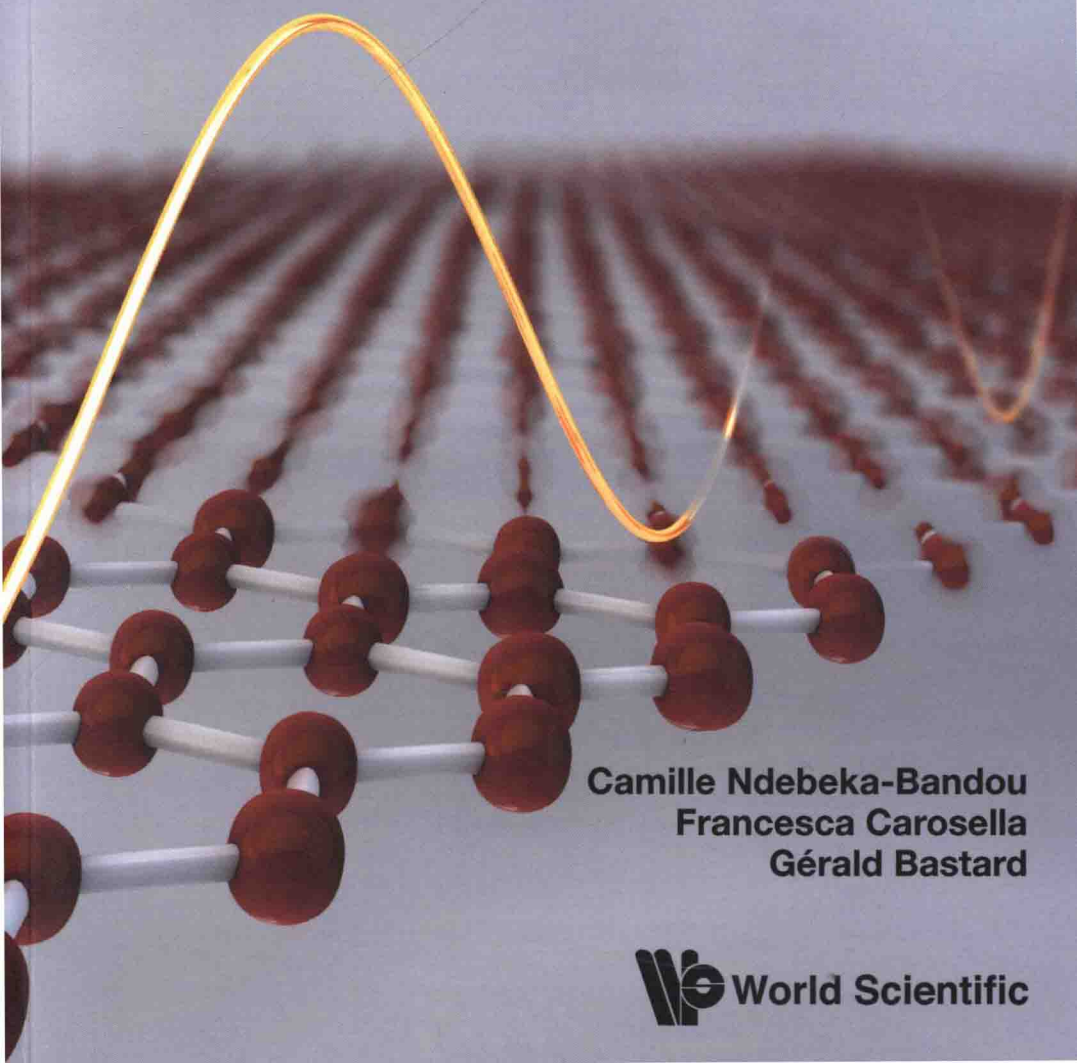



Advanced Textbooks in Physics

Quantum States and Scattering in Semiconductor Nanostructures



Camille Ndebeka-Bandou
Francesca Carosella
Gérald Bastard

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Quantum States and Scattering in Semiconductor Nanostructures

This book is an introduction to quantum states and of their scattering in semiconductor nanostructures. Written with exercises and detailed solutions, it is designed to enable readers to start modelling actual electron states and scattering in nanostructures. It first looks at practical aspects of quantum states and emphasises the variational and perturbation approaches. Following this there is analysis of quasi two-dimensional materials, including discussion of the eigenstates of nanostructures, scattering mechanisms and their numerical results.

Focussing on practical applications, this book moves away from standard discourse on theory and provides students of physics, nanotechnology and materials science with the opportunity to fully understand the electronic properties of nanostructures.

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Carosella
Bastard



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Foreword

The last 30 years have witnessed the prevalent use of — “quantum heterostructures” — in microelectronics (Field Effect Transistor) and optoelectronics (Quantum Well Lasers, Quantum Cascade Lasers). Advances in the area of nanostructures require a good knowledge of elementary quantum mechanics and simultaneously a feeling of how a certain electronic function is best realised (the “bandgap engineering”). Yet, the usual teaching of quantum mechanics focuses on general principles and the applications that are proposed to the students to understand this formal apparatus often fall within the field of atomic physics. The theories of quantum measurements, the decoherence, are also highlighted and for good reasons, as the measurement theory has recently been the topics of major discoveries. To be checked it requires the simpler possible quantum objects: ultra-diluted two-level systems as realised in atomic physics. However, these questions (time evolution of systems with very few degree of freedom) are irrelevant to the understanding of the electron states in semiconductor nanostructures where one deals most often with extended states perturbed by static defects or inelastic interactions with phonons.

The book is written for students and engineers who have already been exposed to elementary quantum mechanics and statistical physics. That is why our book presents an applied version of quantum mechanics that is very short on general physical questions but has the target of predicting the electron whereabouts in existing semiconductor heterostructures. To give an example, we note that many of the nanostructures display an effective cylindrical symmetry around an

axis. That is why we shall devote very little space below to the spherically symmetric problems, so prevalent in atomic physics. A great deal of attention will be given to the scattering of extended states while usual courses emphasize discrete electronic states and their coupling to external fields. Our aim is to bring our readers to answer basic but relevant questions for devices; “where are the electrons in the structures, how do their energy levels vary when this or that parameter changes, what is the order of magnitude of the coupling between the electrons and the static defects or the phonons?”.

While computers allow a numerical solution of a large number of problems in nanostructures, we believe it remains very important that the researchers/engineers working on these materials can analytically handle simplified cases.

To this end, we propose more than 50 exercises/problems (with solutions) where the readers will train him/herself to analytically approach actual situations. We have created most of these exercises (or we believe to have done so). Some of them have been used in different Master programmes at École Normale Supérieure (International Centre for Fundamental Physics), Pierre et Marie Curie (Sciences des Matériaux et des Nano-objets) and Paris Diderot Universities (Dispositifs Quantiques) and in different Universities abroad (TU Wien, IIS Tokyo, HKUST). We have retained a handful of classic exercises (for instance, the variational estimate of the hydrogen binding energy using a Gaussian trial function [1]) because of their pedagogical values despite the fact that some of them can be found on the Web or in specific textbooks.

The exercises/problems can be quite short with the aim of training the reader to do the calculations automatically or can be long if a certain question needs to be discussed more thoroughly. The set of exercises can be split roughly into five parts: (a) basic quantum mechanics (1)–(16); (b) energy levels in 1D structures (17)–(25); (c) (static)perturbation theory (26)–(32); (d) time-dependent problems (33)–(44); (e) scattering (45)–(54). Problems involving one-dimensional (1D) localised states are discussed more thoroughly than usually found in textbooks because a (huge) number of actual heterostructures display 1D bound states.

Throughout the years, discussions with colleagues have helped us to clarify many aspects of semiconductor heterostructures. We are much indebted to Prs K. Unterrainer, G. Strasser, J. Wang, L. Esaki, K. Hirakawa, Y. Arakawa, H. Sakaki, A. Wacker, C. Sirtori, A. Vasanelli, Y. Guldner, C. Delalande, Ph. Roussignol, J.M. Berroir, P. Voisin, M. Voos and, above all, R. Ferreira.

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contributions to the absorption spectrum lineshape and linewidth of disordered quantum cascade structures; the electronic properties of Dirac superlattices and of heterostructures with no-common anions.

Camille Ndebeka-Bandou is a Post-doctoral Researcher and Teaching Assistant at the Institute of Quantum Electronics of the Swiss Federal Institute of Technology (ETH Zürich) and she does her research in the Quantum Optoelectronics Group of Pr. Jérôme Faist. Dr. Ndebeka-Bandou was born in 1987 in Guadeloupe. She studied Physics in Paris (France), graduated at the Université Paris Diderot in 2011 and obtained her PhD degree in 2014 with a theoretical study of the electron states in disordered semiconductor heterostructures, notably the quantum cascade structures. Since 2014, she has carried out various research projects at ETH Zürich such as the theoretical investigation of the loss mechanisms in terahertz quantum cascade lasers, the ultra-strong coupling in metamaterial cavities and the theoretical study of the bulk states in disordered two-dimensional topological insulators.

Contents

<i>Foreword</i>	v
<i>About the Authors</i>	ix
Part I Practical Quantum Mechanics	1
I.1 Schrödinger Equation	3
I.1.1 Eigenvalues, eigenvector of linear operators	6
I.1.2 Adjoint operator	7
I.1.3 Hermitian operators: observables	8
I.1.4 Unitary operators	8
I.1.5 Projectors	9
I.1.6 Commuting operators	9
I.1.7 Two important examples of non-commuting operators	13
I.1.8 Heisenberg inequalities	14
I.1.9 Spin	15
I.1.10 Spin-orbit coupling	16
I.1.11 Density of states	17
I.1.12 Identical particles and Pauli principle	20
I.1.13 Tensorial products	24
I.2 Bound and Extended States	27
I.2.1 Propagating and evanescent states	27
I.2.2 Probability current	30

I.2.3	Boundary conditions	31
I.2.4	Bound states	39
I.2.5	The problem of plane waves	48
I.2.6	Schrödinger equation, time-dependent aspects . . .	52
I.3	Approximate Methods	57
I.3.1	Variational method	57
I.3.2	Perturbation theory	60
I.3.2.1	Non-degenerate perturbation theory	62
I.3.2.2	Degenerate perturbation theory	67
I.3.3	Time-dependent perturbation theory	70
I.3.3.1	Static scatterers	71
I.3.3.2	Time-dependent scattering	77
I.4	Landau Quantisation of Electron Motion in Ideal Semiconductor Bulks and Heterostructures	83
I.4.1	Landau level degeneracy	88
I.4.2	Perturbative estimates of δH_1 and δH_2	89
I.4.3	Magnetic field-dependent density of states	91
I.4.4	A tractable case of lifting of the k_y degeneracy: the crossed E , B fields	93
Part II	The Physics of Heterostructures	95
II.1	Background on Heterostructures	97
II.2	Electrons States in Nanostructures	103
II.2.1	The envelope function approximation	103
II.2.1.1	Introduction	103
II.2.1.2	Electronic states in bulk semiconductors . . .	108
II.2.1.3	Heterostructure states	115
II.2.2	Multiple quantum wells: transfer matrix method	120
II.2.2.1	Multiple quantum wells and superlattices	120

II.2.2.2	Transfer matrix method	122
II.2.3	Double quantum wells	126
II.2.3.1	Tight binding analysis	131
II.2.3.2	Symmetrical double quantum well	134
II.2.4	Holes	135
II.3	Beyond the Ideal World	139
II.3.1	Population, velocity, energy relaxation times through rate equations	147
II.3.2	Rate equations with elastic and inelastic processes	151
II.3.3	Analysis of the relaxation times in rate equations	154
II.3.3.1	Impurity form factor	155
II.3.3.2	Phonon form factors and transition rates	158
II.3.4	Consequence of the Born approximation on the additivity of scattering frequencies	162
II.4	Screening at the Semi-classical Approximation	165
II.4.1	Case of a single subband occupation	166
II.4.2	Case of many subbands occupation	171
II.4.3	Screening of inter-subband matrix elements	173
II.5	Results for Static Scatterers	177
II.5.1	Scattering by static disorder	177
II.5.2	Scattering of composite particles/excitons at the Born approximation	184
II.5.3	Scattering on magnetic impurities	193
II.5.3.1	The “spin”-flip scattering of electrons	193
II.5.3.2	The “spin”-flip scattering of holes	201
II.5.4	Three-body collisions	205
II.5.4.1	FCA in imperfect bulks and heterostructures	206
II.5.4.2	Phonon scattering in the presence of static scatterers	218

II.6	Results for Electron–Phonon Interaction	223
II.6.1	Optical phonon scattering	224
II.6.2	Acoustical phonon scattering	229
II.6.3	Energy loss rate	231
II.7	Beyond the Born Approximation	235
II.7.1	Scattering between Landau levels	238
Part III	Exercises	255
1	Average position and velocity	257
2	Average velocity in a bound state	257
3	Density of states	258
4	Density of states of a camel back shaped dispersion relation	260
5	Heisenberg inequality in a quantum well with infinitely high barriers	263
6	Manipulating Slater determinants	266
7	Pauli principle for two weakly interacting electrons in 1D	267
8	Calculation with Pauli matrices	270
9	Moss–Burstein shift of interband absorption	271
10	Virial theorem	273
11	Absence of degeneracy for the 1D bound states	276
12	Variational method: hydrogen atom	278
13	Variational method: electron in a triangular potential	279
14	Variational method: anharmonic oscillator	280
15	Screened coulombic bound states	281
16	A two-dimensional coulombic problem	287
17	Inter-subband transitions in cubic GaN/AlN quantum wells: information on the conduction band offset	290
18	Asymmetrical square quantum well	294
19	Spherical quantum dots	296
20	Delta quantum well	298