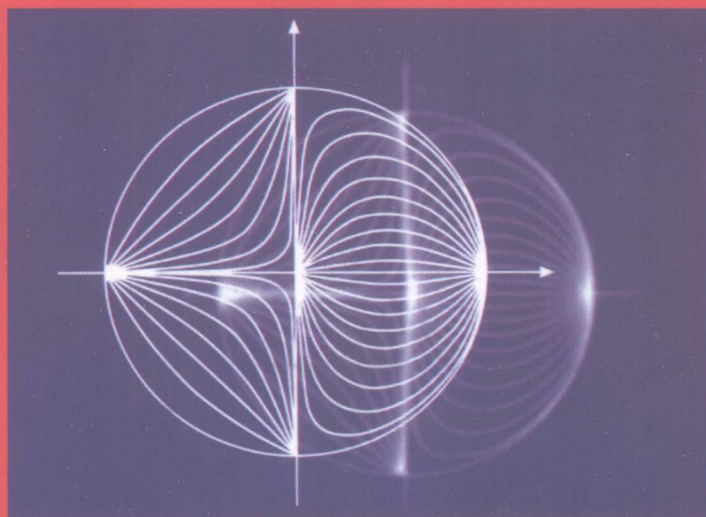Joseph H.Spurk

Fluid
Mechanics
流体力学

Joseph H. Spurk

Fluid Mechanics

With 214 Figures

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Preface to the English Edition

This textbook is the translation of the fourth edition of *Strömungslehre, Einführung in die Theorie der Strömungen*. The German edition has met with a favorable reception in German-speaking countries, showing that there was a demand for a book that emphazises the fundamentals. In the English literature there are books of the same nature, some excellent, and these have indeed influenced me to write this book. However, they cover different ground and are not aimed primarily at mechanical engineering students, which this book is. I have kept the original concept throughout all editions and there is little to say that has not been said in the preface to the first German edition. There is now a companion volume *Solved Problems in Fluid Mechanics,* which alleviates the drawback of the first German edition, namely the absence of problem exercises.

The book has been translated by Katherine Mayes during her stay in Darmstadt, and I had the opportunity to work with her daily. It is for this reason that I am solely responsible for this edition, too. My thanks also go to Prof. L. Crane from Trinity College in Dublin for his assistance with this book. Many people have helped, all of whom I cannot name, but I would like to express my sincere thanks to Ralf Münzing, whose dependable and unselfish attitude has been a constant encouragement during this work.

Darmstadt, January 1997   J. H. Spurk

Preface to the First German Edition

The purpose of this textbook is to give a systematic introduction to fluid mechanics. It is aimed primarily at students of mechanical engineering, but also at engineers, physicists and applied mathematicians. Thus, the book is designed for use in conjunction with a course of lectures, but is also suitable for self study. No previous knowledge of fluid mechanics is necessary.

From the many textbooks of similar intentions, it differs by its emphasis on the foundations of continuum mechanics, which make up a large portion of the representation. Individual branches of fluid mechanics, which are always the result of simplifying assumptions, are then developed according to the guide line "from the general to the specific". The representation fa-

vored by engineers, which starts from simple topics (say from hydrostatics and quasi-onedimensional flow) and progresses to more difficult ones, may pose lesser demands on the student, but is more time consuming, since repetitions are unavoidable. But more important: such an approach obstructs the overview of the subject and lets fluid mechanics appear as a collection of different disciplines, which have scarcely anything in common. This text intends to counteract this impression and stresses the wholeness of fluid mechanics by emphasizing the principles common to all its branches.

The growing importance of numerical methods, which enable the equation of motion to be integrated without the usual restrictions, is best served by encouraging a solid understanding of the fundamentals. The stream filament theory, which so long and rightly has been the core of engineering education in fluid mechanics, can no longer be sufficient.

To achieve the above objective, it seemed proper to divide the text into a part containing the fundamentals and a second part, devoted to the applications of these to particular fields. In the first chapter, the kinematics of flows is treated to the extent needed in later chapter, and as it deemed necessary for the understanding of more advanced textbooks. The second chapter introduces the continuum mechanical formulation of the axioms of mechanics and thermodynamics. It contains the extension of classical thermodynamics to the thermodynamics of irreversible processes. The conceptual interpretation of the balance laws is placed in the forefront and supported by typical examples from mechanical engineering. These two chapters summarize the principles about the behaviour common to all bodies. In the third chapter constitutive equations are discussed, mainly for Newtonian and frictionless fluid. But because of their increasing importance in mechanical engineering, this chapter contains an introduction to non-Newtonian materials. The fourth chapter deals with the general properties of the Navier-Stokes and Eulers equation and introduces their first integrals, Bernoulli's equation and the vortex theorems.

The first four chapters presuppose a certain familiarity with tensor calculus. However, since all equations are developed in Cartesian coordinate systems, only the few rules for Cartesian tensor notation are needed, for which an introduction is given as an appendix.

In choosing the material for the second part, I have been influenced by the needs of mechanical engineers. Of course, with previous knowledge in fluid mechanics, this part can be read without the first part. But since the different branches here evolve from the fundamentals in accord with the structure of the book, the discourse may occasionally differ from the usual and may be new even to the experienced reader.

This book has arisen from lectures, which ran concurrently with tuto-

rials, but for cost reasons it has not been possible to include problems and their solutions. However, any attempt to master such a difficult subject requires the solution of problems and the student is encouraged to work out the many examples in the book from beginning to end.

In keeping with the character of the book, I have not given references, but included a list of the books which I have used in the writing and to which I am indebted. I wish to mention specifically the collected work of L. Prandtl, from which many of the figures have been taken, since these can scarcely be improved upon for clarity and perspicuity.

The notes of my lectures given in the years 1971 to 1985 form the roots of this book. During the restructuring of the teaching curriculum in the Department of Mechanical Engineering at the Technische Hochschule Darmstadt, the number of hours for the compulsory course in fluid mechanics was reduced. This meant that it was necessary to rearrange the material, which eventually led to the complete rewriting of the original manuscript. A large part of this work was carried out during a sabbatical semester at Trinity College, Dublin, Ireland, and I should like to thank Prof. L. Crane for his hospitality throughout this stay. This book has originated directly from this new set of lectures, and my colleagues have assisted greatly in composing and formulating these. In particular I must thank Jürgen Depp, who put great effort into preparing the manuscript for printing, and Ulrich Sauerwein, who checked all the derivations and examples fastidiously and suggested many reductions and improvements. In spite of this assistance, there will be errors remaining, and for these I alone am responsible.

Darmstadt, June 1987   J. H. Spurk

Contents

II   Selected Topics in Fluid Mechanics

1 The Concept of the Continuum and Kinematics

1.1 Properties of Fluids; Continuum Hypothesis

Fluid mechanics is concerned with the behavior of materials which deform without limit under the influence of shearing forces. Even a very small shearing force will deform a fluid body, but the velocity of the deformation will be correspondingly small. This property serves as the definition of a fluid: the shearing forces necessary to deform a fluid body go to zero as the velocity of deformation tends to zero. On the contrary, the behavior of a solid body is such that the deformation itself, not the velocity of deformation, goes to zero when the forces necessary to deform it tend to zero.
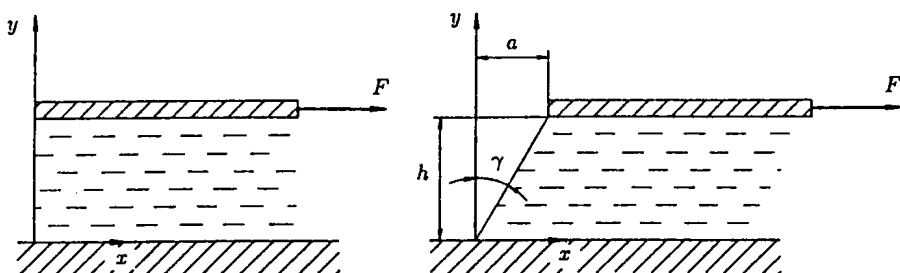
Fig. 1.1. Shearing between two parallel plates

To illustrate this contrasting behavior, consider a material between two parallel plates and adhering to them acted on by a shearing force F (Fig. 1.1).

If the extent of the material in the direction normal to the plane of Fig. 1.1 and in the x-direction is much larger than that in the y-direction, experience shows that for many solids (Hooke's solids), the force per unit area $\tau = F/A$ is proportional to the displacement a and inversely proportional to the distance between the plates h. At least one dimensional quantity typical for the material must enter this relation, and here this is the shear modulus G. The relationship

$$\tau = G\,\gamma \quad (\gamma \ll 1) \tag{1.1}$$

between the shearing angle $\gamma = a/h$ and τ satisfies the definition of a solid: the force per unit area τ tends to zero only when the deformation γ itself goes to zero. Often the relation for a solid body is of a more general form, e. g. $\tau = f(\gamma)$, with $f(0) = 0$.

If the material is a fluid, the displacement of the plate increases continually with time under a constant shearing force. This means there is no relationship between the displacement, or deformation, and the force. Experience shows here that with many fluids the force is proportional to the rate of change of the displacement, that is, to the velocity of the deformation. Again the force is inversely proportional to the distance between the plates. (We assume that the plate is being dragged at constant speed, so that the inertia of the material does not come into play.) The dimensional quantity required is the *shear viscosity* η, and the relationship with $U = \mathrm{d}a/\mathrm{d}t$ now reads:

$$\tau = \eta\,\frac{U}{h} = \eta\,\dot\gamma \,, \tag{1.2}$$

or, if the *shearrate* $\dot\gamma$ is set equal to $\mathrm{d}u/\mathrm{d}y$,

$$\tau(y) = \eta\,\frac{\mathrm{d}u}{\mathrm{d}y} \,. \tag{1.3}$$

$\tau(y)$ is the shear stress on a surface element parallel to the plates at point y. In so-called *simple shearing flow* (*rectilinear shearing flow*) only the x-component of the velocity is nonzero, and is a linear function of y.

The above relationship was known to Newton, and it is sometimes incorrectly used as the definition of a *Newtonian fluid*: there are also *non–Newtonian fluids* which show a linear relationship between the shear stress τ and the shearrate $\dot\gamma$ in this simple state of stress. In general, the relationship for a fluid reads $\tau = f(\dot\gamma)$, with $f(0) = 0$.

While there are many substances for which this classification criterion suffices, there are some which show dual character. These include the glasslike materials which do not have a crystal structure and are structurally liquids. Under prolonged loads these substances begin to flow, that

is to deform without limit. Under short–term loads, they exhibit the be-
havior of a solid body. Asphalt is an oftquoted example: you can walk
on asphalt without leaving footprints (short–term load), but if you remain
standing on it for a long time, you will finally sink in. Under very short-
term loads, e. g. a blow with a hammer, asphalt splinters, revealing its
structural relationship to glass. Other materials behave like solids even
in the long-term, provided they are kept below a certain shear stress, and
then above this stress they will behave like liquids. A typical example of
these substances (*Bingham materials*) is paint: it is this behavior which
enables a coat of paint to stick to surfaces parallel to the force of gravity.

The above definition of a fluid comprises both liquids and gases, since
neither show any resistance to change of shape when the velocity of this
change tends to zero. Now liquids develop a free surface through conden-
sation, and in general do not fill up the whole space they have available
to them, say a vessel, whereas gases completely fill the space available.
Nevertheless, the behavior of liquids and gases is dynamically the same as
long as their volume does not change during the course of the flow.
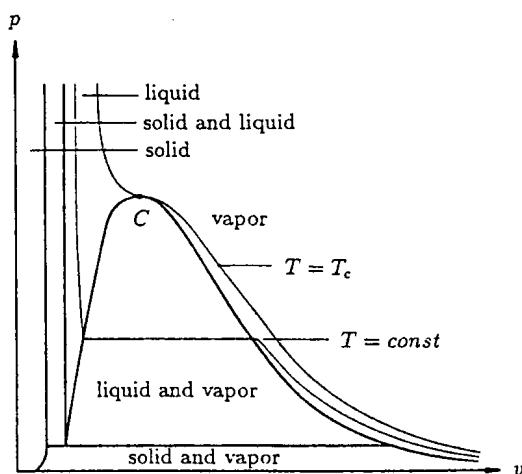
The essential difference
between them lies in
the greater compressibi-
lity of gases. When hea-
ted over the critical tem-
perature T_c, liquid lo-
ses its ability to con-
dense and it is then in
the same thermodyna-
mical state as a gas com-
pressed to the same den-
sity. In this state even
gas can no longer be "ea-
sily" compressed. The
feature we have to ta-
ke note of for the dyna-
mic behavior, therefore,
is not the state of the
fluid (gaseous or liquid)

Fig. 1.2. p–v–diagram

but the resistance it shows to change in volume. Insight into the expected
volume or temperature changes for a given change in pressure can be ob-
tained from a graphical representation of the equation of state for a pure
substance $F(p, T, v) = 0$ in the wellknown form of a p–v–diagram with T
as the parameter (Fig. 1.2). This graph shows that during dynamic pro-

cesses where large changes of pressure and temperature occur, the change of volume has to be taken into account. The branch of fluid mechanics which evolved from the necessity to take the volume changes into account is called *gas dynamics*. It describes the dynamics of flows with large pressure changes as a result of large changes in velocity. There are also other branches of fluid mechanics where the change in volume may not be ignored, among these *meteorology*; there the density changes as a result of the pressure change in the atmosphere due to the force of gravity.

The behavior of solids, liquids and gases described up to now can be explained by the molecular structure, by the thermal motion of the molecules, and by the interactions between the molecules. Microscopically the main difference between gases on the one hand, and liquids and solids on the other is the mean distance between the molecules.

With gases, the spacing at standard temperature and pressure (273.2 K; 1.013 bar) is about ten effective molecular diameters. Apart from occasional collisions, the molecules move along a straight path. Only during the collision of, as a rule, two molecules, does an interaction take place. The molecules first attract each other weakly, and then as the interval between them becomes noticeably smaller than the effective diameter, they repel strongly. The *mean free path* is in general larger than the mean distance, and can occasionally be considerably larger.

With liquids and solids the mean distance is about one effective molecular diameter. In this case there is always an interaction between the molecules. The large resistance which liquids and solids show to volume changes is explained by the repulsive force between molecules when the spacing becomes noticeably smaller than their effective diameter. Even gases have a resistance to change in volume, although at standard temperature and pressure it is much smaller and is proportional to the kinetic energy of the molecules. When the gas is compressed so far that the spacing is comparable to that in a liquid, the resistance to volume change becomes large, for the same reason as referred to above.

Real solids show a crystal structure: the molecules are arranged in a lattice and vibrate about their equilibrium position. Above the melting point, this lattice disintegrates and the material becomes liquid. Now the molecules are still more or less ordered, and continue to carry out their oscillatory motions although they often exchange places. The high mobility of the molecules explains why it is easy to deform liquids with shearing forces.

It would appear obvious to describe the motion of the material by integrating the equations of motion for the molecules of which it consists: for computational reasons this procedure is impossible since in general the

number of molecules in the material is very large. But it is impossible in
principle anyway, since the position and momentum of a molecule cannot
be simultaneously known (Heisenberg's Uncertainty Principle) and thus
the initial conditions for the integration do not exist. In addition, detailed
information about the molecular motion is not readily usable and therefore
it would be necessary to average the molecular properties of the motion
in some suitable way. It is therefore far more appropriate to consider
the average properties of a cluster of molecules right from the start. For
example the macroscopic, or continuum, velocity

$$\vec{u} = \frac{1}{n} \sum_1^n \vec{c_i} \; , \tag{1.4}$$

where $\vec{c_i}$ are the velocities of the molecules and n is the number of molecules
in the cluster. This cluster will be the smallest part of the material that
we will consider, and we call it a *fluid particle*. To justify this name, the
volume which this cluster of molecules occupies must be small compared to
the volume occupied by the whole part of the fluid under consideration. On
the other hand, the number of molecules in the cluster must be large enough
so that the averaging makes sense, i. e. so that it becomes independent of
the number of molecules. Considering that the number of molecules in one
cubic centimeter of gas at standard temperature and pressure is 2.7×10^{19}
(*Loschmidt's number*), it is obvious that this condition is satisfied in most
cases.

Now we can introduce the most important property of a *continuum*,
its *mass density* ρ. This is defined as the ratio of the sum of the molecular
masses in the cluster to the occupied volume, with the understanding that
the volume, or its linear measure, must be large enough for the density
of the fluid particle to be independent of its volume. In other words, the
mass of a fluid particle is a smooth function of the volume.

On the other hand the linear measure of the volume must be small
compared to the macroscopic length of interest. It is appropriate to assume
that the volume of the fluid particle is infinitely small compared to the
whole volume occupied by the fluid. This assumption forms the basis of the
continuum hypothesis. Under this hypothesis we consider the fluid particle
to be a *material point* and the density (or other properties) of the fluid to be
continuous functions of place and time. Occasionally we will have to relax
this assumption on certain curves or surfaces, since discontinuities in the
density or temperature, say, may occur in the context of some idealizations.
The part of the fluid under observation consists then of infinitely many
material points, and we expect that the motion of this continuum will be
described by partial differential equations. However the assumptions which

have led us from the material to the idealized model of the continuum are not always fulfilled. One example is the flow past a space craft at very high altitudes, where the air density is very low. The number of molecules required to do any useful averaging then takes up such a large volume that it is comparable to the volume of the craft itself.

Continuum theory is also inadequate to describe the structure of a shock (see Chapter 9), a frequent occurrence in compressible flow. Shocks have thicknesses of the same order of magnitude as the mean free path, so that the linear measures of the volumes required for averaging are comparable to the thickness of the shock.

We have not yet considered the role the thermal motion of molecules plays in the continuum model. This thermal motion is reflected in the macroscopic properties of the material and is the single source of viscosity in gases. Even if the macroscopic velocity given by (1.4) is zero, the molecular velocities $\vec{c}_i$ are clearly not necessarily zero. The consequence of this is that the molecules migrate out of the fluid particle and are replaced by molecules drifting in. This exchange process gives rise to the macroscopic fluid properties called *transport properties*. Obviously, molecules with other molecular properties (e. g. mass) are brought into the fluid particle. Take as an example a gas which consists of two types of molecule, say O_2 and N_2. Let the number of O_2 molecules per unit volume in the fluid particle be larger than that of the surroundings. The number of O_2 molecules which migrate out is proportional to the number density inside the fluid particle, while the number which drift in is proportional to that of the surroundings. The net effect is that more O_2 molecules drift in than drift out and so the O_2 number density adjusts itself to the surroundings. From the standpoint of continuum theory the process described above represents the *diffusion*.

If the continuum velocity $\vec{u}$ in the fluid particle as given by (1.4) is larger than that of the surroundings, the molecules which drift out bring their molecular velocities which give rise to $\vec{u}$ with them. Their replacements have molecular velocities with a smaller part of the continuum velocity $\vec{u}$. This results in momentum exchange through the surface of the fluid particle which manifests itself as a force on this surface. In the simple shearing flow (Fig. 1.1) the force per unit area on an surface element parallel to the plates is given by (1.3). The sign of this shear stress is such as to even out the velocity. However nonuniformity of the velocity is maintained by the force on the upper plate, and thus the momentum transport is also maintained. From the point of view of continuum theory, this momentum transport is the source of the internal friction, i. e. the *viscosity*. The molecular transport of momentum accounts for internal friction only in the