

OPTICAL SCIENCES

W. Koechner

# Solid-State Laser Engineering

## 固态激光工程

第5版

Fifth Revised and Updated Edition

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Walter Koechner

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Fifth Revised and Updated Edition

With 472 Figures and 56 Tables

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藏书章

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## Preface to the Fifth Edition

This book, written from an industrial vantage point, provides a detailed discussion of solid-state lasers, their characteristics, design and construction, and practical problems. The title *Solid-State Laser Engineering* has been chosen because the emphasis is placed on engineering and practical considerations of solid-state lasers. I have tried to enhance the description of the engineering aspects of laser construction and operation by including numerical and technical data, tables, and curves.

The book is mainly intended for the practicing scientist or engineer who is interested in the design or use of solid-state lasers, but the response from readers has shown that the comprehensive treatment of the subject makes the work useful also to students of laser physics who want to supplement their theoretical knowledge with the engineering aspects of lasers. Although not written in the form of a college textbook, the book might be used in an advanced college course on laser technology.

The aim was to present the subject as clearly as possible. Phenomenological descriptions using models were preferred to an abstract mathematical presentation, even though many simplifications had then to be accepted. Results are given in most cases without proof since I have tried to stress the application of the results rather than the derivation of the formulas. An extensive list of references is cited for each chapter to permit the interested reader to learn more about a particular subject.

Again, gratified by the wide acceptance of the previous edition of *Solid-State Laser Engineering*, I have updated and revised the fifth edition to include developments and concepts which have emerged during the last several years. Since the publication of the fourth edition, continued dramatic changes have taken place in the development of solid-state lasers. Today, systems range from tiny, diode-pumped micro-chip lasers to stadium-sized Nd:glass lasers under construction at the National Ignition Facility. The combination of diode-pump sources with innovative pump and resonator designs has dramatically improved beam quality obtainable from solid-state lasers. Spectral coverage, and output power at different wavelengths, have been considerably increased as a result of the emergence of improved nonlinear crystals. Also, table-top femtosecond laser sources have become a reality. At the high end of the power range, flashlamp-pumped Nd:YAG lasers up to the 5 kW level are employed for welding applications, and a number of diode-pumped lasers with outputs in the kW range have been demonstrated at various laboratories. For military and spaceborne systems, where compact pack-

aging and low power consumption is of critical importance, diode-pumped lasers have become an enabling technology for many applications.

Like in previous editions, obsolete material has been deleted and new information has been added. In particular, the following areas have been expanded:

- In the chapter on laser materials (Chap. 2), several crystals such as Nd:YVO<sub>4</sub>, Yb:YAG and Tm:YAG are discussed in more detail. Although developed many years ago, only in combination with diode-pumping have these crystals become attractive for a number of applications.
- Recent developments of high-voltage and current semiconductor switches are incorporated in the chapter on pump sources (Chap. 6). These devices provide controllable on-off switches for flashlamps under certain operating conditions.
- The chapter on passive Q-switches (Chap. 8) has been rewritten because the emergence of crystals doped with absorbing ions or containing color centers has greatly improved the durability and reliability of passive Q-switches.
- The chapter on nonlinear devices (Chap. 10) has been expanded to include periodically-poled crystals such as LiNbO<sub>3</sub>. Incorporation of these crystals in OPO's has considerably increased the output of mid-infrared lasers.
- The chapter on laser-induced damage to optical components (Chap. 11) has been completely rewritten to include the latest findings and results of coatings and bulk damage. Also, a section on self-focusing as it relates to high-brightness solid-state lasers has been added. Self-focusing, a major design consideration in large Nd:glass lasers, and an effect exploited in Kerr lens mode-locking, can be an issue in high brightness diode-pumped lasers.

The material presented in this book reflects the author's experience gained in directing solid-state laser R&D over a 30-year period. This book would not have been possible without the many contributions to the field of laser engineering that have appeared in the open literature and which have been used here as the basic source material. I apologize to any of my colleagues whose work has not been acknowledged or adequately represented in this book.

It is impossible to describe all the new materials, laser configurations, and pumping schemes which have been developed over the last several years. In doing so, the book would merely become a literature survey with commentaries. Readers interested in specific devices are referred to the original literature. Very good sources of information are the IEEE Journal of Quantum Electronics, Optics Letters, and the conference proceedings of the CLEO and Solid-State Laser Conferences.

My special thanks are due to Mindy Levenson, for typing the new material, and to the editor, Dr. H. Lotsch, for his support in preparing the new edition for printing.

None of the editions of this book could have been written without the encouragement, patience and support of my wife Renate.

Herndon, VA  
May 1999

*Walter Koechner*

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# 1. Introduction

In this introductory chapter we shall outline the basic ideas underlying the operation of solid-state lasers. In-depth treatments of laser physics can be found in a number of excellent textbooks [1.1].

## 1.1 Optical Amplification

To understand the operation of a laser we have to know some of the principles governing the interaction of radiation with matter.

Atomic systems such as atoms, ions, and molecules can exist only in discrete energy states. A change from one energy state to another, called a transition, is associated with either the emission or the absorption of a photon. The wavelength of the absorbed or emitted radiation is given by Bohr's frequency relation

$$E_2 - E_1 = h\nu_{21} , \quad (1.1)$$

where  $E_2$  and  $E_1$  are two discrete energy levels,  $\nu_{21}$  is the frequency, and  $h$  is Planck's constant. An electromagnetic wave whose frequency  $\nu_{21}$  corresponds to an energy gap of such an atomic system can interact with it. To the approximation required in this context, a solid-state material can be considered an ensemble of very many identical atomic systems. At thermal equilibrium, the lower energy states in the material are more heavily populated than the higher energy states. A wave interacting with the substance will raise the atoms or molecules from lower to higher energy levels and thereby experience absorption.

The operation of a laser requires that the energy equilibrium of a laser material be changed such that energy is stored in the atoms, ions, or molecules of this material. This is achieved by an external pump source which transfers electrons from a lower energy level to a higher one. The pump radiation thereby causes a "population inversion." An electromagnetic wave of appropriate frequency, incident on the "inverted" laser material, will be amplified because the incident photons cause the atoms in the higher level to drop to a lower level and thereby emit additional photons. As a result, energy is extracted from the atomic system and supplied to the radiation field. The release of the stored energy by interaction with an electromagnetic wave is based on stimulated or induced emission.

Stated very briefly, when a material is excited in such a way as to provide more atoms (or molecules) in a higher energy level than in some lower level, the material will be capable of amplifying radiation at the frequency corresponding

to the energy level difference. The acronym "laser" derives its name from this process: "Light Amplification by Stimulated Emission of Radiation."

A quantum mechanical treatment of the interaction between radiation and matter demonstrates that the stimulated emission is, in fact, completely indistinguishable from the stimulating radiation field. This means that the stimulated radiation has the same directional properties, same polarization, same phase, and same spectral characteristics as the stimulating emission. These facts are responsible for the extremely high degree of coherence which characterizes the emission from lasers. The fundamental nature of the induced or stimulated emission process was already described by A. Einstein and M. Planck.

In solid-state lasers, the energy levels and the associated transition frequencies result from the different quantum energy levels or allowed quantum states of the electrons orbiting about the nuclei of atoms. In addition to the electronic transitions, multiatom molecules in gases exhibit energy levels that arise from the vibrational and rotational motions of the molecule as a whole.

## 1.2 Interaction of Radiation with Matter

Many of the properties of a laser may be readily discussed in terms of the absorption and emission processes which take place when an atomic system interacts with a radiation field. In the first decade of this century Planck described the spectral distribution of thermal radiation, and in the second decade Einstein, by combining Planck's law and Boltzmann statistics, formulated the concept of stimulated emission. Einstein's discovery of stimulated emission provided essentially all of the theory necessary to describe the physical principle of the laser.

### 1.2.1 Blackbody Radiation

When electromagnetic radiation in an isothermal enclosure, or cavity, is in thermal equilibrium at temperature  $T$ , the distribution of radiation density  $\rho(\nu) d\nu$ , contained in a bandwidth  $d\nu$ , is given by Planck's law

$$\rho(\nu) d\nu = \frac{8\pi\nu^2 d\nu}{c^3} \frac{h\nu}{e^{h\nu/kT} - 1}, \quad (1.2)$$

where  $\rho(\nu)$  is the radiation density per unit frequency [ $\text{Js}/\text{cm}^3$ ],  $k$  is Boltzmann's constant, and  $c$  is the velocity of light. The spectral distribution of thermal radiation vanishes at  $\nu = 0$  and  $\nu \rightarrow \infty$ , and has a peak which depends on the temperature.

The factor

$$\frac{8\pi\nu^2}{c^3} = p_n \quad (1.3)$$

in (1.2) gives the density of radiation modes per unit volume and unit frequency interval. The factor  $p_n$  can also be interpreted as the number of degrees of freedom associated with a radiation field, per unit volume, per unit frequency interval. The expression for the mode density  $p_n$  [modes  $\text{s/cm}^3$ ] plays an important role in connecting the spontaneous and the induced transition probabilities.

For a uniform, isotropic radiation field, the following relationship is valid

$$W = \frac{\varrho(\nu)c}{4}, \quad (1.4)$$

where  $W$  is the blackbody radiation [ $\text{W/cm}^2$ ] which will be emitted from an opening in the cavity of the blackbody. Many solids radiate like a blackbody. Therefore, the radiation emitted from the surface of a solid can be calculated from (1.4).

According to the Stefan-Boltzmann equation, the total black body radiation is

$$W = \sigma T^4, \quad (1.5)$$

where  $\sigma = 5.68 \times 10^{-12} \text{ W/cm}^2 \text{ K}^4$ . The emitted radiation  $W$  has a maximum which is obtained from Wien's displacement law

$$\frac{\lambda_{\max}}{\mu\text{m}} = \frac{2893}{T/\text{K}}. \quad (1.6)$$

For example, a blackbody at a temperature of 5200 K has its radiation peak at 5564 Å, which is about the center of the visible spectrum.

A good introduction to the fundamentals of radiation and its interaction with matter can be found in [1.2].

### 1.2.2 Boltzmann's Statistics

According to a basic principle of statistical mechanics, when a large collection of similar atoms is in thermal equilibrium at temperature  $T$ , the relative populations of any two energy levels  $E_1$  and  $E_2$ , such as the ones shown in Fig. 1.1, must be related by the Boltzmann ratio

$$\frac{N_2}{N_1} = \exp\left(\frac{-(E_2 - E_1)}{kT}\right), \quad (1.7)$$

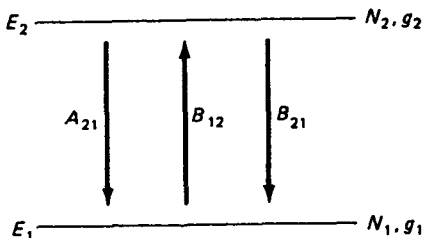


Fig. 1.1. Two energy levels with population  $N_1$ ,  $N_2$  and degeneracies  $g_1$ ,  $g_2$ , respectively

where  $N_1$  and  $N_2$  are the number of atoms in the energy levels  $E_1$  and  $E_2$ , respectively. For energy gaps large enough that  $E_2 - E_1 = h\nu_{21} \gg kT$ , the ratio is close to zero, and there will be very few atoms in the upper energy level at thermal equilibrium. The thermal energy  $kT$  at room temperature ( $T \approx 300$  K) corresponds to an energy gap  $h\nu$  with  $\nu \approx 6 \times 10^{12}$  Hz, which is equivalent in wavelength to  $\lambda \approx 50 \mu\text{m}$ . Therefore, for any energy gap whose transition frequency  $\nu_{21}$  lies in the near-infrared or visible regions, the Boltzmann exponent will be very small at normal temperatures. The number of atoms in any upper level will then be very small compared to the lower levels. For example, in ruby the ground level  $E_1$  and the upper laser level  $E_2$  are separated by an energy gap corresponding to a wavelength of  $\lambda \approx 0.69 \mu\text{m}$ . Since  $h = 6.6 \times 10^{-34}$  Ws<sup>2</sup>, then  $E_2 - E_1 = h\nu = 2.86 \times 10^{-19}$  Ws. With  $k = 1.38 \times 10^{-23}$  Ws K and  $T = 300$  K, it follows that  $N_2/N_1 \approx \exp(-69)$ . Therefore at thermal equilibrium virtually all the atoms will be in the ground level.

Equation (1.7) is valid for atomic systems having only non-degenerate levels. If there are  $g_i$  different states of the atom corresponding to the energy  $E_i$ , then  $g_i$  is defined as the degeneracy of the  $i$ th energy level.

We recall that atomic systems, such as atoms, ions, molecules, can exist only in certain stationary states, each of which corresponds to a definite value of energy and thus specifies an energy level. When two or more states have the same energy, the respective level is called degenerate, and the number of states with the same energy is the multiplicity of the level. All states of the same energy level will be equally populated, therefore the number of atoms in levels 1 and 2 is  $N_1 = g_1 N'_1$  and  $N_2 = g_2 N'_2$ , where  $N'_1$  and  $N'_2$  refer to the population of any of the states in levels 1 and 2, respectively. It follows then from (1.7) that the populations of the energy levels 1 and 2 are related by the formula

$$\frac{N_2}{N_1} = \frac{g_2}{g_1} \frac{N'_2}{N'_1} = \frac{g_2}{g_1} \exp \left( \frac{-(E_2 - E_1)}{kT} \right). \quad (1.8)$$

At absolute zero temperature, Boltzmann's statistics predicts that all atoms will be in the ground state. Thermal equilibrium at any temperature requires that a state with a lower energy be more densely populated than a state with a higher energy. Therefore  $N_2/N_1$  is always less than unity for  $E_2 > E_1$  and  $T > 0$ . This means that optical amplification is not possible in thermal equilibrium.

### 1.2.3 Einstein's Coefficients

We can most conveniently introduce the concept of Einstein's  $A$  and  $B$  coefficients by loosely following Einstein's original derivation. To simplify the discussion, let us consider an idealized material with just two nondegenerate energy levels, 1 and 2, having populations of  $N_1$  and  $N_2$ , respectively. The total number of atoms in these two levels is assumed to be constant

$$N_1 + N_2 = N_{\text{tot}}. \quad (1.9)$$

Radiative transfer between the two energy levels which differ by  $E_2 - E_1 = h\nu_{21}$  is allowed. The atom can transfer from state  $E_2$  to the ground state  $E_1$  by emitting energy; conversely, transition from state  $E_1$  to  $E_2$  is possible by absorbing energy. The energy removed or added to the atom appears as quanta of  $h\nu_{21}$ . We can identify three types of interaction between electromagnetic radiation and a simple two-level atomic system:

**Absorption.** If a quasimonochromatic electromagnetic wave of frequency  $\nu_{21}$  passes through an atomic system with energy gap  $h\nu_{21}$ , then the population of the lower level will be depleted at a rate proportional both to the radiation density  $\varrho(\nu)$  and to the population  $N_1$  of that level

$$\frac{\partial N_1}{\partial t} = -B_{12}\varrho(\nu)N_1, \quad (1.10)$$

where  $B_{12}$  is a constant of proportionality with dimensions  $\text{cm}^3/\text{s}^2 \text{ J}$ .

The product  $B_{12}\varrho(\nu)$  can be interpreted as the probability per unit frequency that transitions are induced by the effect of the field.

**Spontaneous Emission.** After an atom has been raised to the upper level by absorption, the population of the upper level 2 decays spontaneously to the lower level at a rate proportional to the upper level population.

$$\frac{\partial N_2}{\partial t} = -A_{21}N_2, \quad (1.11)$$

where  $A_{21}$  is a constant of proportionality with the dimensions  $\text{s}^{-1}$ . The quantity  $A_{21}$ , being a characteristic of the pair of energy levels in question, is called the spontaneous transition probability because this coefficient gives the probability that an atom in level 2 will spontaneously change to a lower level 1 within a unit of time.

Spontaneous emission is a statistical function of space and time. With a large number of spontaneously emitting atoms there is no phase relationship between the individual emission processes; the quanta emitted are incoherent. Spontaneous emission is characterized by the lifetime of the electron in the excited state, after which it will spontaneously return to the lower state and radiate away the energy. This can occur without the presence of an electromagnetic field.

Equation (1.11) has a solution

$$N_2(t) = N_2(0) \exp\left(\frac{-t}{\tau_{21}}\right), \quad (1.12)$$

where  $\tau_{21}$  is the lifetime for spontaneous radiation of level 2. This radiation lifetime is equal to the reciprocal of the Einstein's coefficient,

$$\tau_{21} = A_{21}^{-1}. \quad (1.13)$$

In general, the reciprocal of the transition probability of a process is called its lifetime.

**Stimulated Emission.** Emission takes place not only spontaneously but also under stimulation by electromagnetic radiation of appropriate frequency. In this case, the atom gives up a quantum to the radiation field by "induced emission" according to

$$\frac{\partial N_2}{\partial t} = -B_{21}\varrho(\nu_{21})N_2, \quad (1.14)$$

where  $B_{21}$  again is a constant of proportionality.

Radiation emitted from an atomic system in the presence of external radiation consists of two parts. The part whose intensity is proportional to  $A_{21}$  is the spontaneous radiation; its phase is independent of that of the external radiation. The part whose intensity is proportional to  $\varrho(\nu)B_{21}$  is the stimulated radiation; its phase is the same as that of the stimulating external radiation.

The probability of induced transition is proportional to the energy density of external radiation in contrast to spontaneous emission. In the case of induced transition there is a firm phase relationship between the stimulating field and the atom. The quantum which is emitted to the field by the induced emission is coherent with it.

But we shall see later, the useful parameter for laser action is the  $B_{21}$  coefficient; the  $A_{21}$  coefficient represents a loss term and introduces into the system photons that are not phase-related to the incident photon flux of electric field. Thus the spontaneous process represents a noise source in a laser.

If we combine absorption, spontaneous, and stimulated emission, as expressed by (1.10, 11, and 14), we can write for the change of the upper and lower level populations in our two-level model

$$\frac{\partial N_1}{\partial t} = -\frac{\partial N_2}{\partial t} = B_{21}\varrho(\nu)N_2 - B_{12}\varrho(\nu)N_1 + A_{21}N_2. \quad (1.15)$$

The relation

$$\frac{\partial N_1}{\partial t} = -\frac{\partial N_2}{\partial t} \quad (1.16)$$

follows from (1.9).

In thermal equilibrium, the number of transitions per unit time from  $E_1$  to  $E_2$  must be equal to the number of transitions from  $E_2$  to  $E_1$ . Certainly, in thermal equilibrium

$$\frac{\partial N_1}{\partial t} = \frac{\partial N_2}{\partial t} = 0. \quad (1.17)$$

Therefore we can write

$$\begin{array}{ccccc} N_2 A_{21} & + & N_2 \varrho(\nu) B_{21} & = & N_1 \varrho(\nu) B_{12} \\ \text{Spontaneous} & & \text{Stimulated} & & \text{Absorption} \\ \text{emission} & & \text{emission} & & \end{array} \quad (1.18)$$



Using the Boltzmann equation (1.8) for the ratio  $N_2/N_1$ , we then write the above expression as

$$\rho(\nu_{21}) = \frac{(A_{21}/B_{21})}{(g_1/g_2)(B_{12}/B_{21}) \exp(h\nu_{21}/kT) - 1} \quad (1.19)$$

Comparing this expression with the black body radiation law (1.2), we see that

$$\frac{A_{21}}{B_{21}} = \frac{8\pi\nu^2 h\nu}{c^3} \quad \text{and} \quad B_{21} = \frac{g_1 B_{12}}{g_2} \quad (1.20)$$

The relations between the  $A$ 's and  $B$ 's are known as Einstein's relations. The factor  $8\pi\nu^2/c^3$  in (1.20) is the mode density  $p_n$  given by (1.3).

In solids the speed of light is  $c = c_0/n$ , where  $n$  is the index of refraction and  $c_0$  is the speed of light in vacuum.

For a simple system with no degeneracy, that is, one in which  $g_1 = g_2$ , we see that  $B_{21} = B_{12}$ . Thus, the Einstein coefficients for stimulated emission and absorption are equal. If the two levels have unequal degeneracy, the probability for stimulated absorption is no longer the same as that for stimulated emission.

### 1.2.4 Phase Coherence of Stimulated Emission

The stimulated emission provides a phase-coherent amplification mechanism for an applied signal. The signal extracts from the atoms a response that is directly proportional to, and phase-coherent with, the electric field of the stimulating signal. Thus the amplification process is phase-preserving. The stimulated emission is, in fact, completely indistinguishable from the stimulating radiation field. This means that the stimulated emission has the same directional properties, same polarization, same phase, and same spectral characteristics as the stimulating emission. These facts are responsible for the extremely high degree of coherence which characterizes the emission from lasers. The proof of this fact is beyond the scope of this elementary introduction, and requires a quantum mechanical treatment of the interaction between radiation and matter. However, the concept of induced transition, or the interaction between a signal and an atomic system, can be demonstrated, qualitatively, with the aid of the classical electron-oscillator model.

Electromagnetic radiation interacts with matter through the electric charges in the substance. Consider an electron which is elastically bound to a nucleus. One can think of electrons and ions held together by spring-type bonds which are capable of vibrating around equilibrium positions. An applied electric field will cause a relative displacement between electron and nucleus from their equilibrium position. They will execute an oscillatory motion about their equilibrium position. Therefore, the model exhibits an oscillatory or resonant behavior and a response to an applied field. Since the nucleus is so much heavier than the electron, we assume that only the electron moves. The most important model for understanding the interaction of light and matter is that of the harmonic oscillator. We take as