STATISTICAL ANALYSIS OF RELIABILITY DATA

M.J. Crowder,
A.C. Kimber,
R.L. Smith
and
T.J. Sweeting

CRC Press
Taylor & Francis Group
A CHAPMAN & HALL BOOK

Statistical Analysis of Reliability Data

M.J. Crowder, A.C. Kimber, R.L. Smith and T.J. Sweeting

Reliability is the study of the failure of systems. These systems can involve mechanical or electrical machinery, computer software, weapons or materials. Reliability theory is concerned with modelling the failure mechanisms of systems.

Statistics enters reliability theory when failure is an inherently unpredictable phenomenon, in which case, statements about reliability can only be made in terms of probabilities. Reliability is not a physically measurable quantity like electrical resistance or elastic modulus and can only be assessed via a statistical analysis of data collected from past experience or experimentation.

This book describes statistical techniques used for the assessment of reliability. Aimed at readers who have had a first course in statistical methods, it develops the specific techniques used in reliability data analysis from a modern computer-oriented viewpoint. Techniques covered include probability plotting, maximum likelihood and Bayesian methods, proportional hazards modelling and the analysis of repairable systems. Some of these techniques are already familiar to those working with survival data in a medical context, but this book describes the differences as well as the similarities when they are applied in reliability. Several data sets are included in full. The final chapter includes the subject of load-sharing systems, highlighting a class of models that has not previously been covered in book form.

The book will be useful to industrial statisticians, students and teachers of applied statistics, engineers and computer scientists wishing to extend their knowledge of statistics as it is applied to their disciplines.

M.J. Crowder is Senior Lecturer, **A.C. Kimber** is Lecturer, and **T.J. Sweeting** is Reader in Statistics all at the University of Surrey, UK. **R.L. Smith** is Professor of Statistics at the University of North Carolina, USA.

ISBN 978-1-138-46915-0

9 781138 469150

CRC Press
Taylor & Francis Group
an informa business
www.crcpress.com

STATISTICAL ANALYSIS OF RELIABILITY DATA
Crowder, Kimber, Smith and Sweeting

CRC Press

STATISTICAL ANALYSIS OF RELIABILITY DATA

M.J. Crowder
Senior Lecturer
University of Surrey,
UK

A.C. Kimber
Lecturer
University of Surrey,
UK

R.L. Smith
Professor of Statistics
University of North Carolina,
USA

and

T.J. Sweeting
Reader in Statistics
University of Surrey,
UK

CRC Press
Taylor & Francis Group
Boca Raton London New York

CRC Press is an imprint of the
Taylor & Francis Group, an **informa** business
A CHAPMAN & HALL BOOK

CRC Press
Taylor & Francis Group
6000 Broken Sound Parkway NW, Suite 300
Boca Raton, FL 33487-2742

First issued in hardback 2017

ISBN 13: 978-0-4125-9480-9 (pbk)
ISBN 13: 978-1-1384-6915-0 (hbk)

Visit the Taylor & Francis Web site at
http://www.taylorandfrancis.com

and the CRC Press Web site at
http://www.crcpress.com

Library of Congress Cataloging-in-Publication Data

Catalog record is available from the Library of Congress

STATISTICAL ANALYSIS OF RELIABILITY DATA

CHAPMAN & HALL/CRC
Texts in Statistical Science Series

Analysis of Failure and Survival Data
Peter J. Smith

The Analysis and Interpretation of Multivariate Data for Social Scientists
David J. Bartholomew, Fiona Steele, Irini Moustaki, and Jane Galbraith

The Analysis of Time Series — An Introduction, Sixth Edition
Chris Chatfield

Applied Bayesian Forecasting and Time Series Analysis
A. Pole, M. West and J. Harrison

Applied Nonparametric Statistical Methods, Third Edition
P. Sprent and N.C. Smeeton

Applied Statistics — Handbook of GENSTAT Analysis
E.J. Snell and H. Simpson

Applied Statistics — Principles and Examples
D.R. Cox and E.J. Snell

Bayes and Empirical Bayes Methods for Data Analysis, Second Edition
Bradley P. Carlin and Thomas A. Louis

Bayesian Data Analysis, Second Edition
Andrew Gelman, John B. Carlin, Hal S. Stern, and Donald B. Rubin

Beyond ANOVA — Basics of Applied Statistics
R.G. Miller, Jr.

Computer-Aided Multivariate Analysis, Third Edition
A.A. Afifi and V.A. Clark

A Course in Categorical Data Analysis
T. Leonard

A Course in Large Sample Theory
T.S. Ferguson

Data Driven Statistical Methods
P. Sprent

Decision Analysis — A Bayesian Approach
J.Q. Smith

Elementary Applications of Probability Theory, Second Edition
H.C. Tuckwell

Elements of Simulation
B.J.T. Morgan

Epidemiology — Study Design and Data Analysis
M. Woodward

Essential Statistics, Fourth Edition
D.A.G. Rees

A First Course in Linear Model Theory
Nalini Ravishanker and Dipak K. Dey

Interpreting Data — A First Course in Statistics
A.J.B. Anderson

An Introduction to Generalized Linear Models, Second Edition
A.J. Dobson

Introduction to Multivariate Analysis
C. Chatfield and A.J. Collins

Introduction to Optimization Methods and their Applications in Statistics
B.S. Everitt

Large Sample Methods in Statistics
P.K. Sen and J. da Motta Singer

Markov Chain Monte Carlo — Stochastic Simulation for Bayesian Inference
D. Gamerman

Mathematical Statistics
K. Knight

Modeling and Analysis of Stochastic Systems
V. Kulkarni

Modelling Binary Data, Second Edition
D. Collett

Modelling Survival Data in Medical Research, Second Edition
D. Collett

Multivariate Analysis of Variance and Repeated Measures — A Practical Approach for Behavioural Scientists
D.J. Hand and C.C. Taylor

Multivariate Statistics — A Practical Approach
B. Flury and H. Riedwyl

Practical Data Analysis for Designed Experiments
B.S. Yandell

Practical Longitudinal Data Analysis
D.J. Hand and M. Crowder

Practical Statistics for Medical Research
D.G. Altman

Probability — Methods and Measurement
A. O'Hagan

Problem Solving — A Statistician's Guide, Second Edition
C. Chatfield

Randomization, Bootstrap and Monte Carlo Methods in Biology, Second Edition
B.F.J. Manly

Readings in Decision Analysis
S. French

Sampling Methodologies with Applications
Poduri S.R.S. Rao

Statistical Analysis of Reliability Data
M.J. Crowder, A.C. Kimber, T.J. Sweeting, and R.L. Smith

Statistical Methods for SPC and TQM
D. Bissell

Statistical Methods in Agriculture and Experimental Biology, Second Edition
R. Mead, R.N. Curnow, and A.M. Hasted

Statistical Process Control — Theory and Practice, Third Edition
G.B. Wetherill and D.W. Brown

Statistical Theory, Fourth Edition
B.W. Lindgren

Statistics for Accountants
S. Letchford

Statistics for Epidemiology
Nicholas P. Jewell

Statistics for Technology — A Course in Applied Statistics, Third Edition
C. Chatfield

Statistics in Engineering — A Practical Approach
A.V. Metcalfe

Statistics in Research and Development, Second Edition
R. Caulcutt

Survival Analysis Using S — Analysis of Time-to-Event Data
Mara Tableman and Jong Sung Kim

The Theory of Linear Models
B. Jørgensen

Preface

Books on reliability tend to fall into one of two categories. On the one hand there are those that present the mathematical theory at a moderately advanced level, but without considering how to fit the mathematical models to experimental or observational data. On the other hand there are texts aimed at engineers, which do deal with statistics but which largely confine themselves to elementary methods. Somewhere in between these two extremes lie the books on survival data analysis, which are more up to date in their statistical methodology but which tend to be oriented towards medical rather than reliability applications.

We have tried in this book to combine the best features of these different themes: to cover both the probability modelling and the statistical aspects of reliability in a book aimed at engineers rather than professional statisticians, but dealing with the statistics from a contemporary viewpoint. In particular, we see no reason why reliability engineers should be afraid of exploiting the computational power that is present in even very small personal computers these days.

One can discern a number of reasons for the bias in existing texts towards elementary statistical methods. One is that, until quite recently, university degree courses in engineering contained little or no training in statistics. This is changing very rapidly, partly as a result of the increasing mathematical sophistication of engineering generally, and partly because of the more specific current interest in the design of industrial experiments. A second reason is a distrust of the use of complicated mathematics to solve reliability problems. Authors such as O'Connor (1985), on what he termed 'the numbers game', have provided eloquent demonstration of the dangers in following an automated statistical procedure without thinking about the assumptions on which it is based. However, there is another side to the statistical story: modern statistics is concerned not only with automated methods for fitting complicated models to data, but also with an ever growing array of techniques for checking up on the assumptions of a statistical model. Our aim here is to present a balanced account presenting not only the mathematical techniques of model fitting, but also the more intuitive concepts (frequently depending heavily on computer graphics) for judging the fit of a model.

Although we have tried as far as possible to make this book self-contained, the book is really aimed at someone with a background of at least one course

in statistics, at about the level of Chatfield (1983). This will include the more common probability distributions, estimation, confidence intervals and hypothesis testing, and the bare bones of regression and the analysis of variance. Given that degree courses in engineering nowadays usually do include at least one course in elementary statistics, we do not believe that these are unreasonable requirements. The present text may then be thought of as providing a follow-up course aimed at one specific, and very common, area of the application of statistics in engineering. Familiarity with one of the more common statistical packages, such as Minitab or SPSS^{X}, will certainly help the reader appreciate the computational aspects of the book, but we do not assume any prior knowledge of reliability theory or life data analysis.

Chapters 1 and 2 present elementary material on probability models and simple statistical techniques. Although some of this is revision material, these chapters also introduce a number of concepts, such as the Kaplan–Meier estimate, which are not often found in introductory courses in statistics. The core of the book starts with Chapter 3, in which numerical maximum likelihood and model-checking techniques are described from the point of view of fitting appropriate distributions to samples of independent, identically distributed observations, but allowing censoring. In Chapter 4, these methods are extended to parametric regression models. The ideas presented here are familiar enough to applied statisticians, but they are not as yet widely used in reliability. This chapter includes several substantial examples which, apart from serving to illustrate the mathematical techniques involved, touch on many of the practical issues in application to reliability data. Chapter 5 is about another class of regression models: semiparametric models based on the proportional hazards assumption. Although by now widely used on medical data, these methods have not reached the same level of acceptance in reliability. In many respects, they may be thought of as an alternative to the parametric techniques in Chapter 4.

Chapter 6 is about Bayesian methods. At the present time there is lively debate over the place of Bayesian statistics in reliability theory. Whether Bayesian statistics will eventually supplant classical statistics, as its more vigorous proponents have been proclaiming for the past forty years, is something still to be seen, but it is certainly our view that reliability engineers should have an awareness of the Bayesian approach and this is what we have tried to provide here. Chapters 7 and 8 deal with more specialized themes: Chapter 7 on multivariate reliability distributions and Chapter 8 on particular methods appropriate for repairable systems. Finally, Chapter 9 presents an overview of models for system reliability, divided between the theory of coherent systems and a more specialized topic, not previously covered in a book of this nature, that of load-sharing systems.

Within this wide scope of topics it is inevitable that the mathematical level of the book varies appreciably. Chapters 1 and 2 are quite elementary but parts

of the later chapters, especially Chapter 9, are considerably more advanced. To aid the reader trying to judge the level of the material, we have indicated with an asterisk (*) certain more advanced sections which can be omitted without loss of continuity.

One area we have not tried to cover is experimental design. The revolution in engineering design associated with the name of Taguchi has focused considerable attention on the proper design of industrial experiments. To do justice to this here would have taken us too far from our main theme, but we certainly do regard experimental design as an important part of reliability, especially in connection with the more advanced analytic techniques described in Chapters 4 and 5. We would encourage the reader who has had some exposure to these ideas to think of the ways in which they could be useful in reliability analysis.

Much of the book relies on numerical techniques to fit complex models, and a natural question is how best to put these into practice. At one stage during the writing of the book, we were considering making a suite of programmes available on disk, but in the end we had to admit that the task of 'cleaning up' our programs to the extent that they could be used as a computer package would have held up the writing of the book even longer than the four years it has taken us to get this far; therefore we abandoned that project, and have contented ourselves with general advice, at appropriate points, about the algorithms to be used. Apart from Chapter 6, the main numerical technique is non-linear optimization, and there are numerous published subroutines for this, for example in the NAG or IMSL libraries or books such as Nash (1979) or Press *et al.* (1986). Our advice to the reader is to become familiar with one such subroutine, preferably one that does not require derivatives (NAG's *E04CGF* is an example) and to program up likelihood functions as required. An advantage of this, over providing a package to do everything in one go, is that it will force the reader to think about how numerical optimization techniques actually work, and thereby to pay attention to such things as appropriate scaling of the variables. The one exception to this is Chapter 6 on Bayes methods, since, except where conjugate priors are employed, this requires numerical integration. Some comments about exact and approximate methods for that are included in Chapter 6.

It is a pleasure to thank those who have helped us in the preparation of this book. Mike Bader of the University of Surrey and Leigh Phoenix of Cornell University have been valuable research collaborators, and the data in Tables 4.8.1 and 4.8.2 (based on experimental work by Mark Priest, at Surrey), 7.3.2 (from Bashear Gul-Mohammad) have come from the collaboration with Bader. Frank Gerstle gave us a copy of Gerstle and Kunz (1983) cited in sections 4.9, 4.10 and 5.4. Stephen M. Stigler of the University of Chicago drew our attention to the Buffon data of Section 4.10. Part of Chapter 4 is based on the paper by Smith (1991). Linda Wolstenholme, Karen Young and Leigh Phoenix

provided us with very valuable comments on preliminary drafts of the manuscript. We also thank Sally Fenwick and Marion Harris for typing substantial portions of the manuscript, and Elizabeth Johnston of Chapman & Hall for her great expertise in bringing the project, despite many delays, to a successful conclusion. Finally, we gratefully acknowledge the Wolfson Foundation (R.L.S.) and the S.E.R.C. for financial support.

Martin J. Crowder
Alan C. Kimber
Richard L. Smith
Trevor J. Sweeting
Guildford.

Contents