge.org

First published 1993
Reprinted 1994, 1997, 2000
Second edition 2002

Ott, Edward.
Chaos in dynamical systems/Edward Ott.
p. cm.
Includes bibliographical references and index.
ISBN 0 521 81196 1 – ISBN 0 521 01084 5 (pbk.)
1. Chaotic behaviour in systems. I. Title.

Q172.5.C45 O87 2002
003'.857–dc21 2001052863

ISBN 0 521 81196 1 hardback
ISBN 0 521 01084 5 paperback

Preface to the first edition

Although chaotic dynamics had been known to exist for a long time, its importance for a broad variety of applications began to be widely appreciated only within the last decade or so. Concurrently, there has been enormous interest both within the mathematical community and among engineers and scientists. The field continues to develop rapidly in many directions, and its implications continue to grow. Naturally, such a situation calls for textbooks to serve the need of providing courses to students who will eventually utilize concepts of chaotic dynamics in their future careers. A variety of chaos texts now exists. In my teaching of several courses on chaos, however, I found that the existing texts were not altogether suitable for the type of course I was giving, with respect to both level and coverage of topics. Hence I was motivated to prepare and circulate notes for my class, and these notes led to this book. The book is intended for use in a graduate course for scientists and engineers. Accordingly, any mathematical concepts that such readers may not be familiar with (e.g., measure, Cantor sets, etc.) are introduced and informally explained as needed. While the intended readers are not mathematicians, there is a greater emphasis on basic mathematical concepts than in most other books that address the same audience. The style is pedagogical, and it is hoped that the very interesting, sometimes difficult, concepts that are the backbone for studies of chaos are made clear. The coverage is broad, including such topics as multifractals, quantum chaos, embedding, chaotic scattering, etc. Thus the book can also serve as a reference for workers in the field. There is too much in this book for a single one semester course. Hence it is expected that a teacher would select parts in designing a course; for example, one choice might be to base a one semester introductory course on Chapters 1–4, possibly supplemented by a few sections from later chapters. The author has also taught more advanced courses that utilized material now contained in Chapters 7, 9 and 10,* supplemented by readings from current research papers.

I wish to thank my students and colleagues who read and commented on various versions and parts of the manuscript. Special thanks in this regard are owed to George Schmidt, Artur Lopes, Mingzhou Ding and Reinhold Blümel. I also wish to thank Denise Best and, especially, Patsy Keehn for their expert typing of the manuscript. Finally, I thank my wife, Mary, and my children, William and Susan, for their patience and support while this book was being prepared.

January 1992 Edward Ott
College Park

* Chapter 10 of the first edition corresponds to Chapter 11 of this second edition.

Preface to the second edition

This second edition updates and expands the first edition. The most important change is a new chapter on control and synchronization of chaos (Chapter 10). Further additions have been made throughout the book, including new material on riddled basins of attraction, phase locking of globally coupled oscillators, fractal aspects of fluid advection by Lagrangian chaotic flows, magnetic dynamos and strange nonchaotic attractors. Also, twenty-eight new homework problems for students have been added.

February 2002 | Edward Ott
College Park

Contents

Chapter 1
Introduction and overview

1.1 Some history

Chaotic dynamics may be said to have started with the work of the French mathematician Henri Poincaré at about the turn of the century. Poincaré's motivation was partly provided by the problem of the orbits of three celestial bodies experiencing mutual gravational attraction (e.g., a star and two planets). By considering the behavior of orbits arising from *sets* of initial points (rather than focusing on *individual* orbits), Poincaré was able to show that very complicated (now called chaotic) orbits were possible. Subsequent noteworthy early mathematical work on chaotic dynamics includes that of G. Birkhoff in the 1920s, M. L. Cartwright and J. E. Littlewood in the 1940s, S. Smale in the 1960s, and Soviet mathematicians, notably A. N. Kolmogorov and his coworkers. In spite of this work, however, the possibility of chaos in real physical systems was not widely appreciated until relatively recently. The reasons for this were first that the mathematical papers are difficult to read for workers in other fields, and second that the theorems proven were often not strong enough to convince researchers in these other fields that this type of behavior would be important in their systems. The situation has now changed drastically, and much of the credit for this can be ascribed to the extensive numerical solution of dynamical systems on digital computers. Using such solutions, the chaotic character of the time evolutions in situations of practical importance has become dramatically clear. Furthermore, the complexity of the dynamics cannot be blamed on unknown extraneous experimental effects, as might be the case when dealing with an actual physical system.

In this chapter, we shall provide some of the phenomenology of chaos and will introduce some of the more basic concepts. The aim is to provide a motivating overview[1] in preparation for the more detailed treatments to be pursued in the rest of this book.

1.2 Examples of chaotic behavior

Most students of science or engineering have seen examples of dynamical behavior which can be fully analyzed mathematically and in which the system eventually (after some transient period) settles either into periodic motion (a limit cycle) or into a steady state (i.e., a situation in which the system ceases its motion). When one relies on being able to specify an orbit analytically, these two cases will typically (and falsely) appear to be the only important motions. The point is that chaotic orbits are also very common but cannot be represented using standard analytical functions. Chaotic motions are neither steady nor periodic. Indeed, they appear to be very complex, and, when viewing such motions, adjectives like wild, turbulent, and random come to mind. In spite of the complexity of these motions, they commonly occur in systems which themselves are not complex and are even surprisingly simple. (In addition to steady state, periodic and chaotic motion, there is a fourth common type of motion, namely quasiperiodic motion. We defer our discussion of quasiperiodicity to Chapter 6.)

Before giving a definition of chaos we first present some examples and background material. As a first example of chaotic motion, we consider an experiment of Moon and Holmes (1979). The apparatus is shown in Figure 1.1. When the apparatus is at rest, the steel beam has two stable steady-state equilibria: either the tip of the beam is deflected toward the left magnet or toward the right magnet. In the experiment, the horizontal position of the apparatus was oscillated sinusoidally with time. Under certain conditions, when this was done, the tip of the steel beam was observed to oscillate in a very irregular manner. As an indication of this very irregular behavior, Figure 1.2(a) shows the output signal of a strain gauge attached to the beam (Figure 1.1). Although the apparatus appears to be very simple, one might attribute the observed complicated motion to complexities in the physical situation, such as the excitation of higher order vibrational modes in the beam, possible noise in the sinusoidal shaking device, etc. To show that it is not necessary to invoke such effects, Moon and Holmes considered a simple model for their experiment, namely, the forced Duffing equation in the following form,

$$\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} + \nu\frac{\mathrm{d}y}{\mathrm{d}t} + (y^3 - y) = g \sin t. \tag{1.1}$$

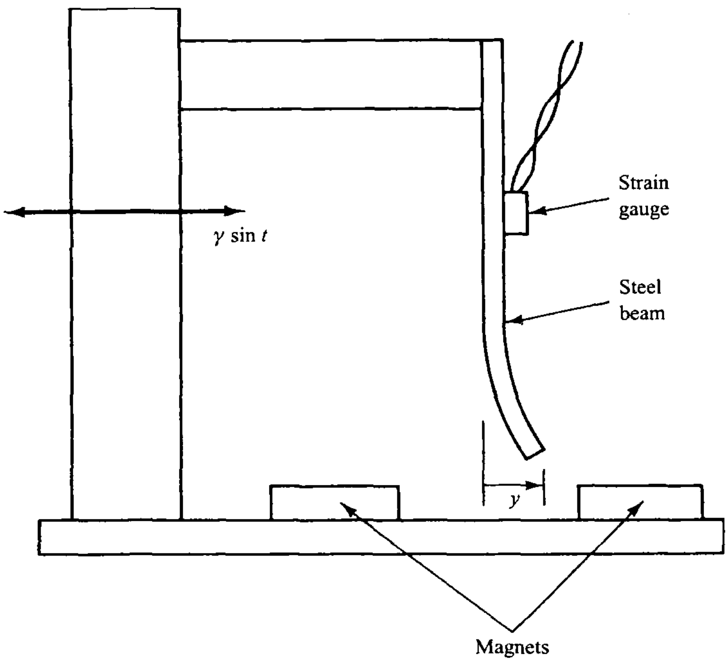

Figure 1.1 The apparatus of Moon and Holmes (1979).

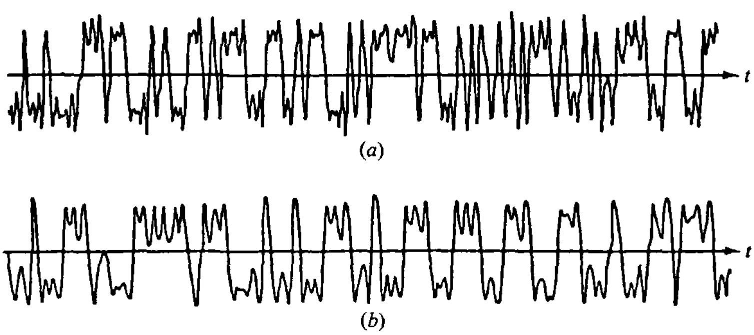

Figure 1.2 (a) Signal from the strain gauge. (b) Numerical solution of Eq. (1.1) (Moon and Holmes, 1979).

In Eq. (1.1), the first two terms represent the inertia of the beam and dissipative effects, while the third term represents the effects of the magnets and the elastic force. The sinusoidal term on the right-hand side represents the shaking of the apparatus. In the absence of shaking ($g = 0$), Eq. (1.1) possesses two stable steady states, $y = 1$ and $y = -1$, corresponding to the two previously mentioned stable steady states of the beam. (There is also an unstable steady state $y = 0$.) Figure 1.2(b) shows the results of a digital computed numerical solution of Eq. (1.1) for a particular choice of ν and g. We observe that the results of the physical experiment are qualitatively similar to those of the numerical solution.

Thus, it is unnecessary to invoke complicated physical processes to explain the observed complicated motion.

As a second example, we consider the experiment of Shaw (1984) illustrated schematically in Figure 1.3. In this experiment, a slow steady inflow of water to a 'faucet' was maintained. Water drops fall from the faucet, and the times at which successive drops pass a sensing device are recorded. Thus, the data consists of the discrete set of times $t_1, t_2, \ldots, t_n, \ldots$ at which drops were observed by the sensor. From these data, the time intervals between successive drops can be formed, $\Delta t_n \equiv t_{n+1} - t_n$. When the inflow rate to the faucet is sufficiently small, the time intervals Δt_n are all equal. As the inflow rate is increased, the time interval sequence becomes periodic with a short interval Δt_a followed by a longer interval Δt_b, so that the sequence of time intervals is of the form $\ldots, \Delta t_a, \Delta t_b, \Delta t_a, \Delta t_b, \Delta t_a, \ldots$. We call this a period two sequence since $\Delta t_n = \Delta t_{n+2}$. As the inflow rate is further increased, periodic sequences of longer and longer periods were observed, until, at sufficiently large inflow rate, the sequence $\Delta t_1, \Delta t_2, \Delta t_3, \ldots$ apparently has no regularity. This irregular sequence is argued to be due to chaotic dynamics.

As a third example, we consider the problem of chaotic Rayleigh–Benard convection, originally studied theoretically and computationally in the seminal paper of Lorenz (1963) and experimentally by, for example, Ahlers and Behringer (1978), Gollub and Benson (1980), Bergé *et al.* (1980) and Libchaber and Maurer (1980). In Rayleigh–Benard convection, one considers a fluid contained between two rigid plates and subjected to gravity, as shown in Figure 1.4. The bottom plate is maintained at a higher temperature $T_0 + \Delta T$ than the temperature T_0 of the top plate. As a result, the fluid near the warmer lower plate expands, and buoyancy creates a tendency for this fluid to rise. Similarly, the cooler

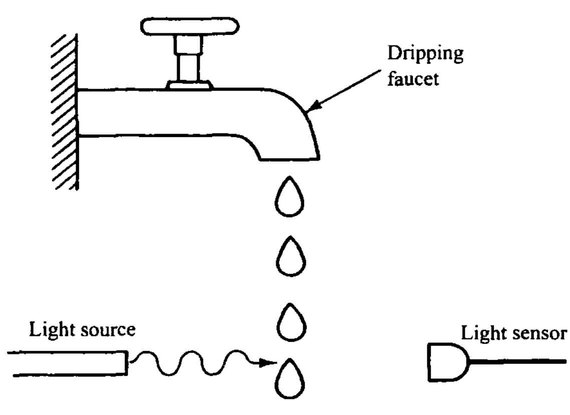

Figure 1.3 Schematic illustration of the experiment of Shaw (1984).

more dense fluid near the top plate has a tendency to fall. While Lorenz's equations are too idealized a model to describe the experiments accurately, in the case where the experiments were done with vertical bounding side-walls situated at a spacing of two to three times the distance between the horizontal walls, there was a degree of qualitative correspondence between the model and the experiments. In particular, in this case, for some range of values of the temperature difference ΔT, the experiments show that the fluid will execute a *steady* convective cellular flow, as shown in the figure. At a somewhat larger value of the temperature difference, the flow becomes time-dependent, and this time dependence is chaotic. This general behavior is also predicted by Lorenz's paper.

From these simple examples, it is clear that chaos should be expected to be a very common basic dynamical state in a wide variety of systems. Indeed, chaotic dynamics has by now been shown to be of potential importance in many different fields including fluids,[2] plasmas,[3] solid state devices,[4] circuits,[5] lasers,[6] mechanical devices,[7] biology,[8] chemistry,[9] acoustics,[10] celestial mechanics,[11] etc.

In both the dripping faucet example and the Rayleigh–Benard convection example, our discussions indicated a situation as shown schematically in Figure 1.5. Namely, there was a system parameter, labeled p in Figure 1.5, such that, at a value $p = p_1$, the motion is observed to be nonchaotic, and at another value $p = p_2$, the motion is chaotic. (For the faucet example, p is the inflow rate, while for the example of Rayleigh–Benard convection, p is the temperature difference ΔT.) The natural question raised by Figure 1.5 is *how does chaos come about as the parameter p is varied continuously from* p_1 *to* p_2? That is, how do the dynamical motions of the system evolve with continuous variation of p from p_1 and p_2? This question of the *routes to chaos*[12] will be considered in detail in Chapter 8.

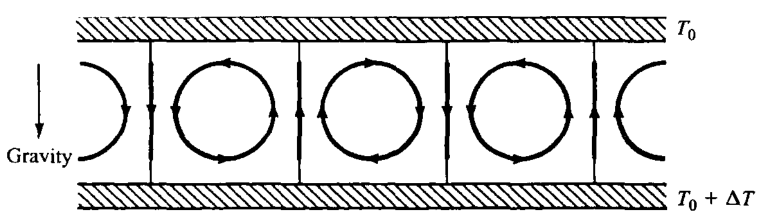

Figure 1.4 Rayleigh–Benard convection.

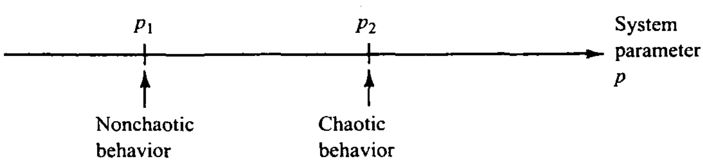

Figure 1.5 Schematic illustration of the question of the transition to chaos with variation of a system parameter.