Engineering Mechanics

Dynamics

Joseph F. Shelley

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ENGINEERING MECHANICS: DYNAMICS

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Preface

Engineering mechanics is the study of the effects that forces produce on bodies. It has two major subdivisions: statics, in which the bodies are at rest or are moving with constant velocity; and dynamics, in which the bodies move with varying velocity—so that acceleration becomes a necessary part of the description of dynamics problems. As a general concept, acceleration, which is a vector quantity, is the time rate of change of velocity. The absence of acceleration effects distinguishes statics from dynamics. Dynamics is the first engineering course in the fields of aeronautical, civil, and mechanical engineering and technology. A thorough understanding of the basic principles of this body of knowledge is a necessary prerequisite to further study in mechanical design and analysis, vibrations, control systems, and fluid dynamics.

The subject of dynamics rests upon a very limited number of simply stated physical laws—notably Newton's second law and the law of conservation of energy. Despite this apparent simplicity, dynamics is an inherently complex subject. The central objective is to identify from the physical problem the type of force-motion relation that exists, and then to use the appropriate techniques to obtain the final solution.

In preparing this work, I have attempted the following:

- 1. To present the basic theory in a straightforward manner.
- 2. To point out, and emphasize, the potential pitfalls that exist in certain areas of the subject matter.
- To illustrate the particular theory with one or more example problems.

- 4. To present a summary section, at the end of each chapter, on the major concepts in the chapter.
- 5. To provide with each chapter a set of problem assignments, of varying degrees of complexity, that illustrates the different aspects of the theory.

An attempt has been made, wherever possible, to give a physical, or "hardware," flavor to the example problems and problem assignments, with major emphasis on obtaining useful technical solutions. In many cases, the effect on the solution of varying one, or more, of the initial conditions of the problem is shown and discussed. In the more difficult areas of the subject matter, every attempt has been made to anticipate the reader's questions. Throughout the text, the units are equally divided between the U. S. Customary System (USCS) and the International System (SI). Answers to odd-numbered problem assignments are provided at the end of the text.

The general organization of the text is as follows. Chapter 15 considers the motion of particles without regard for the forces that produce these motions, referred to as the subject of kinematics. Chapter 16 introduces particle dynamics, where the relation between the force that acts on a particle and the resulting motion is obtained. Chapter 17 presents the kinematics of plane motion of a rigid body. Chapter 18 digresses from the main stream of dynamics to introduce centroids and mass moments of inertia of rigid bodies. The results from Chapters 17 and 18 are then used in Chapter 19, the dynamics of rigid bodies in plane motion, where the relation between the forces and moments that act on a body and the resulting translational and angular motions are obtained. Chapter 20 considers the use of work-energy methods to obtain solutions to problems in particle and rigid-body dynamics.

Chapter 21 considers the highly significant technical problem of impact of bodies, and impulse-momentum solutions. Chapter 22 covers three-dimensional dynamic motion of rigid body with emphasis on rotating unbalance, dynamic balancing, and gyroscopic moments. Chapter 23 is a treatment of damped and undamped free and forced vibration of a single degree of freedom mechanical systems. Chapter 24, the final chapter, considers the motion of a body subjected to viscous, or quadratic resistance, drag forces. Chapters 15 through 21 contain the material usually found in a first course in dynamics. The remaining chapters may be studied in any order, depending on the emphasis desired.

The mathematics used in this text includes trigonometry, algebra, and elementary differential and integral calculus. The derivative is used principally in the definitions of velocity and acceleration; and the integral is used primarily to interpret the area under displacement, velocity and acceleration curves. A very limited number of problems require formal differentiation or integration techniques for their solution.

It is difficult to acknowledge formally all the people who have offered comments and suggestions, and who have influenced this work. The following individuals, however, deserve special mention. Professor Kenneth Schneider, California State Polytechnic University at Pomona, served as consulting editor on this project from its inception. The barriers of distance between Pomona and the author's home in Princeton, New Jersey did not prevent the editor and the author from having numerous discussions and meetings at all stages of the work. Professor Schneider's constructive suggestions and insights are woven throughout the entire fabric of this book. Professors Peter Basch, Pratt Institute, Brooklyn, New York; John Pautz, Middlesex County College, Edison, New Jersey; Emil Yanchula, George Brown University, Canada; and Dean Gerald Seeley, Tri-State University, Angola, Indiana formally served as reviewers. The work was also reviewed by Professors Chia- Ching Feng and Clay Carlisle of Trenton State College. Invaluable comments, from the point of view of the student, were offered by Kenneth Lore and David Davidson, undergraduates at Trenton State College. The opinion of colleagues in the profession was also sought, and comments on the manuscript were offered by Dr. Frederick Tepper, team leader, Mechanical Analysis Staff, U. S. Army Research and Development Command, Dover, New Jersey, and Dr. Donald Mack, Program Manager for Technical Education, General Electric Company, Fairfield, Connecticut. James Scullin, a friend and consulting engineer, performed invaluable service in working out all the problem assignments as a check on the author's computations. Finally, the manuscript was typed completely, correctly, and cheerfully by Anna Waite.

The author also wishes to express his appreciation to the McGraw-Hill Book Company for their high level of enthusiasm and support for the project. B. J. Clark served as Editor-in-Chief and Julienne V. Brown as editor. Felix Cooper fashioned the illustrations with fertile imagination and deft fingers. Jack Maisel, as editing supervisor, attended to the details both large and small required to transform the author's manuscript to its final book form, and his accomplishment was to make the whole greater than the sum of its parts.

Preparation of a work such as this, and its companion volume Engineering Mechanics: Statics, is a very subjective exercise in creation. It reflects many judgments on the part of the author with respect to organization of material and emphasis of topics. As with any other textbook, it receives its ultimate review by the readership only after publication. This author welcomes comments of any kind from the readers, and an attempt will be made in all cases to acknowledge such communications.

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