国际著名力学图书 —— 影印版系列 2

Engineering Mechanics

DYNAMICS Second Edition

工程力学

动力学 第2版

Andrew Pytel · Jaan Kiusalaas

清华大学出版社
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Engineering Mechanics

Dynamics(Second Edition)

影印版序

工程力学(包括理论力学和材料力学)是许多工程学科的基础,被列为我国高等院校理工科大学生所必修的基础课。学习工程力学需要有清晰的物理概念和形象的几何直观,准确地理解基本原理和基本方法,熟练地掌握数学推理,因此工程力学是大学阶段较难学好的课程之一。另一方面,正因为它具有科学严密性和应用灵活性紧密结合的魅力,是培养学生综合研究素质的训练园地,工程力学课程又为许多大学生所青睐。

本书由美国宾夕法尼亚州立大学的两位教授所著,包括静力学和动力学两部分,本册为动力学部分。书中对动力学基本概念和基本原理的叙述简明、清晰,通过例题的讨论(共133个)帮助学生掌握应用原理的方法和解题技巧,附有大量习题(共1102个)供学生训练,每章后面还配有供学生自我检查的测试题,是一本典型的美式教材,出版后被美国多所大学采用。作者并不追求理论上的系统、完整,而更重视动力学基本原理的熟练应用。作者强调在开始进行具体计算以前首先要从整体上分析和把握问题的求解思路,判断问题中的未知量数和独立方程数,这是学好工程力学的经验之谈。习题经过精心挑选,有应用基本原理可以直接求解的,也有来自工程实际、较有趣而更复杂的。动力学方程较为复杂,大多数找不到解析题,书中引进了数值解法和相应的计算机程序。

本书共分10章,内容包括:动力学引论,质点动力学——直角坐标系,质点动力学——曲线坐标系,质点的功-能原理和冲量原理,质系动力学,刚体的平面运动学,刚体的平面动力学——力-质量-加速度法,刚体的平面动力学——功-能法和冲量法,刚体的三维动力学,振动。其中带星号(*)的章节是要求较高的补充教材,由教师决定取舍。配有刚体运动相对速度方程的证明,数值微分以及质量矩与惯性积三个附录。最后附有双号习题的答案和名词检索。汇总第一版出版以来新的教学经验和读者的建议,第二版对若干章节进行了修改,增加了例题,更新了习题。

我国目前的工程力学课程一般按研究对象(刚体和变形体)划分为理论力学和材料力学两部分。美国则按运动状态划分为静力学和动力学两部分,其中动力学相当于我国理论力学中运动学和动力学的教学内容。

本书是美国现有工程力学教材中值得一读的优秀教材之一,内容丰富、语言流畅,可作为我国高等工业院校工程力学和理论力学课程的主要英文参考书。

陆明万
清华大学工程力学系

To Jean, Leslie, Lori, John, Nicholas

and

To Judy, Nicholas, Jennifer, Timothy

Preface

Statics and dynamics form the foundation of many engineering disciplines and are, therefore, essential to the training of an engineer. Mastery of these subjects requires a clear understanding of the principles and experience in applying the principles to a wide range of situations. Because the applications require reasoning rather than memorization, students perceive statics and dynamics as difficult courses, thus making their teaching particularly challenging.

This textbook on dynamics and its companion volume on statics were developed over many years of teaching. The salient features of *Dynamics* are:

- Each sample problem contains a comprehensive discussion of the reasoning behind its solution. We also emphasize the importance of comparing the number of available equations with the number of unknowns.
- The selection of homework problems strives for a balanced presentation. It includes many "textbook" problems that illustrate the principles involved in a straightforward manner. There are also numerous problems of direct engineering relevance, which are more interesting and challenging.
- The first chapter (Chapter 12) on particle dynamics integrates kinematics and kinetics. We found this arrangement superior to the traditional approach of devoting the opening chapter exclusively to kinematics. Introducing the student immediately to practical problems, where the equations of motion are derived and not given, provides a stronger motivation for learning. Moreover, the student is introduced early to the all-important role played by the free-body diagrams in dynamics.
- Numerical methods of solving equations of motion are integrated seamlessly into the text, with emphasis being on computer applications, not on programming. The incorporation of numerical methods into the course is optional. The instructor is at liberty to skip it entirely or cover only some of the material. We found that students' interest in dynamics is significantly increased when they are exposed to numerical solutions. Even if the material is skipped, its presence in the text alerts students to the fact that most dynamics problems, even the comparatively simple ones, cannot be solved analytically.

The book contains several optional topics, which are marked by an asterisk (*). This material can be omitted without jeopardizing the presentation of other parts of *Dynamics*. An asterisk is also used to mark problems that require advanced reasoning. Topics and problems associated with numerical methods are preceded by an icon representing a computer disk ( ).

New to the Second Edition We received considerable feedback from the users of the first edition that we found very helpful when we prepared the second edition. As the result of the comments and suggestions made by the reviewers, we made the following changes:

- The number of introductory problems was increased.
- New problem sets were added to the end of Chapter 11 (Introduction to Dynamics) and to Chapter 15 (Dynamics of Particle Systems).
- Additional sample problems appear in various chapters.
- We changed several chapter introductions in order to place the topics in better perspective.
- We rewrote a few articles in a more concise form.
- Some improvements were made to notation and terminology.
- Chapter 16 (Planar Kinematics of Rigid Bodies) was reorganized to conform better with the traditional presentation.
- The discussion of numerical solutions now introduces the concept of replacing a second-order differential equation by equivalent first-order equations. This step brings the topic more in line with the available software. (A computer disk is no longer enclosed in the book.)

Ancillary *Study Guide to Accompany Pytel and Kiusalaas, Engineering Mechanics: Dynamics, Second Edition,* J. L. Pytel, 1999. The goal of this study guide is to help students master the problem-solving skills required in their study of dynamics. Students are prompted to interact with the material as they work through "guided" problems. The study guide contains additional problems accompanied by complete solutions.

Acknowledgments Statics and dynamics is a mature field of study that has been built over many generations. Therefore, any new textbook pays tacit homage to the books that preceded it and to their authors. We are grateful to the following reviewers for their valuable suggestions during preparation of the first edition: Anil K. Bajaj, Purdue University; Duane Casteneda, University of Alabama–Birmingham; Scott G. Danielson, North Dakota State University; Richard N. Downer, University of Vermont; Howard Epstein, University of Connecticut; Ralph E. Flori, University of Missouri–Rolla; Robert A. Freeman, University of Texas–Austin; Li-Sheng Fu, Ohio State University; S. C. Gambrell, Jr., University of Alabama; Norman W. Garrick, University of Connecticut; Edward E. Hornsey, University of Missouri–Rolla; Cecil O. Huey, Clemson University; Vernal H. Kenner, Ohio State University; Thomas J. Kosic, Texas A&M University; E. Harry Law, Clemson University; Dahsin Liu, Michigan State University; Mohammad Mahinfalah, North Dakota State University; Mark Mear, University of Texas–Austin; Satish Nair, University of Missouri–Columbia; Saeed B. Niku, California Polytechnic University; William W. Predebon, Michigan Tech University; Robert Price, Louisiana Tech University; R. K. Raju, Auburn University; Robert Seabloom, University of Washington–Seattle; John M. Vance, Texas A&M University; Dennis Vandenbrink, Western Michigan University; and Arthur N. Willoughby, Morgan State University.

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We specifically recognize our indebtedness to Dr. Christine Masters, who checked the solutions to the homework problems.

Andrew Pytel
Jaan Kiusalaas

Contents

*Indicates optional articles

11
Introduction to Dynamics

11.1 Introduction

Classical dynamics studies the motion of bodies using the principles established by Newton and Euler.* The organization of this text is based on the subdivisions of classical dynamics shown in Fig. 11.1.

The first part of the text deals with dynamics of particles. A *particle* is a mass point; it possesses a mass but has no size. The particle is an approximate model of a body whose dimensions are negligible in comparison with all other dimensions that appear in the formulation of the problem. For example, in studying the motion of the earth around the sun, it is permissible to consider the earth as a particle, because its diameter is much smaller than the dimensions of the orbit.

The second part of the text is devoted mainly to dynamics of *rigid bodies.* A body is said to be rigid if the distance between any two material points of the body remains constant, that is, if the body does not deform. Because any body undergoes some deformation when loads are applied to it, a truly rigid body does not exist. However, in many applications the deformation is so small (relative to the dimensions of the body) that the rigid-body idealization is a good approximation.

As seen in Fig. 11.1, the main branches of dynamics are kinematics and kinetics. *Kinematics* is the study of the geometry of motion. It is not concerned with the causes of motion. *Kinetics,* on the other hand, deals with the relationships between the forces acting on the body and the resulting motion. Kinematics is not only an important topic in its own right but is also a prerequisite to kinetics. Therefore, the study of dynamics always begins with the fundamentals of kinematics.

Kinematics can be divided into two parts as shown in Fig. 11.1: absolute motion and relative motion. The term *absolute motion* is used when the motion is described with respect to a fixed reference frame (coordinate system). Relative motion, on the other hand, describes the motion with respect to a moving coordinate system.

Figure 11.1 also lists the three main methods of kinetic analysis. The *force-mass-acceleration* (FMA) *method* is a straightforward application of the

*Sir Isaac Newton is credited with laying the foundation of classical mechanics with the publication of *Principia* in 1687. However, the laws of motion as we use them today were developed by Leonhard Euler and his contemporaries more than sixty years later. In particular, the laws for the motion of finite bodies are attributable to Euler.

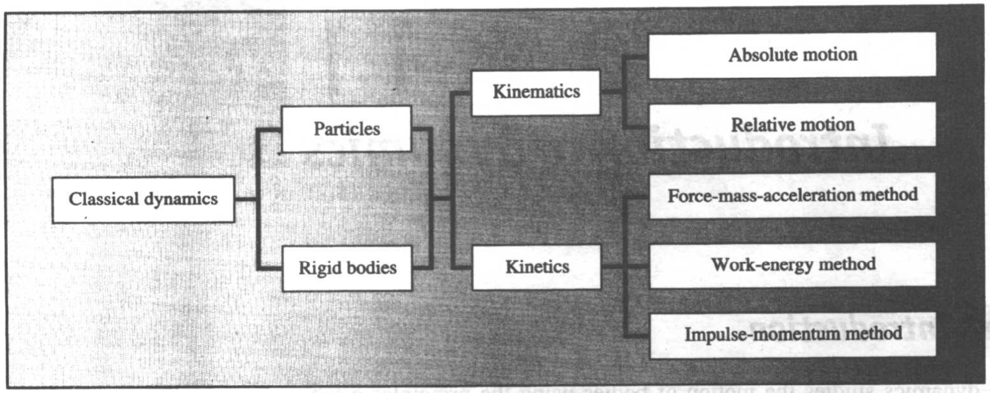

Fig. 11.1

Newton-Euler laws of motion, which relate the forces acting on the body to its mass and acceleration. These relationships, called the ***equations of motion***, must be integrated twice in order to obtain the velocity and the position as functions of time.

The ***work-energy*** and ***impulse-momentum methods*** are integral forms of Newton-Euler laws of motion (the equations of motion are integrated with respect to position or time). In both methods the acceleration is eliminated by the integration. These methods can be very efficient in the solution of problems concerned with velocity-position or velocity-time relationships.

The purpose of this chapter is to review the basic concepts of Newtonian mechanics: displacement, velocity, acceleration, Newton's laws, and units of measurement.

11.2 Derivatives of Vector Functions

A knowledge of vector calculus is a prerequisite for the study of dynamics. Here we discuss the derivatives of vectors; integration is introduced throughout the text as needed.

The vector **A** is said to be a vector function of a scalar parameter u if the magnitude and direction of **A** depend on u. (In dynamics, time is frequently chosen to be the scalar parameter.) This functional relationship is denoted by $\mathbf{A}(u)$. If the scalar variable changes from the value u to $(u + \Delta u)$, the vector **A** will change from $\mathbf{A}(u)$ to $\mathbf{A}(u + \Delta u)$. Therefore, the change in the vector **A** can be written as

$$\Delta\mathbf{A} = \mathbf{A}(u + \Delta u) - \mathbf{A}(u) \tag{11.1}$$

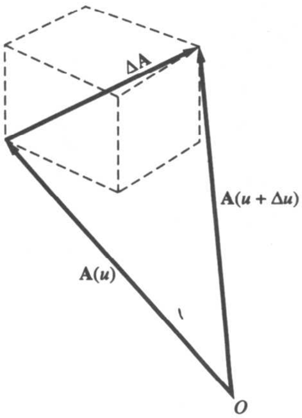

Fig. 11.2

As seen in Fig. 11.2, $\Delta\mathbf{A}$ is due to a change in both the magnitude and the direction of the vector **A**.

The derivative of **A** with respect to the scalar u is defined as

$$\frac{d\mathbf{A}}{du} = \lim_{\Delta u \to 0} \frac{\Delta\mathbf{A}}{\Delta u} = \lim_{\Delta u \to 0} \frac{\mathbf{A}(u + \Delta u) - \mathbf{A}(u)}{\Delta u} \tag{11.2}$$

assuming that the limit exists. This definition resembles the derivative of the scalar function $y(u)$, which is defined as

$$\frac{dy}{du} = \lim_{\Delta u \to 0} \frac{\Delta y}{\Delta u} = \lim_{\Delta u \to 0} \frac{y(u + \Delta u) - y(u)}{\Delta u} \tag{11.3}$$

Caution In dealing with a vector function, the magnitude of the derivative $|d\mathbf{A}/du|$ must not be confused with the derivative of the magnitude $d|\mathbf{A}|/du$. In general, these two derivatives will not be equal. For example, if the magnitude of a vector **A** is constant, then $d|\mathbf{A}|/du = 0$. However, $|d\mathbf{A}/du|$ will not equal zero unless the direction of **A** is also constant.

The following useful identities can be derived from the definitions of derivatives (**A** and **B** are assumed to be vector functions of the scalar u, and m is also a scalar):

$$\frac{d(m\mathbf{A})}{du} = m\frac{d\mathbf{A}}{du} + \frac{dm}{du}\mathbf{A} \tag{11.4}$$

$$\frac{d(\mathbf{A} + \mathbf{B})}{du} = \frac{d\mathbf{A}}{du} + \frac{d\mathbf{B}}{du} \tag{11.5}$$

$$\frac{d(\mathbf{A} \cdot \mathbf{B})}{du} = \mathbf{A} \cdot \frac{d\mathbf{B}}{du} + \frac{d\mathbf{A}}{du} \cdot \mathbf{B} \tag{11.6}$$

$$\frac{d(\mathbf{A} \times \mathbf{B})}{du} = \mathbf{A} \times \frac{d\mathbf{B}}{du} + \frac{d\mathbf{A}}{du} \times \mathbf{B} \tag{11.7}$$

11.3 *Position, Velocity, and Acceleration of a Particle*

a. *Position*

Consider the motion of a particle along a smooth path as shown in Fig. 11.3. The position of the particle at time t is specified by the *position vector* $\mathbf{r}(t)$, which is the vector drawn from a fixed origin O to the particle. Let the location of the particle be A at time t, and B at time $t + \Delta t$, where Δt is a finite time

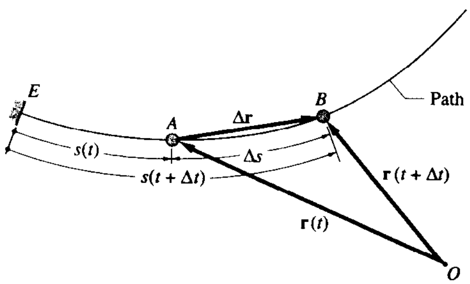

Fig. 11.3

interval. The corresponding change in the position vector of the particle,

$$\Delta \mathbf{r} = \mathbf{r}(t + \Delta t) - \mathbf{r}(t) \tag{11.8}$$

is called the *displacement vector* of the particle.

As indicated in Fig. 11.3, the position of the particle at time t can also be specified by the *path coordinate* $s(t)$, which is the length of the path between a fixed point E and the particle. The change in path length during the time interval Δt is

$$\Delta s = s(t + \Delta t) - s(t) \tag{11.9}$$

Caution The change in path length should not be confused with the *distance traveled* by the particle. The two are equal only if the direction of motion does not change during the time interval. If the direction of motion changes during Δt, then the distance traveled will be larger than Δs.

b. *Velocity*

The velocity of the particle at time t is defined as

$$\mathbf{v}(t) = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t} = \dot{\mathbf{r}}(t) \tag{11.10}$$

where the overdot denotes differentiation with respect to time. Because the velocity is the derivative of the vector function $\mathbf{r}(t)$, it is also a vector. From Fig. 11.3 we observe that $\Delta \mathbf{r}$ becomes tangent to the path at A as $\Delta t \to 0$. Consequently, the *velocity vector is tangent to the path* of the particle.

We also deduce from Fig. 11.3 that $|\Delta \mathbf{r}| \to \Delta s$ as $\Delta t \to 0$. Therefore, the magnitude of the velocity, also known as the *speed* of the particle, is

$$v(t) = \lim_{\Delta t \to 0} \frac{|\Delta \mathbf{r}|}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \dot{s}(t) \tag{11.11}$$

The dimension of velocity is $[L/T]$, so the unit of velocity is m/s or ft/s.

c. *Acceleration*

The velocity vectors of the particle at A (time t) and B (time $t + \Delta t$) are shown in Fig. 11.4(a). Note that both vectors are tangent to the path. The change in the velocity during the time interval Δt, shown in Fig. 11.4(b), is

$$\Delta \mathbf{v} = \mathbf{v}(t + \Delta t) - \mathbf{v}(t) \tag{11.12}$$

The acceleration of the particle at time t is defined as

$$\mathbf{a}(t) = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t} = \dot{\mathbf{v}}(t) = \ddot{\mathbf{r}}(t) \tag{11.13}$$

The acceleration is a vector of dimension $[L/T^2]$; hence its unit is m/s^2 or ft/s^2.

Caution The acceleration vector is generally *not tangent to the path* of the particle. The direction of the acceleration coincides with $\Delta \mathbf{v}$ as $\Delta t \to 0$, which, as seen in Fig. 11.4(b), is not necessarily in the same direction as $\mathbf{v}$.

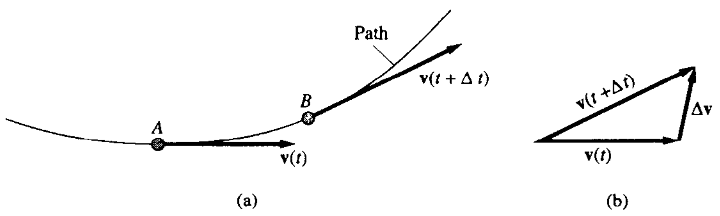

Fig. 11.4

11.4 *Newtonian Mechanics**

a. *Scope of Newtonian mechanics*

In 1687, Sir Isaac Newton (1642–1727) published his celebrated laws of motion in *Principia (Mathematical Principles of Natural Philosophy).* Without a doubt, this work ranks among the most influential scientific books ever published. We should not think, however, that its publication immediately established classical mechanics. Newton's work on mechanics dealt primarily with celestial mechanics and was thus limited to particle motion. Another two hundred or so years elapsed before rigid-body dynamics, fluid mechanics, and the mechanics of deformable bodies were developed. Each of these areas required new axioms before it could assume a usable form.

Nevertheless, Newton's work is the foundation of classical, or Newtonian, mechanics. His efforts have even influenced two other branches of mechanics born at the beginning of the twentieth century: relativistic and quantum mechanics. *Relativistic mechanics* addresses phenomena that occur on a cosmic scale (velocities approaching the speed of light, strong gravitational fields, etc.). It removes two of the most objectionable postulates of Newtonian mechanics: the existence of a fixed or inertial reference frame and the assumption that time is an absolute variable, "running" at the same rate in all parts of the universe. (There is evidence that Newton himself was bothered by these two postulates.) *Quantum mechanics* is concerned with particles on the atomic or subatomic scale. It also removes two cherished concepts of classical mechanics: determinism and continuity. Quantum mechanics is essentially a probabilistic theory; instead of predicting an event, it determines the likelihood that an event will occur. Moreover, according to this theory, the events occur in discrete steps (called *quanta*) rather than in a continuous manner.

Relativistic and quantum mechanics, however, have by no means invalidated the principles of Newtonian mechanics. In the analysis of the motion of bodies encountered in our everyday experience, both theories converge on the equations of Newtonian mechanics. Thus the more esoteric theories actually reinforce the validity of Newton's laws of motion.

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*This article, which is the same as Art. 1.2 in *Statics*, is repeated here because of its relevance to our study of dynamics.

b. *Newton's laws for particle motion*

Using modern terminology, Newton's laws of particle motion may be stated as follows.

1. If a particle is at rest (or moving with constant velocity), it will remain at rest (or continue to move with constant velocity) unless acted on by a force.
2. A particle acted on by a force will accelerate in the direction of the force. The magnitude of the acceleration is proportional to the magnitude of the force and inversely proportional to the mass of the particle.
3. For every action, there is an equal and opposite reaction; that is, the forces of interaction between two particles are equal in magnitude and opposite in direction.

Although the first law is simply a special case of the second law, it is customary to state the first law separately because of its importance to the subject of statics.

c. *Inertial reference frames*

When applying Newton's second law, attention must be paid to the coordinate system in which the accelerations are measured. An *inertial reference* frame (also known as a Newtonian or Galilean reference frame) is defined to be any rigid coordinate system in which Newton's laws of particle motion relative to that frame are valid with an acceptable degree of accuracy. In most design applications used on the surface of the earth, an inertial frame can be approximated with sufficient accuracy by attaching the coordinate system to the earth. In the study of earth satellites, a coordinate system attached to the sun usually suffices. For interplanetary travel, it is necessary to use coordinate systems attached to the so-called fixed stars.

It can be shown that any frame that is translating with constant velocity relative to an inertial frame is itself an inertial frame. It is a common practice to omit the word *inertial* when referring to frames for which Newton's laws obviously apply.

d. *Units and dimensions*

The standards of measurement are called *units*. The term *dimension* refers to the type of measurement, regardless of the units used. For example, kilogram and foot/second are units, whereas mass and length/time are dimensions. Throughout this text we use two standards of measurement: the U.S. Customary system and the SI system (from *Système international d'unités*). In the *U.S. Customary system* the base (fundamental) dimensions are force $[F]$, length $[L]$, and time $[T]$. The corresponding base units are pound (lb), foot (ft), and second (s). The base dimensions in the *SI system* are mass $[M]$, length $[L]$, and time $[T]$, and the base units are kilogram (kg), meter (m), and second (s). All other dimensions or units are combinations of the base quantities. For example, the dimension of velocity is $[L/T]$, the unit being ft/s, m/s, and so on.

A system with the base dimensions $[FLT]$ (such as the U.S. Customary system), is called a *gravitational system*. If the base dimensions are $[MLT]$ (as in the SI system), the system is known as an *absolute system*. In each system of