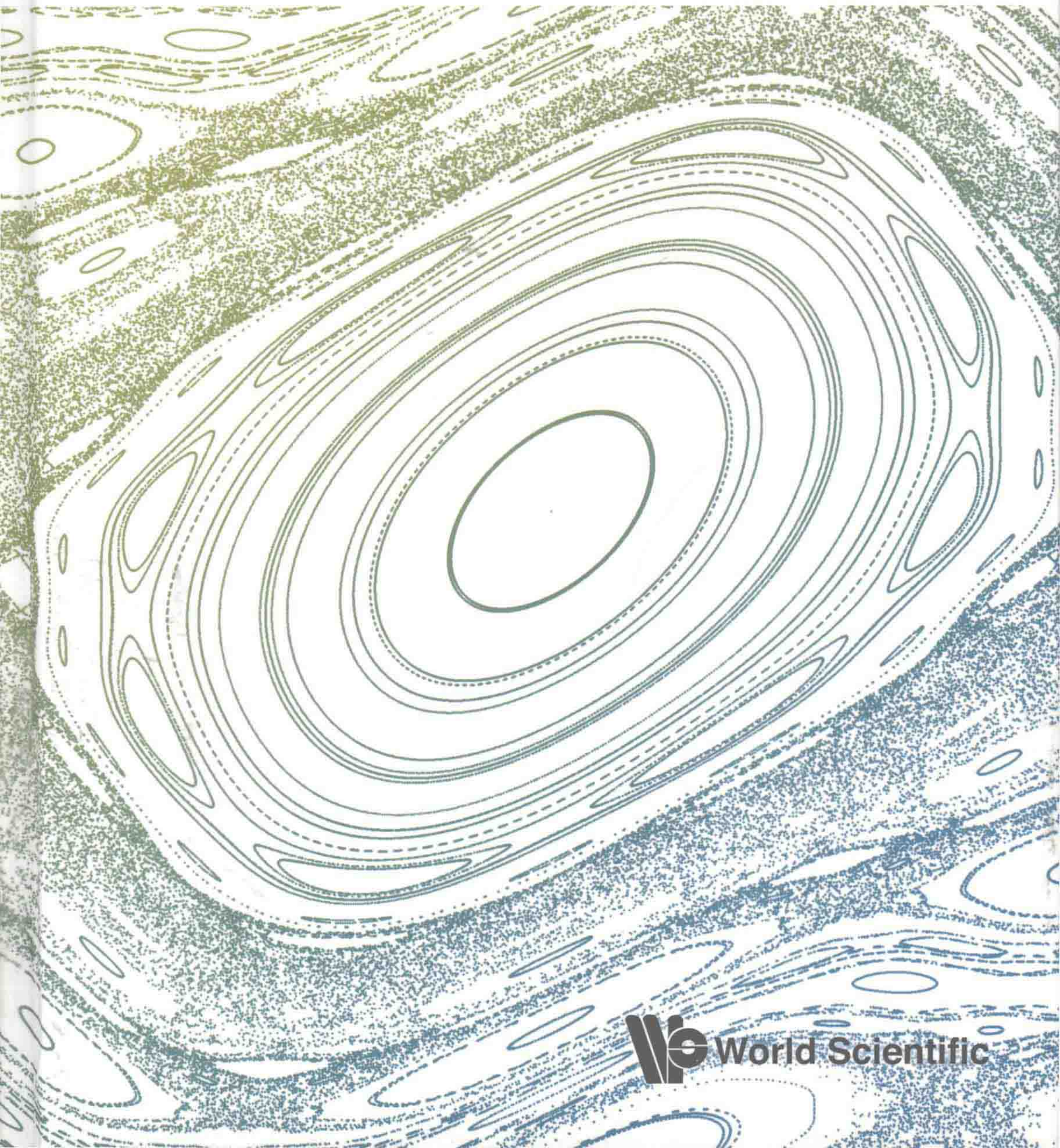
THE KAM STORY

A Friendly Introduction to the Content, History, and Significance of Classical Kolmogorov–Arnold–Moser Theory

H Scott Dumas



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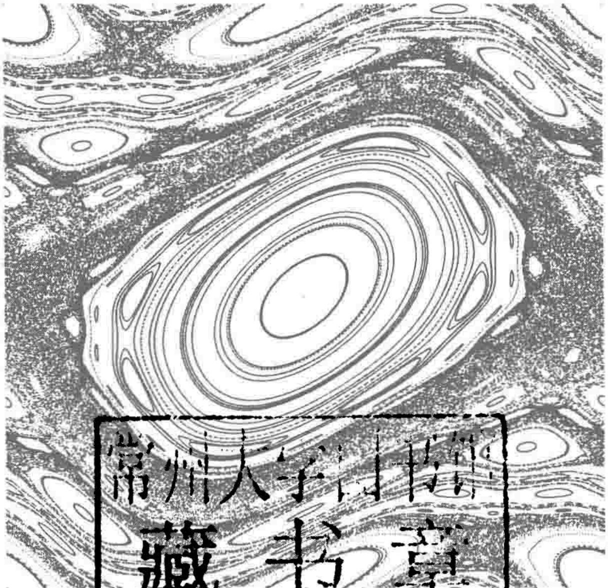

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To Warren Farnholtz

Preface

In many ways classical mechanics serves as the bedrock of physical science, yet surprisingly, it has crucial features that are not widely known. Many people know something about 'chaos theory'—how mathematical models of certain deterministic classical systems fail to predict the evolution of those systems in a practical sense. If they're interested in history, they also know that much of chaos theory was understood by mathematicians almost a century before it was popularized by way of computer models in the last third of the 20th century. But there is a deeper, more interesting story that is not well known outside a circle of experts, and the aim of this book is to tell this story to a wider audience.

The story in a nutshell is this: Right from the start, after enunciating his laws of mechanics and gravitation, Isaac Newton ran into difficulties using those laws to describe the motion of three bodies moving under mutual gravitational attraction (the so-called 'three body problem'). For the next two centuries, these difficulties resisted solution, as the best minds in mathematics and physics concentrated on solving other, increasingly complex model systems in classical mechanics (in the abstract mathematical setting, to 'solve' a system means showing that its trajectories move linearly on so-called 'invariant tori'). But toward the end of the 19th century, using his own new methods, Henri Poincaré confronted Newton's difficulties head-on and discovered an astonishing form of 'unsolvability,' or chaos, at the heart of the three body problem. This in turn led to a paradox. According to Poincaré and his followers, most classical systems should be chaotic; yet observers and experimentalists did not see this in nature, and mathematicians working with model systems could not (quite) prove it to be true either. The paradox persisted for more than a half-century, until Andrey Kolmogorov unraveled it by announcing that, against all expectation, many of the invariant tori from solvable systems remain intact in chaotic systems. These tori make most systems into hybrids—they are a strange, fractal mixture of regularity and chaos. This stunning announcement was later affirmed with rigorous mathematical proofs by Vladimir Arnold and Jürgen Moser, and the names Kolmogorov, Arnold, and Moser were combined in the acronym KAM, by which the theory has since been known. Thus the true picture of classical mechanics—often thought to have been essentially sketched in the 17th century—was not complete until the latter part of the 20th century. And although the mathematical theory is indeed mostly complete, certain applications to problems in physics (especially in celestial and statistical mechanics) have been developed only with great difficulty, and some remain controversial and uncertain even today.

To compare the practical impact of KAM theory to that of relativity or quantum theory is not realistic (to be frank, the practical impact of KAM theory has been limited). Yet in the history of ideas and the philosophy of science, it is not a stretch to rank KAM theory alongside the revolutions in modern physics. But KAM theory—and the paradox that precipitated it—also had the misfortune of playing out over roughly the same interval during which the revolutions of modern physics took place. Not surprisingly, in that period, physicists abandoned classical mechanics to the few hardy mathematicians who remained interested in it. The physicists returned with wondrous stories of their exploits in quantum mechanics, relativity, and nuclear physics. The time has come for mathematicians to tell their tales from this period in a broad setting, too.

When I asked specialists why none of them had yet written a broad overview of KAM theory, they invariably answered that, with several different 'schools' having descended from the original founders of the theory, it would be awkward for any one individual to take up that task. In other words, KAM theory is still slightly controversial, and the experts are understandably touchy about each other's contributions. Since I don't belong to any particular school, I am prepared to step into the breach, or break the ice. I hope the experts will follow me, not with pitchforks, but with first-hand accounts, corrections, and further detail.

H.S. Dumas, December 2013

Acknowledgments

Like its subject, this book has long roots, and there are many people to thank for its development. My interest in history of science began early and was fueled in my undergraduate days by contact with Albert Van Helden and Salomon Bochner. I heard of KAM theory from Asim Barut at the beginning of graduate school, and was inspired to learn more through discussions with my mentors Jim Ellison, Bill Sáenz, Tom Kyner, Vageli Coutsias, François Golse, and especially Pierre Lochak, who taught me much of what I know about the subject and pointed me toward the Russian and Italian literature. Later, I had the good fortune to meet and talk with Jürgen Moser and Vladimir Arnold, along with some of their students, which only magnified my interest.

Once I formed the idea of this book, I was strongly encouraged by Hildeberto Cabral, Jie Chen, Chuck Wells, and especially Teresa Stuchi. When I needed to find more detail about Weierstrass and Poincaré, Alain Albouy directed me to original sources; he and Alain Chenciner later helped me understand those sources in context. Many people looked at early versions of the draft and encouraged me, including Mathias Vogt, Ken Meyer, Qiu Dong Wang, Patricia Yanguas, Jesús Palacián, Klaus Heinemann, Ning Zhong, and Bing Yu Zhang. Once written, the manuscript would not have found a good publisher without the key endorsements of Reuben Hersh, Jacques Féjoz, and Steve Wiggins.

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