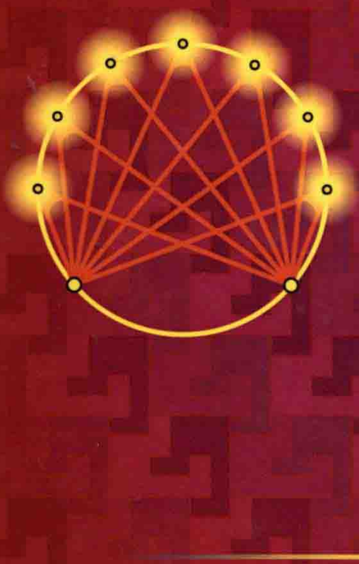
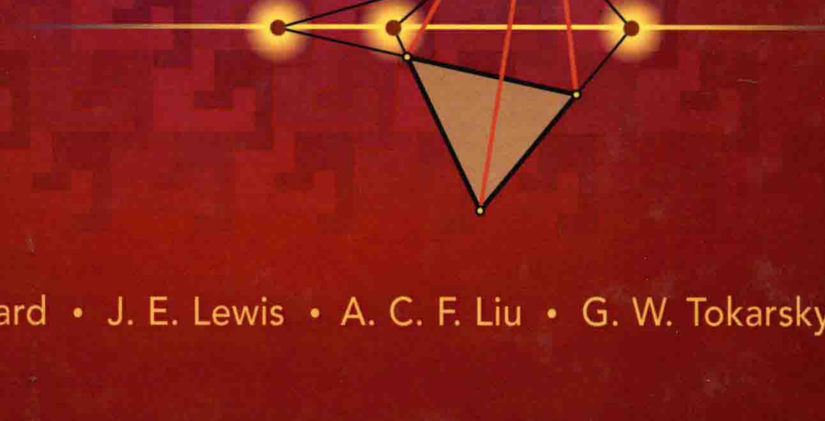
Classical Geometry

Euclidean, Transformational, Inversive, and Projective





I. E. Leonard • J. E. Lewis • A. C. F. Liu • G. W. Tokarsky

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CLASSICAL GEOMETRY

PREFACE

It is sometimes said that geometry should be studied because it is a useful and valuable discipline, but in fact many people study it simply because geometry is a very enjoyable subject. It is filled with problems at every level that are entertaining and elegant, and this enjoyment is what we have attempted to bring to this textbook.

This text is based on class notes that we developed for a three-semester sequence of undergraduate geometry courses that has been taught at the University of Alberta for many years. It is appropriate for students from all disciplines who have previously studied high school algebra, geometry, and trigonometry.

When we first started teaching these courses, our main problem was finding a suitable method for teaching geometry to university students who have had minimal experience with geometry in high school. We experimented with material from high school texts but found it was not challenging enough. We also tried an axiomatic approach, but students often showed little enthusiasm for proving theorems, particularly since the early theorems seemed almost as self-evident as the axioms. We found the most success by starting early with problem solving, and this is the approach we have incorporated throughout the book.

The geometry in this text is synthetic rather than Cartesian or coordinate geometry. We remain close to classical themes in order to encourage the development of geometric intuition, and for the most part we avoid abstract algebra although we do demonstrate its use in the sections on transformational geometry.

Part I is about Euclidean geometry; that is, the study of the properties of points and lines that are invariant under isometries and similarities. As well as many of the usual topics, it includes material that many students will not have seen, for example, the theorems of Ceva and Menelaus and their applications. Part I is the basis for Parts II and III.

Part II discusses the properties of Euclidean transformations or isometries of the plane (translations, reflections, and rotations and their compositions). It also introduces groups and their use in studying transformations.

Part III introduces inversive and projective geometry. These subjects are presented as natural extensions of Euclidean geometry, with no abstract algebra involved.

We would like to acknowledge our late colleagues George Cree and Murray Klamkin, without whose inspiration and encouragement over the years this project would not have been possible.

Finally, we would like to thank our families for their patience and understanding in the preparation of the textbook. In particular, I. E. Leonard would like to thank Sarah for proofreading the manuscript numerous times.

ED, TED, ANDY, AND GEORGE

Edmonton, Alberta, Canada

January, 2014

CONTENTS

PART I

EUCLIDEAN GEOMETRY

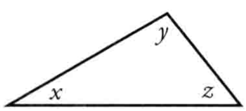
CHAPTER 1

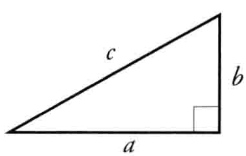
CONGRUENCY

1.1 Introduction

Assumed Knowledge

This text assumes a bit of knowledge on the part of the reader. For example, it assumes that you know that the sum of the angles of a triangle in the plane is 180° ($x + y + z = 180^\circ$ in the figure below), and that in a right triangle with hypotenuse c and sides a and b, the Pythagorean relation holds: $c^2 = a^2 + b^2$.





Classical Geometry: Euclidean, Transformational, Inversive and Projective, First Edition. By
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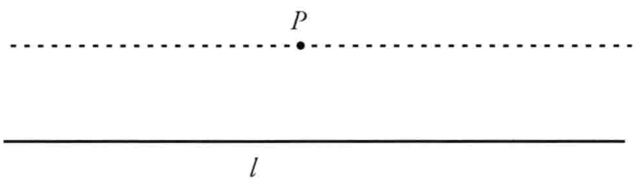
We use the word ***line*** to mean *straight line*, and we assume that you know that two lines either do not intersect, intersect at exactly one point, or completely coincide. Two lines that do not intersect are said to be ***parallel***.

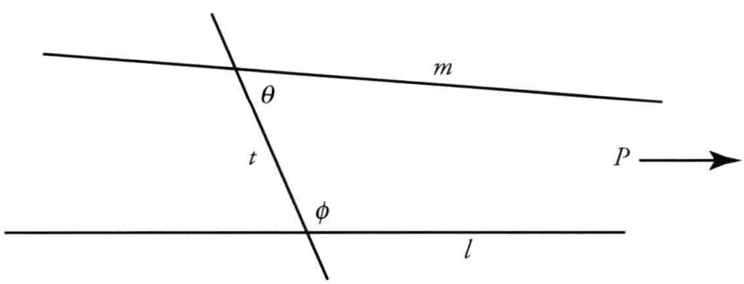
We also assume certain knowledge about parallel lines, namely, that you have seen some form of the ***parallel axiom***:

Given a line l and a point P in the plane, there is exactly one line through P parallel to l.



The preceding version of the parallel axiom is often called ***Playfair's Axiom***. You may even know something equivalent to it that is close to the original version of the ***parallel postulate***:

Given two lines l and m, and a third line t cutting both l and m and forming angles ϕ and θ on the same side of t, if $\phi + \theta < 180°$, then l and m meet at a point on the same side of t as the angles.



The subject of this part of the text is Euclidean geometry, and the above-mentioned parallel postulate characterizes Euclidean geometry. Although the postulate may seem to be obvious, there are perfectly good geometries in which it does not hold.