世界优秀教材
中国版

理科类系列教材

PEARSON
Addison Wesley

双语版

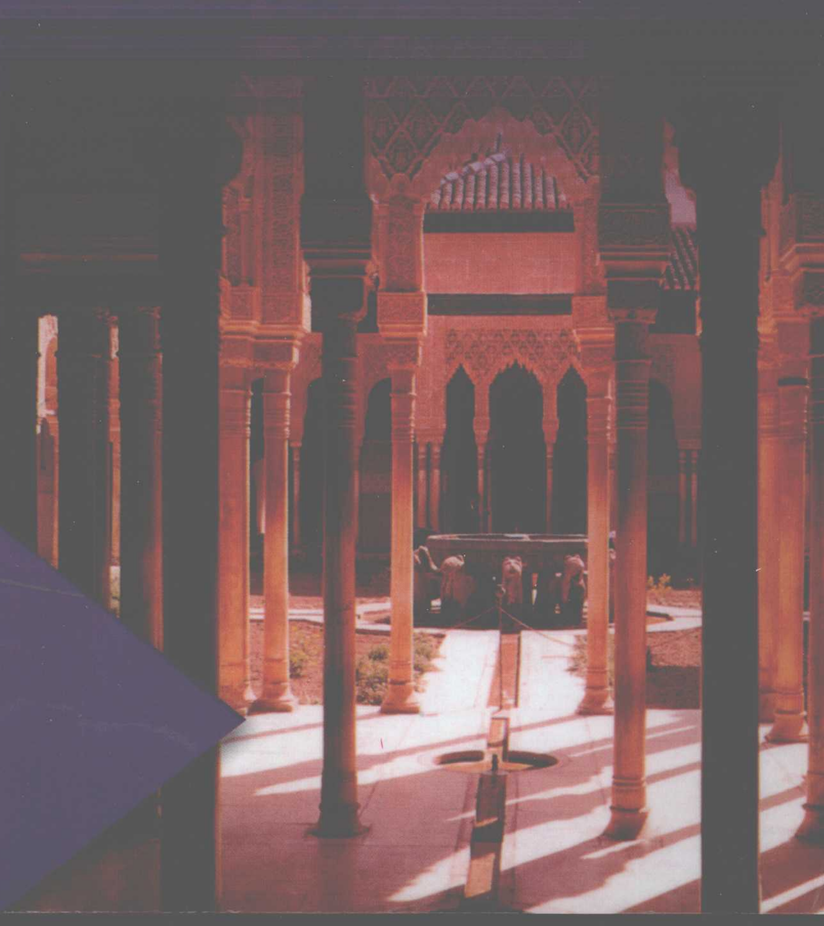
A History of Mathematics

An Introduction (Second Edition)

数学史通论（第2版）

□ VICTOR J. KATZ 著

□ 李文林 王丽霞 译



高等教育出版社
Higher Education Press

世界优秀教材 中国版

理科类系列教材

双语版

A History of Mathematics

An Introduction (Second Edition)

数学史通论（第2版）

VICTOR J. KATZ 著

李文林 王丽霞 译

高等教育出版社
Higher Education Press

图字：01-2007-6090 号

图书在版编目（CIP）数据

数学史通论：第 2 版：双语版：汉英对照/（美）卡兹（Katz, V. J.）著；李文林，王丽霞译．—2 版．—北京：高等教育出版社，2008.6
书名原文：A History of Mathematics: An Introduction
ISBN 978-7-04-023870-9

Ⅰ．数…　Ⅱ．①卡…②李…③王…　Ⅲ．数学史－高等学校－教材－汉、英　Ⅳ．011

中国版本图书馆 CIP 数据核字（2008）第 068968 号

出版发行	高等教育出版社	购书热线	010-58581118
社　　址	北京市西城区德外大街 4 号	免费咨询	800-810-0598
邮政编码	100120	网　　址	http://www.hep.edu.cn
总　　机	010-58581000		http://www.hep.com.cn
		网上订购	http://www.landraco.com
经　　销	蓝色畅想图书发行有限公司		http://www.landraco.com.cn
印　　刷	北京新华印刷厂	畅想教育	http://www.widedu.com
		版　　次	1980 年 10 月第 1 版
开　　本	850×1168　1/16		2008 年 6 月第 2 版
印　　张	56.5	印　　次	2008 年 6 月第 1 次印刷
字　　数	1 400 000	定　　价	64.00 元

本书如有缺页、倒页、脱页等质量问题，请到所购图书销售部门联系调换。

物料号　23870-00

TIME LINE FOR THE HISTORY OF MATHEMATICS

3000 B.C.E. — 1000 B.C.E. — 0 — 800

3000 B.C.E. – 1000 B.C.E.

3000–2000 B.C.E.

Egypt: Beginnings of hieroglyphic writing of numbers; Building of pyramids at Giza

Iraq: Beginnings of cuneiform writing of numbers in Mesopotamia

2000–1000 B.C.E.

Egypt: Rhind and Moscow papyri written; Ideas of linear equations, volumes, areas

Iraq: Cuneiform mathematical tablets written with Pythagorean theorem, quadratic equations, systems of equations

1000 B.C.E. – 0

1000–500 B.C.E.

India: Square root calculations, Pythagorean theorem

China: Rod numerals, Pythagorean theorem

Greece: Beginnings of theoretical geometry

500–300 B.C.E.

Greece: Plato, Aristotle, and axiomatic mathematics; Discovery of incommensurability; Eudoxus and proportionality

Egypt: Euclid and the *Elements*

300--0 B.C.E.

China: Square and cube roots, systems of linear equations

Italy: Archimedes and theoretical physics

Egypt: Apollonius and conic sections

Turkey: Hipparchus and trigonometry

0 – 800

0–200 C.E.

Egypt: Heron and practical mathematics; Ptolemy and astronomy

Jordan: Nicomachus and number theory

Israel: Nehemiah and practical geometry

200–400

Egypt: Diophantus and number theory; Hypatia and commentaries

China: Liu Hui and mathematical surveying techniques

400–800

Italy: Boethius and elementary mathematics

Mexico: Development of Mayan numeration and astronomy

India: Aryabhata and trigonometry; Brahmagupta and indeterminate analysis; Development of Hindu-Arabic decimal place-value number system

China: First tangent tables

800 –

800–1000

India: Development of algebraic techniques

Iraq: Al-Khwārizmī and first algebra text

Egypt: Abū Kāmil and advanced algebraic techniques

United States: Geometry used in architectural designs in Mississippian civilization

Spain: Gerbert learns Arabic number system

3000 B.C.E. — 1000 B.C.E. — 0 — 800

1000 | 1200 | 1600 | 1800

1000–1200

Iraq: Al-Karajī, induction, and Pascal triangle

Egypt: Ibn al-Haytham, sums of powers, and volumes of paraboloids

Iran: Omar Khayyam and the geometric solution of cubic equations

India: Al-Birūni and spherical trigonometry; Bhaskara and the Pell equation

China: Pascal triangle used to solve equations

Spain: Arabic works translated in Latin; Abraham ibn Ezra and combinatorics

Italy: Leonardo of Pisa and introduction of Islamic mathematics

United States: Astronomical alignments in Anasazi buildings in the Southwest

Zimbabwe: Construction of Great Zimbabwe structures

1200–1400

Iran: Nasīr al-Dīn al-Tūsī and trigonometry

France: Jordanus and advanced algebra; Levi ben Gerson and induction; Oresme and kinematics

England: Velocity, acceleration, and the mean speed theorem

China: Chinese remainder theorem; Solution of polynomial equations

Peru: Quipus used for record keeping

1400–1600

India: Discovery of power series for sine, cosine, and arctangent

Italy: Algebraic solution of the cubic equation

Germany: Perspective and geometry

England: New algebra and trigonometry texts

Poland: Copernicus and the heliocentric system

France: Viète and algebraic symbolism

1600–1700

Kepler, Newton, and celestial physics

Descartes, Fermat, and analytic geometry

Napier, Briggs, and logarithms

Girard, Descartes, and the theory of equations

Pascal, Fermat, and elementary probability

Pascal, Desargues, and projective geometry

Newton, Leibniz, and the invention of calculus

1700–1800

Development of techniques for solving ordinary and partial differential equations

Development of the calculus of functions of several variables

Attempts to give logically correct foundations to the calculus

Lagrange and the analysis of the solution of polynomial equations

1800–1900

Algebraic number theory

Galois theory

Groups and fields

Quaternions and the discovery of noncommutative algebra

Theory of matrices

The arithmetization of analysis

Development of complex analysis

Vector analysis

Differential geometry

Non-Euclidean geometry

Projective geometry

Foundations of geometry

1900–2000

Set theory

Growth of topology

Algebraization of mathematics

Influence of computers

1000 | 1200 | 1600 | 1800

出版说明

本书为高等教育出版社“世界优秀教材中国版”系列教材之一.

为了更好地优化、整合世界优秀教育资源,并通过本土化使其最大程度地发挥作用,丰富我国的教育资源,促进我国的教学改革,提高我国高等教育的教学质量,高等教育出版社决定出版“世界优秀教材中国版”系列教材.

“世界优秀教材中国版”系列教材具有以下特征:

1. 从全球各知名教育出版社精选最好的内容资源进行本土化改造,形成新的系列教材;

2. 由国内一流学者根据我国高等学校的专业设置、课程体系及教学要求,对所选资源进行英文改编或中文改编,使之更具教学适用性;

3. 围绕纸质版主教材,形成包括多媒体及网络资源与服务的整体教学资源集成方案,力争为广大师生提供最优的教学资源与信息服务.

希望该系列教材的出版能为我国高等学校教学改革和教育资源建设作出贡献.

高等教育出版社

2005 年 5 月

译 者 序

本书是美国数学史家维克多·卡兹(Victor J. Katz)所著 ***A History of Mathematics, An Introduction***(Second Edition)的双语本.

在上世纪80年代以来国外出版的诸多数学通史中,卡兹这部著作是较有影响、颇获好评的一本.与已有的数学通史著作相比,该书具有若干鲜明的特点,卡兹教授本人在第二版序言中对此有所总结和介绍,而笔者在此特别希望强调的则是以下几点:

(一)该书充分地反映和利用了1980年代以来数学史研究领域的最新成果.

(二)该书十分重视数学发展的多元文化根源,按作者自己的说法是,"花了特别的工夫来讨论数学在世界上除欧洲以外一些地区的发展."

(三)该书在适应数学教学需要方面从内容到形式都有较周详的设计和安排,以面向"未来的中小学教师"和"未来的大学数学教师".

最后这一点使该书对于我国当前的数学教学改革具有特别的参考意义,同时也使该书的读者范围大大扩展.为适应国内数学教学与数学史研究的需要,高等教育出版社曾于2004年出版了该书的中译本《数学史通论》(李文林、邹建成、胥鸣伟等译).此次出版的双语本,呈现了英文原著的全部内容,同时摘译每一章节中的关键段落和语句并对照编排于相应的英文旁.中文摘译部分可以作为英文原著的导读,它们整合起来也构成一篇简明的数学发展史纲要;而那些希望品尝原汁原味的读者则可直接阅读英文原文.由于数学史涉及人类文化的广泛方面,我们相信这样的双语本数学史著作将是适宜的双语教学读本,并能在全面素质教育中发挥积极作用.

笔者感谢高等教育出版社为本书的出版所作的努力.由于时间仓促,本书在摘译、编排等方面难免存在缺点和疏漏,欢迎读者批评指正.

李文林

2008年1月于北京中关村

To Phyllis,
for long talks,
long walks,
and afternoon naps

Preface

APPROACH AND GUIDING PHILOSOPHY

In *A Call For Change: Recommendations for the Mathematical Preparation of Teachers of Mathematics*, the Mathematical Association of America's (MAA) Committee on the Mathematical Education of Teachers recommends that all prospective teachers of mathematics in schools

> develop an appreciation of the contributions made by various cultures to the growth and development of mathematical ideas; investigate the contributions made by individuals, both female and male, and from a variety of cultures, in the development of ancient, modern, and current mathematical topics; [and] gain an understanding of the historical development of major school mathematics concepts.

According to the MAA, knowledge of the history of mathematics shows students that mathematics is an important human endeavor. Mathematics was not discovered in the polished form of our textbooks, but often developed in intuitive and experimental fashion out of a need to solve problems. The actual development of mathematical ideas can be effectively used in exciting and motivating students today.

This new textbook in the history of mathematics grew out of the conviction that not only prospective school teachers of mathematics but also prospective college teachers of mathematics need a background in history to teach the subject more effectively to their students. It is therefore designed for junior or senior mathematics majors who intend to teach in college or high school and thus concentrates on the history of those topics typically covered in an undergraduate curriculum or in elementary or high school. Because the history of any given mathematical topic often provides excellent ideas for teaching the topic, there is sufficient detail in each explanation of a new concept for the future (or present) teacher of mathematics to develop a classroom lesson or series of lessons based on history. In fact, many of the problems ask the reader to develop a particular lesson. My hope is that the student and prospective teacher will gain from this book a knowledge of how we got here from there, a knowledge that will provide a deeper understanding of many of the important concepts of mathematics.

DISTINGUISHING FEATURES

Flexible Organization

Although the chief organization of the book is by chronological period, within each period the material is organized topically. By consulting the detailed subsection headings, the reader can choose to follow a particular theme throughout history. For example, to study equation solving one could consider ancient Egyptian and Babylonian methods, the geometrical solution methods of the Greeks, the numerical methods of the Chinese, the Islamic solution methods for cubic equations by use of conic sections, the Italian discovery of an algorithmic solution of cubic and quartic equations, the work of Lagrange in developing criteria for methods of solution of higher degree polynomial equations, the work of Gauss in solving cyclotomic equations, and the work of Galois in using permutations to formulate what is today called Galois theory.

Focus on Textbooks

There is an emphasis throughout the book on the important textbooks of various periods. It is one thing to do mathematical research and discover new theorems and techniques. It is quite another to elucidate these in a way that others can learn them. In nearly every chapter, therefore, there is a discussion of one or more important texts of the time. These will be the works from which students learned the important ideas of the great mathematicians. Today's students will see how certain topics were treated and will be able to compare these treatments to those in current texts and see the kinds of problems students of years ago were expected to solve.

Astronomy and Mathematics

Two chapters are devoted entirely to mathematical methods, that is, to the ways in which mathematics was used to solve problems in other areas of endeavor. A substantial part of both of these chapters, one for the Greek period and one for the Renaissance, deals with astronomy. In fact, in ancient times astronomers and mathematicians were usually the same people. It is crucial to the understanding of a substantial part of Greek mathematics to understand the Greek model of the heavens and how mathematics was used in applying this model to give predictions. Similarly, we will discuss the Copernicus-Kepler model of the heavens and see how mathematicians of the Renaissance applied mathematics to its study.

Non-Western Mathematics

A special effort has been made to consider mathematics developed in parts of the world other than Europe. Thus, there is substantial material on mathematics in China, India, and the Islamic world. There is also an "interchapter" in which a comparison is made of the mathematics in the major civilizations at about the turn of the fourteenth century. That comparison is followed by a discussion of the mathematics of various other societies around the world. The reader will see how certain mathematical ideas have occurred in many places, although not perhaps in the context of what we in the West call "mathematics."

Topical Exercises

Each chapter contains many exercises, collected by topic for easy access. Some of the exercises are simple computational ones while others help to fill the gaps in the mathematical arguments presented in the text. **For Discussion** exercises are open-ended questions for discussion, which may involve some research to find answers. Many of these ask students to think about how they would use historical material in the classroom. (Answers to most of the computational exercises are provided in the answer section.) Even if readers do not attempt many of the exercises, they should at least read them to gain a fuller understanding of the material of the chapter.

Focus Essays

Biographies For easy reference, many biographies of the mathematicians whose work is discussed are in separate boxes. In particular, although women have for various reasons not participated in large numbers in mathematical research, biographies of several important women mathematicians are included, women who succeeded, usually against heavy odds, in contributing to the mathematical enterprise.

Special Topics There are also boxes on special topics scattered throughout the book. These include such items as a treatment of the question of the Egyptian influence on Greek mathematics, a discussion of the idea of a function in the work of Ptolemy, and a comparison of various notions of continuity. There are also boxes containing important definitions collected together for easy reference.

Additional Pedagogy Each chapter begins with a relevant quotation and a description of an important mathematical "event." At the end of each chapter, a brief chronology of the mathematicians discussed will help students organize their knowledge. Each chapter also contains an annotated list of references to both primary and secondary sources from which the students can obtain more information. Finally, there is a **time line** of the history of mathematics in the inside front cover and a **map** in the inside back cover indicating the location of some of the important places mentioned in the text. Finally, given that students may have difficulty pronouncing the names of some mathematicians, the **index** has a special feature—a **phonetic pronunciation guide**.

PREREQUISITES

A working knowledge of one year of calculus is sufficient to understand the first twelve chapters of the text. The mathematical prerequisites for the later chapters are somewhat more demanding, but the titles of the various sections indicate clearly what kind of mathematical knowledge is required. For example, a full understanding of Chapters 14 and 15 will require that the student has studied abstract algebra.

NEW FOR THIS EDITION

The generally friendly reception to the first edition of this book encouraged me to maintain the basic organization and content. Nevertheless, I have attempted to make a number of improvements, both in clarity and in content, based on the comments of many users of the first edition as well as new discoveries in the history of mathematics which have appeared in the recent literature. There are minor changes in virtually every section, but the major changes include new material on combinatorics in the Islamic tradition, Newton's derivation of his system of the world, linear algebra in the nineteenth and twentieth centuries, and statistical ideas in the nineteenth century. I have attempted to correct all errors of fact without introducing new ones, but would appreciate notes from anyone who discovers any remaining errors. There are new problems in every chapter, some of them easier ones, and the references to the literature have been updated wherever possible. There are also a few new stamps as illustrations. One should note, however, that any portraits on these stamps—or indeed elsewhere—purporting to represent mathematicians before the sixteenth century are fictitious. There are no known representations of any of these people that have credible evidence of being authentic.

COURSE FLEXIBILITY

There is far more material in this text than can be included in a typical one-semester course in the history of mathematics. In fact, there is adequate material for a full year course, the first half being devoted to the period through the invention of calculus in the late seventeenth century and the second half covering the mathematics of the eighteenth, nineteenth, and twentieth centuries. For those instructors who have only one semester, however, there are several ways to use this book. First, one could cover most of the first twelve chapters and simply conclude with calculus. Second, one could choose to follow one or two particular themes through history. Some possible themes with the appropriate section numbers are

Equation Solving: 1.4, 1.8, 1.9, 2.4.3, 2.5, 5.2, 6.3, 6.4, 6.7, 6.8, 7.2, 8.3, 9.3, 9.4, 11.2, 14.2.4, 15.2

Ideas of Calculus: 2.3.2, 2.3.3, 2.4.9, 3.2, 3.3, 7.2.4, 7.4.4, 8.4, 10.5, 12, 13, 16.1, 16.2, 16.3, 16.4

Concepts of Geometry: 1.5, 1.8, 2.1.2, 2.2, 2.4, 3.3, 3.4, 3.5, 4.3, 5.3, 7.4, 8.1, 10.1, 11.1, 11.5, 14.3, 17, 18.2

Trigonometry, Astronomy, and Surveying: 1.6, 4.1, 4.2, 6.2, 6.6, 7.5, 8.1, 10.2, 10.3, 12.5.6, 13.1.3

Combinatorics, Probability, and Statistics: 6.8, 7.3, 8.2, 11.3, 14.1, 16.5

Linear Algebra: 1.4, 14.2.2, 14.2.4, 15.5, 17.4, 18.3.3, 18.4.7

Number Theory: 2.1.1, 2.4.7, 5.1, 11.4, 14.2.3, 15.1

Modern Algebra: 6.8, 7.2, 8.3, 9.1, 9.2, 14.2, 15.2, 15.3, 15.4, 18.3, 18.4.4, 18.4.6, 18.4.8

Third, one could cover in detail most of the first ten chapters and then pick selected ideas from the later chapters, again following a particular theme. One could also assign various sections for individual or small-group reading assignments and reports.

ACKNOWLEDGMENTS

Like any book, this one could not have been written without the help of many people. The following people contributed to the first edition and their input continues to impact the text: Marcia Ascher (Ithaca College), J. Lennart Berggren (Simon Fraser University), Robert Kreiser (A.A.U.P.), Robert Rosenfeld (Nassau Community College), and John Milcetich (University of the District of Columbia).

Many people made detailed suggestions for the second edition. Although I have not followed every one of them (and may come to regret that), I sincerely appreciate the thought they gave to improving the book. These people include Ivor Grattan-Guinness, Kim Plofker, Eleanor Robson, Richard Askey, William Anglin, Claudia Zaslavsky, Rebekka Struik, William Ramaley, Joseph Albree, Calvin Jongsma, David Fowler, John Stillwell, Christian Thybo, Jim Tattersall, Judith Grabiner, Tony Gardiner, Ubi D'Ambrosio, Dirk Struik, and David Rowe. My heartfelt thanks to all of them.

The many reviewers of sections of the manuscript have also provided great help with their detailed critiques and have made this a much better book than it otherwise could have been.

First Edition Reviewers: Duane Blumberg, University of Southwestern Louisiana; Walter Czarnec, Framingham State University; Joseph Dauben, Herbert Lehman College–CUNY; Harvey Davis, Michigan State University; Joy Easton, West Virginia University; Carl FitzGerald, University of California–San Diego; Basil Gordon, University of California–Los Angeles; Mary Gray, American University; Branko Grunbaum, University of Washington; William Hintzman, San Diego State University; Barnabas Hughes, California State University–Northridge; Israel Kleiner, York University; David E. Kullman, Miami University; Robert L. Hall, University of Wisconsin, Milwaukee; Richard Marshall, Eastern Michigan University; Jerold Mathews, Iowa State University; Willard Parker, Kansas State University; Clinton M. Petty, University of Missouri–Columbia; Howard Prouse, Mankato State University; Helmut Rohrl, University of California–San Diego; David Wilson, University of Florida; and Frederick Wright, University of North Carolina–Chapel Hill.

Second Edition Reviewers: Salvatore Anastasio, State University of New York, New Platz; Bruce Crauder, Oklahoma State University; Walter Czarnec, Framingham State College; William England, Mississippi State University; David Jabon, Eastern Washington University; Charles Jones, Ball State University; Michael Lacey, Indiana University; Harold Martin, Northern Michigan University; James Murdock, Iowa State University; Ken Shaw, Florida State University; Sverre Smalo, University of California, Santa Barbara; Domina Eberle Spencer, University of Connecticut; Jimmy Woods, North Georgia College.

I have also benefited greatly from conversations with many historians of mathematics at various forums. In particular, those who have regularly attended the annual History of Mathematics seminars, organized by Uta Merzbach, former curator of mathematics at the National Museum of American History, may well recognize some of the ideas discussed there. The book has also profited from discussions over the years with, among others, Charles Jones (Ball State University), V. Frederick Rickey (Bowling Green State University), Florence Fasanelli (MAA), Israel Kleiner (York University), Abe Shenitzer (York University), Ubiratan D'Ambrosio (Univ. Estadual de Campinas), and Frank Swetz (Pennsylvania State University). My students in History of Mathematics (and other) classes at the University of the District of Columbia have also helped me clarify many of my ideas. Naturally, I welcome any additional comments and correspondence from students and colleagues elsewhere in an effort to continue to improve this book.

Special thanks are due to the librarians at the University of the District of Columbia and especially to Clement Goddard, who never failed to secure any of the obscure books I requested on interlibrary loan. Leslie Overstreet of the Smithsonian Institution Libraries' Special Collections Department was extremely helpful in finding sources for pictures.

Thanks are due to my former editors at HarperCollins, Steve Quigley, Don Gecewicz, and George Duda, who helped form the first edition.

I also want to thank Jennifer Albanese, my new editor at Addison Wesley Longman, for her suggestions and her patience as she pushed this book to completion, as well as Rebecca Malone and Barbara Pendergast, for their efforts in handling the production aspects, and Susan Holbert for preparation of the index.

My family has been very supportive during the many years of writing the book. I thank my parents for their patience and their faith in me. I thank my children Sharon, Ari, and Naomi for help at various times and especially for allowing me to use our computer. And last, I thank my wife Phyllis for long discussions at any hour of the day or night and for being there when I needed her. I owe her much more than I can ever repay.

VICTOR J. KATZ

目　录

第二篇 中世纪的数学:500－1400

第三篇 早期近代数学:1400 - 1700

第四篇 近代数学:1700 - 2000