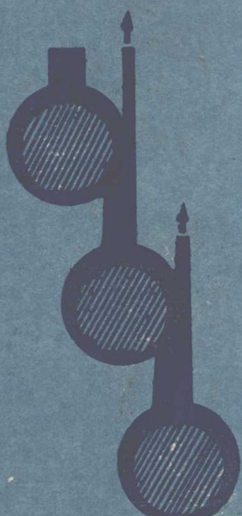
HEAT TRANSFER



HEAT TRANSFER

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New York
THE MACMILLAN COMPANY

PREFACE

The rapid changes occurring in modern technology have made it increasingly important for engineers to be able to work effectively in a great variety of new and unexplored fields as well as in the mature, established areas of conventional engineering. As a result, in the teaching of most engineering subjects it has become necessary to be more concerned with fundamental concepts and their application to the solution of physical problems than with the details of design procedures, testing, etc. For these reasons the author feels that there is a need for a teachable textbook in the field of heat transfer presenting the fundamental approach in a suitably rigorous manner. At the same time, it is essential to show how the mathematically derived results can be applied in order to gain an understanding of heat transfer processes of physical importance. The value of sound engineering intuition and how it supplements the analytical results of theory must also be stressed.

The presentation in this text, then, presupposes a working knowledge of mathematics (at least through differential equations) and the fundamental principles of thermodynamics and fluid dynamics. However, one chapter is devoted to a basic treatment of those topics of viscous fluid dynamics that are pertinent to convective heat transfer. The text is written for use in an introductory course in heat transfer taught to senior or first-year-graduate students. Depending on the extent of the preparation of the students, some of the material involving the more advanced mathematical techniques may be omitted without introducing an imbalance of presentation.

All three modes of heat transfer are treated, and an ample collection of problems is given with each chapter in order to provide the familiarity that can be obtained only by experience. The work on heat conduction treats the cases of one-dimensional conduction that have engineering significance and includes an extensive treatment of extended surface problems. In addition, steady and transient conduction in geometric shapes of practical importance are treated. Some of the mathematical analyses presented in the chapters on heat conduction involve the use of Bessel functions and orthogonal sets of functions. Since these topics may be somewhat beyond the mathematical experience of some students, the appendix includes summaries of the important aspects of these special subjects. For the benefit of readers who are more concerned with the results of the analyses than with the mathematical details of the solution of the heat conduction problems, all results of problems having some practical value have been reduced to relatively simple graphical presentations.

Convective heat transfer receives an extensive treatment which is intended to show how the basic principles of thermodynamics, fluid dynamics, and applied mathematics can be brought together to analyze physical convection phenomena. The importance of boundary layer theory in convective heat transfer is emphasized, and the reduction of the complexity of the mathematical analysis of convection problems by the application of sound physical simplifications is also stressed. Since the analysis of convective heat transfer involves the combination of powerful analytical tools with extensive experimental results, the importance of dimensional analysis in the determination of valuable empirical relations is also demonstrated.

The treatment of radiant heat transfer is limited to the cases of radiant energy exchange between gray, or black, finite surfaces that are separated by a nonabsorbing medium. Cases in which the surfaces are connected by reradiating walls are also considered. The logical determination of radiation configuration factors is treated extensively.

Two chapters are presented that illustrate examples in which more than one mode of heat transfer is taking place. Included in these chapters are analyses of heat exchangers, thermometric errors, and the solution of certain classes of problems by iterative schemes. The treatment of exchangers places emphasis on the application of the basic principles of heat transfer discussed in earlier chapters, and both the "log-mean" and "effectiveness" methods are illustrated. The analysis of thermometric errors includes the effects of conduction, convection, and radiation, and methods are given for predicting the error to be expected in various methods of temperature measurement.

The application of the results of any analysis to the solution of real heat transfer problems will require the knowledge of values of the necessary physical properties of the substances involved. Because of the importance of this aspect of heat transfer, a chapter has been devoted to a brief discussion of the material properties having significance in heat transfer. The tables of the appendix present an extensive collection of the thermal properties of some of the gases, liquids, and solids most frequently encountered in engineering applications of heat transfer.

Throughout the text, acknowledgment has been made to the many authors of the books and journal articles that formed the sources from which much of the special material, physical data, etc. were taken. The author would also like to express his appreciation to Professor Fred Landis of New York University for the valuable comments and suggestions that resulted from his reading of the manuscript. Special acknowledgment is due Miss Leona Hoop in gratitude for her typing of the many drafts of the manuscript.

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CONTENTS

Chapter 1

INTRODUCTION

1.1 GENERAL REMARKS.

In the science of thermodynamics, which deals with energy in its various forms and with its transformation from one form to another, two particularly important *transient* forms are defined, namely work and heat. These energies are termed transient since, by definition, they exist only when there is an exchange of energy between two systems or between a system and its surroundings, i.e., when an energy form of one system (such as kinetic energy, potential energy, internal energy, flow energy, chemical energy, etc.) is transformed into an energy form of another system or of the surroundings. When such an exchange takes place without the transfer of mass from the system and not by means of a temperature difference, the energy is said to have been transferred through the performance of *work*. If, on the other hand, the exchange is due to a temperature difference, then the energy is said to have been transferred by a flow of *heat*. It is this latter form, heat, and the basic physical laws governing its exchange with which this book is concerned. It should be noted that the existence of a temperature difference is a distinguishing feature of the energy form known as heat.

In most instances the problems of engineering importance involving an exchange of energy by the flow of heat are those in which there is a transfer of internal energy (or enthalpy in the case of flow processes) between two systems. The two systems may be different parts of the same body. In general this internal energy transfer is called "heat transfer" although, thermodynamically speaking, this is incorrect. The flow of heat is the mechanism of the transfer of the internal energy, not the quantity transferred. However, it is convenient to use this expression and to speak of heat as "flowing" as has been done here in spite of the implied contradiction with thermodynamics. Indeed, the old and discredited caloric theory of heat in which heat is defined as a weightless, colorless, odorless fluid flowing from one body to another would form an adequate basis for the science of heat transfer.

When such exchanges of internal energy or heat take place, the first law of thermodynamics requires that the heat given up by one body must equal that taken up by the other. The second law of thermodynamics demands that the transfer of heat take place from the hotter system to the colder system.

1.2 THE IMPORTANCE OF HEAT TRANSFER.

The importance of a thorough knowledge of the science of heat transfer and the necessity of being able to analyze, quantatively, problems involving a transfer of heat have become increasingly important as modern technology has become more and more complex. In almost every phase of scientific and engineering work processes involving the exchange of energy through a flow of heat are encountered.

Mechanical and chemical engineers are particularly concerned with problems of heat transfer. Modern power generation involves the production of work from either a combustible fuel or a nuclear reaction. This energy is converted into useful work by means of boilers, turbines, condensers, air heaters, water preheaters, pumps, etc. All of these pieces of apparatus involve a transfer of heat by one means or another, as does almost every piece of apparatus found in a chemical process industry or a petroleum refinery. Certainly the design of the familiar internal combustion engine, the gas turbine, the jet engine, etc., requires a complete understanding of heat transfer for a thorough analysis of the combustion and cooling processes.

The much discussed "thermal barrier" is the problem of finding means of transferring away from the aircraft the enormous amounts of heat produced by the dissipative effect of the viscosity of the air. Indeed, since all processes in nature have been observed to be irreversible, it follows that all natural processes involve a dissipation of the various forms of mechanical energy into thermal energy with consequent heat transfer processes taking place.

The importance of heat transfer in the production of comfort cooling or comfort heating is readily apparent. This influences the design of building structures of all kinds.

1.3 FUNDAMENTAL CONCEPTS AND THE BASIC MODES OF HEAT TRANSFER.

It is customary to categorize the various heat transfer processes into three basic types or modes although, as will become apparent as one studies the subject, it is certainly a rare instance when one encounters a problem of practical importance which does not involve at least two, and sometimes all three, of these modes occurring simultaneously. The three modes are called *conduction*, *convection*, and *radiation*.

Heat conduction is the term applied to the mechanism of internal energy exchange from one body to another, or from one part of a body to another part, by the exchange of the kinetic energy of motion of the molecules by direct communication or by the drift of free electrons in the case of heat conduction in metals. This flow of energy or heat passes from the higher energy molecules to the lower energy ones, i.e., from a high temperature region to a low temperature region. The distinguishing feature of conduction is that it takes place within the boundaries of a body, or across the boundary of a body into another body placed in contact with the first, without an appreciable displacement of the matter comprising the body.

A metal bar heated on one end will, in time, become hot at its other end. This is the simplest illustration of conduction. The laws governing conduction can be expressed in concise mathematical terms, and the analysis of the heat flow can be treated analytically in many instances.

Convection is the term applied to the heat transfer mechanism which occurs in a fluid by the mixing of one portion of the fluid with another portion due to gross movements of the mass of fluid. The actual process of energy transfer from one fluid particle or molecule to another is still one of conduction, but the energy may be transported from one point in space to another by the displacement of the fluid itself.

The fluid motion may be caused by external mechanical means, e.g., by a fan, pump, etc., in which case the process is called "forced convection." If the fluid motion is caused by density differences which are created by the temperature differences existing in the fluid mass, the process is termed "free convection" or "natural convection." The circulation of the water in a pan heated on a stove is an example of free convection. The important heat transfer problems of condensing and boiling are also examples of convection—involving the additional complication of a latent heat exchange.

It is virtually impossible to observe pure heat conduction in a fluid because as soon as a temperature difference is imposed on a fluid, natural convection currents will occur due to the resulting density differences.

The laws of conduction and the fundamental laws of fluid dynamics must both be considered in the analysis of heat convection. The mathematical analysis of such problems is perhaps one of the most complex fields of applied mathematics and has not been developed to as high a degree as it has in the case of heat conduction. There exists a great amount of empirical information on the subject of heat convection.

Thermal radiation is the term used to describe the electromagnetic radiation which has been observed to be emitted at the surface of a body which has been thermally excited. This electromagnetic radiation is emitted in all directions; and when it strikes another body, part may be reflected, part may be transmitted, and part may be absorbed. If the incident radiation is thermal radiation, i.e., if it is of the proper wave length, the absorbed radiation will appear as heat within the absorbing body.

Thus, in a manner completely different from the two modes discussed above, heat may pass from one body to another without the need of a medium of transport between them. In some instances there may be a separating medium, such as air, which is unaffected by this passage of energy. The heat of the sun is the most obvious example of thermal radiation.

There will be a continuous interchange of energy between two radiating bodies, with a net exchange of energy from the hotter to the colder. Even in the case of thermal equilibrium there is an energy exchange occurring, although the net exchange will be zero.

1.4 THE FUNDAMENTAL LAWS OF CONDUCTION.

Thermal Conductivity and Thermal Conductance.

The basic law governing heat conduction may best be illustrated by considering the simple, idealized situation pictured in Fig. 1.1. Consider a plate of material having a surface area A, and a thickness of Δx. Let one side be maintained at a temperature t_1, uniformly over the surface, and the other side at temperature t_2. Let q denote the rate of heat flow (i.e., energy per unit time) through the plate, neglecting any edge effects. Experiment has shown that the rate of heat flow is directly proportional to the area A and the temperature difference $(t_1 - t_2)$ but inversely proportional to the thickness Δx. This proportionality is made an equality by the definition of a constant k.

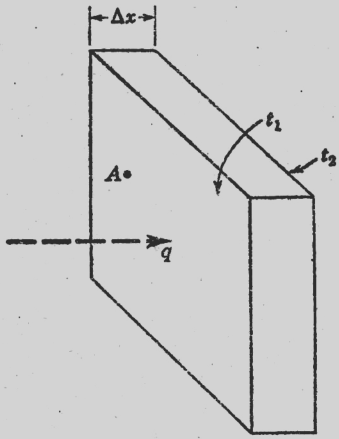

Fig. 1.1

Thus,
$$q = kA\frac{t_1 - t_2}{\Delta x}. \tag{1.1}$$

The constant of proportionality, k, is called the *thermal conductivity* of the material of which the plate is composed. It is a property dependent only on the composition of the material, not on its geometrical configuration. Sometimes a gross quantity, *thermal conductance,* is used to express the heat conducting capacity of a given physical configuration so that if C denotes the thermal conductance,

$$q = CA(t_1 - t_2).$$

Thus, it is seen that thermal conductance is the conductivity of a substance divided by its thickness. It is no longer a physical property but depends, as well, on the geometrical configuration at hand and, thus, is a less general quantity than is thermal conductivity.

Equation (1.1) forms the basis for the fundamental relation of heat conduction. Consider now a homogeneous, isotropic solid as depicted in Figure 1.2. If the solid subjected to certain prescribed boundary temperatures, what is the rate at which heat is conducted across some surface, S, within the solid?

Selecting some point P on the surface S, one can select a wafer of material having an area δA, which is part of the surface S containing P, and having a

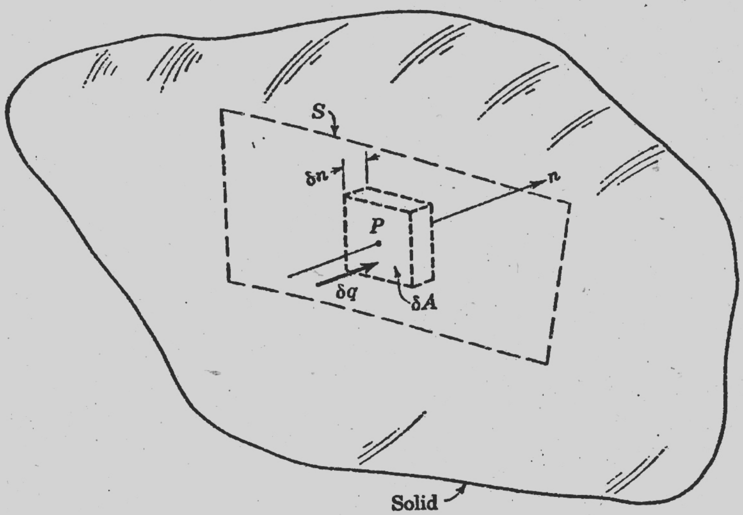

Fig. 1.2

thickness δn in the direction of the normal drawn to the surface at P. If the difference between the temperature of the back face of the wafer and its front face is δt, and if δA is chosen small enough so that δt is essentially uniform over it, then the rate of heat flow across the wafer, δq, is, by Equation (1.1),

$$\delta q = -k\delta A \frac{\delta t}{\delta n}.$$

The minus sign is due to the convention that the heat flow is taken to be positive if δt is negative in the direction of increasing n, the normal displacement. Forming the ratio $\delta q/\delta A$ and allowing the area $\delta A \to 0$, one obtains what is termed the "flux" of heat conducted through the thickness δn at the point P, namely

$$f = \frac{dq}{dA} = -k\frac{\delta t}{\delta n}.$$

Further, allowing $\delta n \to 0$ one arrives at the flux of heat across S at the point P in terms of the "temperature gradient" at P in the n direction, dt/dn:

$$f = -k\frac{dt}{dn}. \tag{1.2}$$

This is called Fourier's conduction law after the French mathematician who first made an extensive analysis of heat conduction. It states that the flux of heat conducted (energy per unit time per unit area) across a surface is proportional to the temperature gradient taken in a direction normal to the surface at the point in question.

Returning to the situation pictured in Figure 1.2, the total rate of heat transferred across the finite surface S would be

$$q = -\int_s k\frac{dt}{dn}dA.$$

Generally speaking, the normal gradient dt/dn may vary over the surface, but in many instances it is possible to select the surface as one on which the gradient is everywhere the same. This is the situation in the case depicted in Figure 1.1 in which every plane normal to Δx is such a surface. In the case of a hollow cylinder with uniform outside and inside surface temperatures, every concentric interior cylindrical surface is isothermal with a uniform temperature gradient normal to it. In such cases then,

$$q = -kA\frac{dt}{dn}, \tag{1.3}$$

where A is the total area of the finite surface.

The General Heat Conduction Equation.

The above relations may be used to develop an equation describing the distribution of the temperature throughout a heat conducting solid. In general, a heat conduction problem consists of finding the temperature at any time and at any point within a specified solid which has been heated to some known initial temperature distribution and whose surface has been subjected to a known set of boundary conditions.

To develop the differential equation governing this problem consider a solid, as shown in Figure 1.3, and select arbitrarily three mutually perpendicular coordinate directions, x, y, and z. Select in the solid a parallelepiped of dimensions Δx, Δy, and Δz. By making an energy balance on this element between the heat conducted in and out of its six faces and the heat stored within, an expression interrelating the temperatures throughout the solid will be obtained.

First, consider heat conduction in the x direction only. Let t denote the temperature at the point P which is located at the geometric center of the element. Let the subscripts "1" and "2" denote the left and right yz faces (shown shaded in Figure 1.3), respectively. The excess rate at which heat is conducted into the element, q_1, over that conducted out, q_2, is

$$q_1 - q_2.$$

Application of Equation (1.3) gives this excess to be

$$q_1 - q_2 = -k\Delta y\Delta z\left[\left(\frac{\partial t}{\partial x}\right)_1 - \left(\frac{\partial t}{\partial x}\right)_2\right],$$

where the partial derivative is used since t may depend on x, y, z, and time. The assumption that the thermal conductivity is independent of x and t has

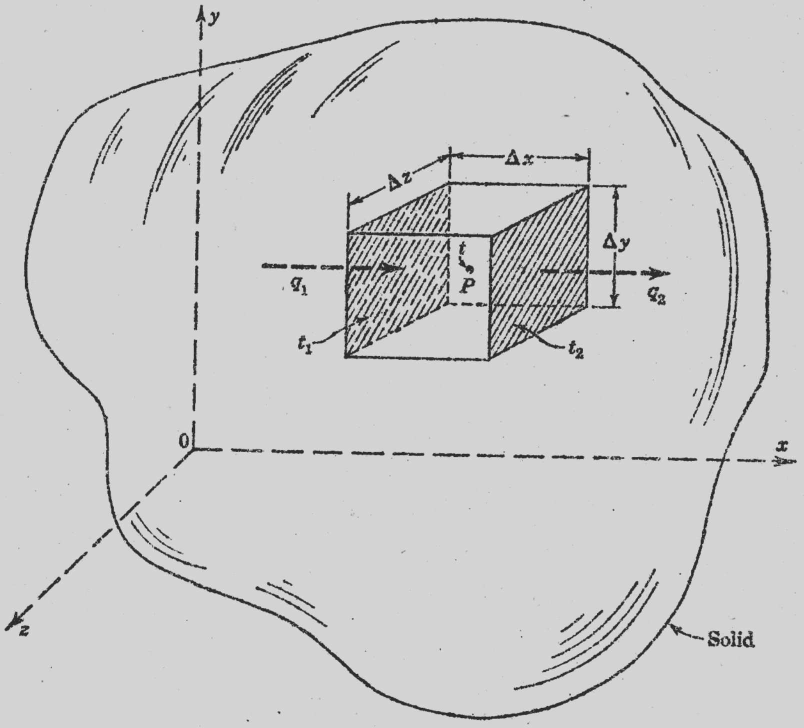

Fig. 1.3

been made. Making a Taylor's expansion of the temperature gradients $(\partial t/\partial x)_1$ and $(\partial t/\partial x)_2$ in terms of the gradient at P, one has

$$\left(\frac{\partial t}{\partial x}\right)_2 = \left(\frac{\partial t}{\partial x}\right) + \frac{\partial}{\partial x}\left(\frac{\partial t}{\partial x}\right)\left(\frac{\Delta x}{2}\right) + \frac{\partial^2}{\partial x^2}\left(\frac{\partial t}{\partial x}\right)\frac{\left(\frac{\Delta x}{2}\right)^2}{2!} + \cdots,$$

$$\left(\frac{\partial t}{\partial x}\right)_1 = \left(\frac{\partial t}{\partial x}\right) + \frac{\partial}{\partial x}\left(\frac{\partial t}{\partial x}\right)\left(\frac{-\Delta x}{2}\right) + \frac{\partial^2}{\partial x^2}\left(\frac{\partial t}{\partial x}\right)\frac{\left(\frac{-\Delta x}{2}\right)^2}{2!} + \cdots,$$

where no subscript denotes derivatives evaluated at P. So, the expression for the heat storage rate due to conduction in the x direction is

$$-k\Delta y\Delta z\left[-\frac{\partial}{\partial x}\left(\frac{\partial t}{\partial x}\right)\Delta x-\frac{\partial^3}{\partial x^3}\left(\frac{\partial t}{\partial x}\right)\frac{2}{3!}\left(\frac{\Delta x}{2}\right)^3+\cdots\right],$$

or

$$k\Delta x\Delta y\Delta z\left[\frac{\partial^2 t}{\partial x^2}+\frac{1}{24}\frac{\partial^4 t}{\partial x^4}(\Delta x)^2+\cdots\right].$$

Similar analyses in the other two coordinate directions give the following expression for the *total* rate of heat storage in the element:

$$k\Delta x\Delta y\Delta z\left[\frac{\partial^2 t}{\partial x^2}+\frac{\partial^2 t}{\partial y^2}+\frac{\partial^2 t}{\partial z^2}+\frac{(\Delta x)^2}{24}\frac{\partial^4 t}{\partial x^4}+\frac{(\Delta y)^2}{24}\frac{\partial^4 t}{\partial y^4}+\frac{(\Delta z)^2}{24}\frac{\partial^4 t}{\partial z^4}+\cdots\right]. \quad (1.4)$$

But, the rate of heat storage may be expressed also in terms of the time rate of change of the average temperature of the element. Letting t_{av} denote this temperature and $\mathbf{t}$ denote time, then the heat storage rate is

$$\Delta x\Delta y\Delta z\rho c_p\frac{\partial t_{av}}{\partial \mathbf{t}}. \quad (1.5)$$

In Equation (1.5) ρ is the density and c_p is the specific heat at constant pressure of the solid. The constant pressure specific heat is used since most heat conduction problems in solids occur under such conditions. Equating Equations (1.4) and (1.5), dividing by the volume $\Delta x\cdot\Delta y\cdot\Delta z$, allowing Δx, Δy, and $\Delta z\rightarrow 0$, and noting that under these conditions $t_{av}\rightarrow t$, one finally obtains

$$\frac{\partial t}{\partial \mathbf{t}}=\frac{k}{\rho c_p}\left(\frac{\partial^2 t}{\partial x^2}+\frac{\partial^2 t}{\partial y^2}+\frac{\partial^2 t}{\partial z^2}\right),$$

or

$$\frac{\partial t}{\partial \mathbf{t}}=\alpha\left(\frac{\partial^2 t}{\partial x^2}+\frac{\partial^2 t}{\partial y^2}+\frac{\partial^2 t}{\partial z^2}\right). \quad (1.6)$$

This last equation is the general heat conduction equation for an isotropic solid with a constant thermal conductivity and describes, in a differential form, the dependence of the temperature in the solid on the coordinates x, y, z, and on time, $\mathbf{t}$.

The quantity

$$\alpha=\frac{k}{\rho c_p} \quad (1.7)$$

is called the *thermal diffusivity* and is seen to be a physical property of the material of which the solid is composed.

As was mentioned, the above analysis assumed that the thermal conductivity k was constant and the same in all directions. In some materials this may not be true due to a possible temperature dependence or a directional dependence of k (in substances such as wood).

Other Coordinate Systems.

The above discussion was carried out in terms of rectangular coordinates. It is often useful to write the equation in cylindrical or spherical coordinates. The results are:

Cylindrical Coordinates, r (radius), z (axis), θ (longitude):

$$\frac{\partial t}{\partial \mathrm{t}} = \alpha\left[\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r} + \frac{1}{r^2}\frac{\partial^2 t}{\partial \theta^2} + \frac{\partial^2 t}{\partial z^2}\right]. \tag{1.8}$$

Spherical Coordinates, r (radius), φ (longitude), θ (colatitude):

$$\frac{\partial t}{\partial \mathrm{t}} = \alpha\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\,\frac{\partial t}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial \varphi^2}\right]. \tag{1.9}$$

The proof of these equations will be left as exercises for the student.

The Steady State.

A particularly useful special case of Equation (1.6) is one which has a very wide range of application in engineering. This is the so-called *steady state* in which there is no dependence on time. The heat conduction equation then reduces to Laplace's equation: In rectangular coordinates this is:

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} = 0. \tag{1.10}$$

1.5 THE FUNDAMENTAL LAWS OF CONVECTION.

The Boundary Layer Concept.

The discussion of Section 1.3 defined "convection" as the term applied to the heat transfer mechanism which takes place in a fluid because of a combination of conduction within the fluid and energy transport which is due to the fluid motion itself—the fluid motion being produced either by artificial means or by density currents.

Since fluid motion is the distinguishing feature of heat convection, it is necessary to understand some of the principles of fluid dynamics in order to describe adequately the processes of convection. When any real fluid moves