Handbook of Geophysical Exploration: Seismic Exploration

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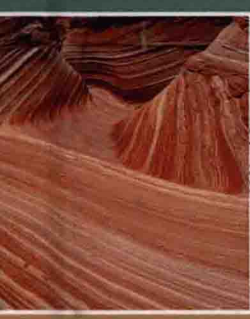
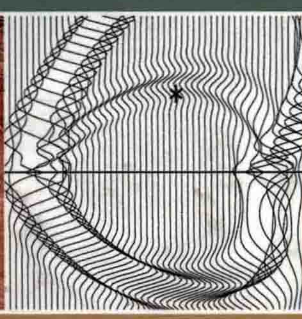
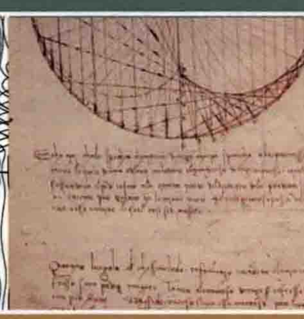
THIRD EDITION

WAVE FIELDS IN REAL MEDIA

Wave Propagation in Anisotropic, Anelastic, Porous and Electromagnetic Media







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Wave Fields in Real Media

Wave Propagation in Anisotropic, Anelastic, Porous and Electromagnetic Media

Third edition

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Wave Fields in Real Media

⟨⟨ L'impeto ⟩⟩ cioè la propagazione della perturbazione del mezzo o, più in generale, di un qualsiasi elemento saliente ⟨⟨ è molto più veloce che ll'acqua, perché molte sono le volte che l'onda fuggie il locho della sua creatione, e ll'acqua non si muove di sito, a ssimilitudine delle onde fatte il maggio nelle biade dal corso de venti, che ssi vede correre l'onde per le campagnie, e le biade non si mutano di lor sito ⟩⟩.

⟨⟨ The impetus ⟩⟩ that is, the propagation of the perturbation of the medium or, more generally, of any salient element ⟨⟨ is much faster than the water, because many are the times that the wave escapes the place of its creation, and water stays in place, as the waves made in May in the corn by the blowing of the wind, so that one can see the running waves in the fields and the corn does not change place ⟩⟩.

Leonardo da Vinci (Del moto e misura dell'acqua)

Preface

This book presents the fundamentals of wave propagation in anisotropic, anelastic and porous media, including electromagnetic waves. This new edition incorporates research work performed during the last seven years on several relevant topics, which have been distributed in the various chapters. The emphasis is on geophysical applications for hydrocarbon exploration, but researchers in the fields of earthquake seismology, rock acoustics and material science – including many branches of acoustics of fluids and solids (acoustics of materials, non-destructive testing, etc.) – may also find this text useful. This book can be considered, in part, a monograph, since much of the material represents my own original work on wave propagation in anisotropic, viscoelastic media. Although it is biased to my scientific interests and applications, I have, nevertheless, sought to retain the generality of the subject matter, in the hope that the book will be of interest and use to a wide readership.

The concepts of anisotropy, anelasticity[1] and poroelasticity in physical media have gained much attention in recent years. The applications of these studies cover a variety of fields, including physics and geophysics, engineering and soil mechanics, underwater acoustics, etc. In particular, in the exploration of oil and gas reservoirs, it is important to predict the rock porosity, the presence of fluids (type and saturation), the preferential directions of fluid flow (anisotropy), the presence of abnormal pore-pressures (overpressure), etc. These microstructural properties and *in situ* rock conditions can be obtained, in principle, from seismic and electromagnetic properties, such as travel times, amplitude information and wave polarization. These measurable quantities are affected by the presence of anisotropy and attenuation mechanisms. For instance, shales are naturally bedded and possess intrinsic anisotropy at the microscopic level. Similarly, compaction and the presence of microcracks and fractures make the skeleton of porous rocks anisotropic. The presence of fluids implies relaxation phenomena, which causes wave dissipation. The use of modelling and inversion for the interpretation of the wave response of reservoir rocks requires an understanding of the relationship between the seismic and electromagnetic properties and the

1. The term anelasticity seems to have been introduced by Zener (1948) to denote materials in which "strain may lag behind stress in periodic vibrations", in which no permanent deformation occurs and wherein the stress–strain relation is linear. Viscoelasticity combines the classical theories of elasticity and Newtonian fluids, but is not restricted to linear behaviour. Since this book deals with linear deformations, anelasticity and viscoelasticity will be synonymous herein.

rock characteristics, such as permeability, porosity, tortuosity, fluid viscosity, stiffness, dielectric permittivity, electrical conductivity, etc.

Wave simulation is a theoretical field of research that began nearly four decades ago, in close relationship with the development of computer technology and numerical algorithms for solving differential and integral equations of several variables. In the field of research known as computational physics, algorithms for solving problems using computers are important tools that provide insight into wave propagation for a variety of applications.

In this book, I examine the differences between an ideal and a real description of wave propagation, where ideal means an elastic (lossless), isotropic and single-phase medium, and real means an anelastic, anisotropic and multi-phase medium. The first realization is, of course, a particular case of the second, but it must be noted that, in general, the real description is not a simple and straightforward extension of the ideal description.

The analysis starts by introducing the constitutive equation (stress–strain relation) appropriate for the particular rheology.[2] This relation and the equations of conservation of linear momentum are combined to give the equation of motion, a second-order or a first-order matrix differential equation in time, depending on the formulation of the field variables. The differential formulation for lossy media is written in terms of memory (hidden) variables or alternatively, fractional derivatives. Biot theory is essential to describe wave propagation in multi-phase (porous) media from the seismic to the ultrasonic frequency range, representative of field and laboratory experiments, respectively. The acoustic–electromagnetic analogy reveals that the different physical phenomena have the same mathematical formulation. For each constitutive equation, a plane-wave analysis is performed in order to understand the physics of the wave propagation (i.e., calculation of phase, group and energy velocities, and quality and attenuation factors). For some cases, it is possible to obtain an analytical solution for transient wave fields in the space-frequency domain, which is then transformed to the time domain by a numerical Fourier transform. The book concludes with a review of the so-called direct numerical methods for solving the equations of motion in the time-space domain. The plane-wave theory and the analytical solutions serve to test the performance (accuracy and limitations) of the modelling codes.

A brief description of the main concepts discussed in this book follows.

Chapter 1: Anisotropic Elastic Media. In anisotropic lossless media, the directions of the wavevector and Umov–Poynting vector (ray or energy-flow vector) do not coincide. This implies that the phase and energy velocities differ. However, some ideal properties prevail: there is no dissipation, the

2. From the Greek ρεῶ – to flow, and λογός – word, science. Today, rheology is the science concerned with the behaviour of real materials under the influence of external stresses.

group-velocity vector is equal to the energy-velocity vector, the wavevector is normal to the wave-front surface, the energy-velocity vector is normal to the slowness surface, plane waves are linearly polarized and the polarization of the different wave modes are mutually orthogonal. Methods used to calculate these quantities and provide the equation of motion for inhomogeneous media are shown. I also consider the seismic properties of finely stratified media composed of anisotropic layers, anomalously polarized media and the best isotropic approximation of anisotropic media. Finally, the analysis of a reflection–transmission problem and analytical solutions along the symmetry axis of a transversely isotropic medium are discussed.

Chapter 2: Viscoelasticity and Wave Propagation. Attenuation is introduced in the form of Boltzmann superposition law, which implies a convolutional relation between the stress and strain tensors through the relaxation and creep matrices. The analysis is restricted to the one-dimensional case, where some of the consequences of anelasticity become evident. Although phase and energy velocities are the same, the group velocity loses its physical meaning. The concept of centrovelocity for non-harmonic waves is discussed. The uncertainty in defining the strain and rate of dissipated-energy densities is overcome by introducing relaxation functions based on mechanical models. The concepts of memory variable and fractional derivative are introduced to avoid time convolutions and obtain a time-domain differential formulation of the equation of motion.

Chapter 3: Isotropic Anelastic Media. The space dimension reveals other properties of anelastic (viscoelastic) wave fields. There is a distinct difference between the inhomogeneous waves of lossless media (interface waves) and those of viscoelastic media (body waves). In the former case, the direction of attenuation is normal to the direction of propagation, whereas for inhomogeneous viscoelastic waves, that angle must be less than $\pi/2$. Furthermore, for viscoelastic inhomogeneous waves, the energy does not propagate in the direction of the slowness vector and the particle motion is elliptical in general. The phase velocity is less than that of the corresponding homogeneous wave (for which planes of constant phase coincide with planes of constant amplitude); critical angles do not exist in general, and, unlike the case of lossless media, the phase velocity and the attenuation factor of the transmitted waves depend on the angle of incidence. There is one more degree of freedom, since the attenuation vector is playing a role at the same level as the wavenumber vector. Snell law, for instance, implies continuity of the tangential components of both vectors at the interface of discontinuity. For homogeneous plane waves, the energy-velocity vector is equal to the phase-velocity vector. The last part of the chapter analyzes the viscoelastic wave equation expressed in terms of fractional time derivatives, and provides expressions of the reflection and transmission coefficients corresponding to a partially welded interface.

Chapter 4: Anisotropic Anelastic Media. In isotropic media there are two well-defined relaxation functions, describing purely dilatational and shear deformations of the medium. The problem in anisotropic media is to obtain the time dependence of the relaxation components with a relatively reduced number of parameters. Fine layering has an "exact" description in the long-wavelength limit. The concept of eigenstrain allows us to reduce the number of relaxation functions to six; an alternative is to use four or two relaxation functions when the anisotropy is relatively weak. Fracture-induced anisotropic attenuation is studied, and harmonic quasi-static numerical experiments are designed to obtain the stiffness components of anisotropic anelastic media. The analysis of SH waves suffices to show that in anisotropic viscoelastic media, unlike the lossless case: the group-velocity vector is not equal to the energy-velocity vector, the wavevector is not normal to the energy-velocity surface, the energy-velocity vector is not normal to the slowness surface, etc. However, an energy analysis shows that some basic fundamental relations still hold: for instance, the projection of the energy velocity onto the propagation direction is equal to the magnitude of the phase velocity. The analysis is extended to qP–qS wave propagation and expressions of the wave velocities, wave surfaces and quality factors are given. It is also shown how to implement the memory-variable approach to recast the equation of motion in full differential form.

Chapter 5: The Reciprocity Principle. Reciprocity is usually applied to concentrated point forces and point receivers. However, reciprocity has a much wider application potential; in many cases, it is not used at its full potential, either because a variety of source and receiver types are not considered or their implementation is not well understood. In this chapter, the reciprocity relations for inhomogeneous, anisotropic, viscoelastic solids, and for distributed sources and receivers are obtained. In addition to the usual relations involving directional forces, it is shown that the reciprocity can also be applied to a variety of source-receiver configurations used in earthquake seismology and seismic reflection and refraction methods. Moreover, reciprocity applied to flexural waves illustrates another applicability of the principle.

Chapter 6: Reflection and Transmission of Plane Waves. The SH and qP–qSV cases illustrate the physics of wave propagation in anisotropic anelastic media. In general, the reflected and transmitted waves are inhomogeneous, i.e., equiphase planes do not coincide with equiamplitude planes. The reflected wave is homogeneous only when the symmetry axis is perpendicular to the interface. If the transmission medium is elastic and the incident wave is homogeneous, the transmitted wave is inhomogeneous of the elastic type, i.e., the attenuation vector is perpendicular to the Umov–Poynting vector. The angle between the attenuation vector and the slowness vector may exceed 90°, but the angle between the attenuation and the Umov–Poynting vector is always less than 90°. If the incidence medium is elastic, the attenuation of the transmitted wave is perpendicular to the interface. The relevant physical phenomena are not related to the propagation direction (slowness vector), but rather to the energy-flow direction (Umov–Poynting vector) – for instance, the characteristics of the elastic type inhomogeneous waves, the existence of critical angles, and the fact that the amplitudes of the reflected and transmitted waves decay in the direction of energy flow despite the fact that they grow in the direction of phase propagation. The applications of the theory include propagation at fluid/solid interfaces, Rayleigh surface waves and propagation through a set of layers.

Chapter 7: Biot Theory for Porous Media. Dynamic porous media behaviour is described by means of Biot theory of poroelasticity. However, many developments in the area of porous media existed before Biot introduced the theory in the mid-50s. These include, for instance, Terzaghi law, Gassmann equation, and the static approach leading to the concept of effective stress, much used in soil mechanics. The simple asperity-deformation model is useful to explain the physics of porous and cracked media under confining and pore-fluid pressures. Moreover, I consider a model for pressure build-up due to kerogen-oil conversion. The dynamical problem is analyzed in detail using Biot approach: that is, the definition of the energy potentials and kinetic energy and the use of Hamilton principle to obtain the equation of motion. The coefficients of the strain energy are obtained by the so-called jacketed and unjacketed experiments. The theory includes anisotropy and dissipation due to viscodynamic and viscoelastic effects. A short discussion involving the complementary energy theorem and volume-average methods serves to define the equation of motion for inhomogeneous media. The interface boundary conditions and the Green function problem are treated in detail, since they provide the basis for the solution of wave propagation in inhomogeneous media. The mesoscopic loss mechanism is described by means of White theory for plane-layered media developed in the mid-70s. The theory is applied to layered and fractured media in order to obtain the five stiffness components of the equivalent transversely isotropic medium. Then, I analyze the physics of diffusion fields resulting from Biot equations. An energy-balance analysis for time-harmonic fields identifies the strain- and kinetic-energy densities, and the dissipated-energy densities due to viscoelastic and viscodynamic effects. The analysis allows the calculation of these energies in terms of the Umov–Poynting vector and kinematic variables, and the generalization of the fundamental relations obtained in the single-phase case (Chapter 4). Measurable quantities, like the attenuation factor and the energy velocity, are expressed in terms of microstructural properties, such as tortuosity and permeability. Finally, I derive Gassmann equation for an anisotropic frame and a solid pore infill.medskip

Chapter 8: The Acoustic–Electromagnetic Analogy. The two-dimensional Maxwell equations are mathematically equivalent to the SH-wave equation based on a Maxwell stress–strain relation, where the correspondence is magnetic field/particle velocity, electric field/stress, dielectric permittivity/elastic compliance, resistivity/viscosity and magnetic permeability/density. It is shown that Fresnel formulae can be obtained from the reflection and transmission coefficients of shear waves. The analogy is extended to three dimensions. Although there is not a complete correspondence, the material properties are mathematically equivalent by using the Debye–Zener analogy. Moreover, an electromagnetic energy-balance equation is obtained from viscoelasticity, where the dielectric and magnetic energies are equivalent to the strain and kinetic energies. Other analogies involve Fresnel wave surface and its elastic-medium's equivalent wave surface, Backus averaging for finely layered media, the time-average equation, the Kramers–Kronig dispersion relations, the reciprocity principle, Babinet principle, Alford rotation, and the diffusion equation describing electromagnetic fields and the behaviour of the Biot quasi-static mode (the second-slow wave) at low frequencies. Finally, useful cross-property relations between elastic-wave velocity and electrical conductivity are discussed in this chapter.

Chapter 9: Numerical Methods. In order to solve the equation of motion by direct methods, the model (the geological layers in exploration geophysics and seismology) is approximated by a numerical mesh; that is, the model is discretized in a finite numbers of points. These techniques are also called grid methods and full-wave equation methods, since the solution implicitly gives the full wave field. Direct methods do not have restrictions on the material variability and can be very accurate when a sufficiently fine grid is used. They are more expensive than analytical and ray methods in terms of computer time, but the technique can easily handle the implementation of different strain–stress laws. Moreover, the generation of snapshots can be an important aid in interpretation. Finite-differences, pseudospectral and finite-element methods are considered in this chapter. The main aspects of the modelling are introduced as follows: (a) time integration, (b) calculation of spatial derivatives, (c) source implementation, including the moment-tensor source, (d) boundary conditions, and (e) absorbing boundaries. All these aspects are discussed and illustrated using the acoustic and SH wave equations. In addition, I discuss the concept of fractional derivative and present the wave-modelling equations for single-phase and porous media in cylindrical coordinates. The pseudospectral algorithms are discussed in more detail.

This book is aimed mainly at graduate students and researchers. It requires a basic knowledge of linear elasticity and wave propagation, and the fundamentals of numerical analysis. The following books are recommended for study in these areas: Love (1944), Kolsky (1953), Born and Wolf (1964), Pilant (1979), Auld (1990a,b), Celia and Gray (1992), Jain (1984), Slawinski (2003), Mainardi (2010) and Schön (2011). At the end of the book, I provide a list of questions about the relevant concepts, a chronological table of the main discoveries and a list of famous scientists, regarding wave propagation and its related fields of research.

Slips and errors that were present in the second edition have been corrected in the present edition. The history of science has been expanded by including additional researchers and discoveries.

Errata for the first and second editions can be found in my homepage at:
http://www.lucabaradello.it/carcione
Errata and comments may be sent to me at the following e-mail addresses:

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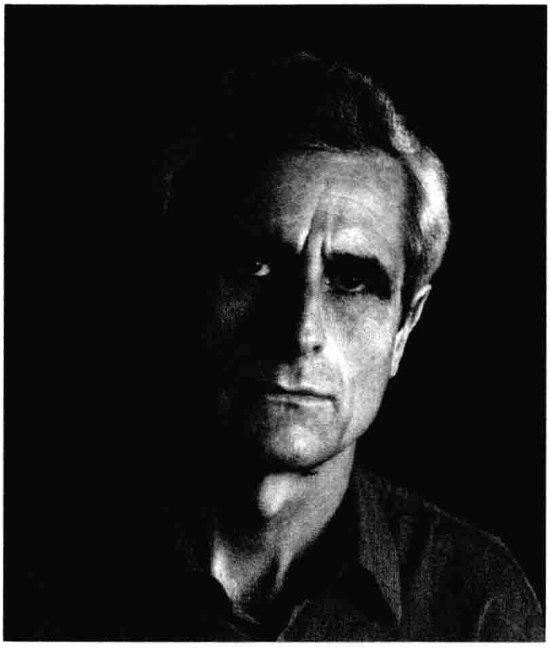
About the Author

José M. Carcione was born in Buenos Aires, Argentina. He received the degree "Licenciado in Ciencias Físicas" from Buenos Aires University in 1978, the degree "Dottore in Fisica" from Milan University in 1984, and the Ph.D. in Geophysics from Tel-Aviv University in 1987. That year he was awarded the Alexander von Humboldt scholarship for a position at the Geophysical Institute of Hamburg University, where he stayed from 1987 to 1989. From 1978 to 1980, he worked at the "Comisión Nacional de Energía Atómica" at Buenos Aires. From 1981 to 1987, he was employed at "Yacimientos Petrolíferos Fiscales", the national oil company of Argentina. Presently, he is a senior geophysicist at the "Istituto Nazionale di Oceanografia e di Geofisica Sperimentale (OGS)" (former "Osservatorio Geofisico Sperimentale") in Trieste. His current research deals with the theory of wave propagation in acoustic and electromagnetic media, numerical modelling and their application to geophysical problems.



Basic Notation

I denote the spatial variables x, y and z of a right-hand Cartesian system by the indices $i, j, k = 1, 2$ and 3, respectively, the position vector by $\mathbf{x}$ or by $\mathbf{r}$, a partial derivative of order m with respect to a variable x_i with ∂_i^m, a first, second and third time derivative with ∂_t, ∂_{tt}^2 and ∂_{ttt}^3, respectively, and a fractional time derivative of order β with ∂_t^β. For clarity in reading and ease in programming, the use of numbers to denote the subindices corresponding to the spatial variables is preferred. The upper case indices $I, J, \ldots = 1, \ldots, 6$ indicate the shortened matrix notation (Voigt notation) where pairs of subscripts (i, j) are replaced by a single number (I or J) according to the correspondence $(11) \rightarrow 1$, $(22) \rightarrow 2$, $(33) \rightarrow 3$, $(23) = (32) \rightarrow 4$, $(13) = (31) \rightarrow 5$, $(12) = (21) \rightarrow 6$. Matrix transposition is denoted by the superscript "$\top$" (it is not indicated in two- and three-components vectors), $\sqrt{-1}$ by i, complex conjugate by the superscript "$*$", the scalar and matrix products by the symbol "$\cdot$", the vector product by the symbol "$\times$", the dyadic product by the symbol "$\otimes$", and unit vectors by $\hat{\mathbf{e}}_i$, $i = 1, 2, 3$ if referring to the Cartesian axes. The identity matrix in n-dimensional space is denoted by $\mathbf{I}_n$. The gradient, divergence, Laplacian and curl operators are denoted by grad $[\,\cdot\,]$, div $[\,\cdot\,]$, $\Delta\,[\,\cdot\,]$ and curl $[\,\cdot\,]$, respectively. The components of the Levi–Civita tensor ϵ_{ijk} are 1 for cyclic permutations of 1, 2 and 3, -1 if two indices are interchanged and 0 if an index is repeated. The operators Re($\cdot$) and Im($\cdot$) take the real and imaginary parts of a complex quantity (in some cases, the subindices R and I are used). The Fourier-transform operator is denoted by $\mathcal{F}\,[\,\cdot\,]$ or a tilde above the function. The convention is

$$\tilde{g}(\omega) = \int_{-\infty}^{\infty} g(t) \exp(-\mathrm{i}\omega t)\mathrm{d}t, \quad g(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{g}(\omega) \exp(\mathrm{i}\omega t)\mathrm{d}t,$$

where t is the time variable and ω is the angular frequency. In other cases, such as the Green function, the transformed pair is denoted by $g(t)$ and $G(\omega)$. The Einstein convention of repeated indices is assumed, but the notation $I(I)$ or $i(i)$ implies no summation. In general, I express vectors and column matrices (arrays) by bold and lower case letters and matrices and tensors by bold and upper case letters. In some cases, for simplicity, the same letter is used to denote different physical quantities.