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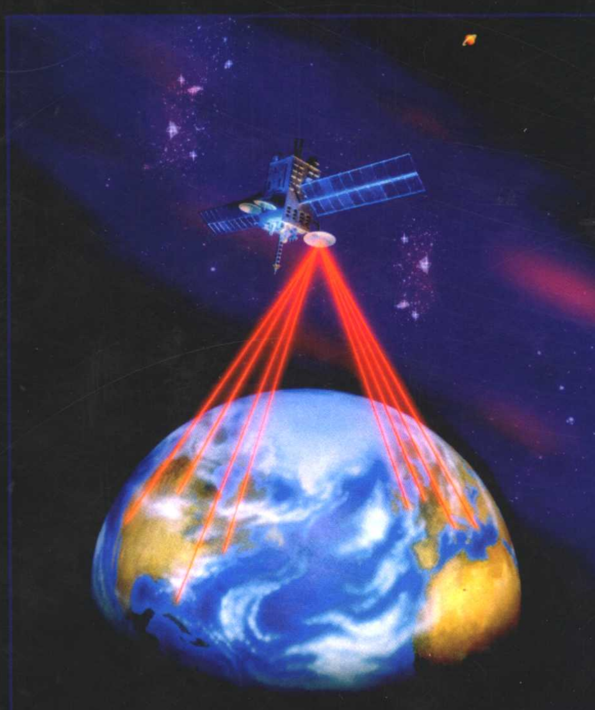
Fundamentals of Complex Analysis

with Applications to Engineering and Science

Third Edition



E. B. Saff · A

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Fundamentals of Complex Analysis with Applications to Engineering and Science

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本书全面介绍复变理论及其在当今工程问题上的应用，理论与实际应用密切结合，对工程类学科的学生来说，这种方式使数学方法更具生动性。

本书的主要特点

- 结合使用MATLAB工具箱：使复杂算术运算及保形映射更加可视化。
- 对复函数在线性分析中的用途的最新阐述：为学生提供了交流电路、运动学及信号处理等应用的另一种视角。
- 茹利亚集：使学生熟悉复分析研究的最新论题。
- 以两种可选的方式给出了柯西定理：提供了更易于可视化、更易于应用到特定情况的方法。
- 对数值保形映射的高可读性阐述：这对现代技术领域中的应用非常重要，与其他数学领域也密切相关。
- 在实际工程问题中的应用：吸引并帮助学生灵活应用数学方法。

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E. B. Saff 1964年于佐治亚理工学院获得学士学位，1968年于马里兰大学获得博士学位，现为范德比尔特大学数学系教授，构造逼近中心教授、主任。他主要从事逼近论、位势论、复分析和数值分析等领域的研究。

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To Loretta and Dawn,
who have added real joy to our complex lives

Preface

The raison d'existence for *Fundamentals of Complex Analysis with Applications to Engineering and Science, 3/e* is our conviction that engineering, science, and mathematics undergraduates who have completed the calculus sequence are capable of understanding the basics of complex analysis and applying its methods to solve engineering problems. Accordingly, we address ourselves to this audience in our attempt to make the fundamentals of the subject more easily accessible to readers who have little inclination to wade through the rigors of the axiomatic approach. To accomplish this goal we have modeled the text after standard calculus books, both in level of exposition and layout, and have incorporated engineering applications throughout the text so that the mathematical methodology will appear less sterile to the reader.

To be more specific about our mode of exposition, we begin by addressing the question most instructors ask first: To what extent is the book self contained, i.e., which results are proved and which are merely stated? Let us say that we have elected to include all the proofs that reflect the spirit of analytic function theory and to omit most of those that involve deeper results from real analysis (such as the convergence of Riemann sums for complex integrals, the Cauchy criterion for convergence, Goursat's generalization of Cauchy's theorem, or the Riemann mapping theorem). Moreover, in keeping with our philosophy of avoiding pedantics, we have shunned the ordered pairs interpretation of complex numbers and retained the more intuitive approach (grounded in algebraic field extensions).

Cauchy's theorem is given two alternative presentations in Chapter 4. The first is based on the deformation of contours, or what is known to topologists as homotopy. We have taken some pains to make this approach understandable and transparent to the novice because it is easy to visualize and to apply in specific situations. The second treatment interprets contour integrals in terms of line integrals and invokes Green's theorem to complete the argument. These parallel developments constitute the two parts of Section 4 in Chapter 4; either one may be read, and the other omitted, without disrupting the exposition (although it should not be difficult to discern our preference, from this paragraph).

Steady state temperature patterns in two dimensions are, in our opinion, the most familiar instances of harmonic functions, so we have principally chosen this interpretation for visualization of the theorems of analytic function theory. This application receives attention throughout the book, with special emphasis in Chapter 7 in the context of conformal mapping. There we draw the distinction between direct methods, wherein a mapping must be constructed to solve a specific problem, and indirect methods that postulate a mapping and then investigate which problems it solves. In doing so we hope to dispel the impression, given in many older books, that all applications of the technique fall in the latter category.

In this third edition L. N. Trefethen and T. Driscoll have updated an appendix that reflects the progress made in recent years on the numerical construction of conformal mappings. A second appendix compiles a listing of some useful mappings having closed form expressions.

Linear systems analysis is another application that recurs in the text. The basic ideas of frequency analysis are introduced in Chapter 3 following the study of the transcendental functions; Smith charts, circuit synthesis, and stability criteria are addressed at appropriate times; and the development culminates in Chapter 8 with the exposition of the analytic-function aspects of Fourier, Mellin, Laplace, Hilbert, and z transforms, including new applications in signal processing and communications. We hope thereby that our book will continue to serve the reader as a reference resource for subsequent coursework in these areas.

Features of the Third Edition

Novel features of the third edition are a discussion of the Riemann sphere, adding substance to the pragmatic concept of the "point at infinity" in complex analysis; an introduction to functional iteration and the picturesque Julia sets that thereby manifest themselves in the complex plane; an early exploration of the enrichment that the complex viewpoint provides in the analysis of polynomials and rational functions; and an introductory survey of harmonic function methods for calculating equilibrium temperatures for simple geometries. Optional sections are indicated with an asterisk so that readers can select topics of special interest. Summaries and suggested readings appear at the end of each chapter. As in previous editions, the text is distinguished by its wealth of worked-out examples that illustrate the theorems, techniques, and applications of complex analysis.

Instructors (and curious students) may benefit from a MATLAB toolbox developed by Francisco Carreras, available by Internet download from the web site

`http://ee.eng.usf.edu/people/snider2.html`

(click on complextools.zip). Instructions for its use are detailed in the file compman.doc. The toolbox provides graphic onscreen visualizations and animations of the algebraic manipulations of complex numbers and the common conformal maps, as well as a introductory guide for designing Joukowski airfoils.

A downloadable .pdf file of the inevitable errata that our helpful readers report to us is also available at this site.

The authors wish to acknowlege our mentors, Joseph L. Walsh and Paul Garabedian, who have inspired our careers, and to express their gratitude to Samuel Garrett, our longtime colleague at the University of South Florida; to acquisitions editor

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Contents

Chapter 1

Complex Numbers

1.1 The Algebra of Complex Numbers

To achieve a proper perspective for studying the system of complex numbers, let us begin by briefly reviewing the construction of the various numbers used in computation.

We start with the rational numbers. These are ratios of integers and are written in the form m/n, $n \neq 0$, with the stipulation that all rationals of the form n/n are equal to 1 (so we can cancel common factors). The arithmetic operations of addition, subtraction, multiplication, and division with these numbers can always be performed in a finite number of steps, and the results are, again, rational numbers. Furthermore, there are certain simple rules concerning the order in which the computations can proceed. These are the familiar commutative, associative, and distributive laws:

Commutative Law of Addition

$$a + b = b + a$$

Commutative Law of Multiplication

$$ab = ba$$

Associative Law of Addition

$$a + (b + c) = (a + b) + c$$

Associative Law of Multiplication

$$a(bc) = (ab)c$$

Distributive Law

$$(a + b)c = ac + bc,$$

for any rationals a, b, and c.

Notice that the rationals are the only numbers we would ever need, to solve equations of the form

$$ax + b = 0.$$

The solution, for nonzero a, is $x = -b/a$, and since this is the ratio of two rationals, it is itself rational.

However, if we try to solve quadratic equations in the rational system, we find that some of them have no solution; for example, the simple equation

$$x^2 = 2 \tag{1}$$

cannot be satisfied by any rational number (see Prob. 29 at the end of this section). Therefore, to get a more satisfactory number system, we extend the concept of "number" by appending to the rationals a new symbol, mnemonically written as $\sqrt{2}$, which is defined to be a solution of Eq. (1)). Our revised concept of a number is now an expression in the standard form

$$a + b\sqrt{2}, \tag{2}$$

where a and b are rationals. Addition and subtraction are performed according to

$$(a + b\sqrt{2}) \pm (c + d\sqrt{2}) = (a \pm c) + (b \pm d)\sqrt{2}. \tag{3}$$

Multiplication is defined via the distributive law with the proviso that the square of the symbol $\sqrt{2}$ can always be replaced by the rational number 2. Thus we have

$$(a + b\sqrt{2})(c + d\sqrt{2}) = (ac + 2bd) + (bc + ad)\sqrt{2}. \tag{4}$$

Finally, using the well-known process of *rationalizing the denominator*, we can put the quotient of any two of these new numbers into the standard form

$$\frac{a + b\sqrt{2}}{c + d\sqrt{2}} = \frac{a + b\sqrt{2}}{c + d\sqrt{2}} \frac{c - d\sqrt{2}}{c - d\sqrt{2}} = \frac{ac - 2bd}{c^2 - 2d^2} + \frac{bc - ad}{c^2 - 2d^2}\sqrt{2}. \tag{5}$$

This procedure of "calculating with radicals" should be very familiar to the reader, and the resulting arithmetic system can easily be shown to satisfy the commutative, associative, and distributive laws. However, observe that the symbol $\sqrt{2}$ has not been absorbed by the rational numbers painlessly. Indeed, in the standard form (2) and in the algorithms (3), (4), and (5) its presence stands out like a sore thumb. Actually, we are only using the symbol $\sqrt{2}$ to "hold a place" while we compute around it using the rational components, except for those occasional opportunities when it occurs squared and we are temporarily relieved of having to carry it. So the inclusion of $\sqrt{2}$ as a number is a somewhat artificial process, devised solely so that we might have a richer system in which we can solve the equation $x^2 = 2$.

With this in mind, let us jump to the stage where we have appended all the real numbers to our system. Some of them, such as $\sqrt[4]{17}$, arise as solutions of more complicated equations, while others, such as π and e, come from certain limit processes.

Each irrational is absorbed in a somewhat artificial manner, but once again the resulting conglomerate of numbers and arithmetic operations satisfies the commutative, associative, and distributive laws.[†]

At this point we observe that we still cannot solve the equation

$$x^2 = -1. \tag{6}$$

But now our experience suggests that we can expand our number system once again by appending a symbol for a solution to Eq. (6); instead of $\sqrt{-1}$, it is customary to use the symbol i. (Engineers often use the letter j.) Next we imitate the model of expressions (2) through (5) (pertaining to $\sqrt{2}$) and thereby generalize our concept of number as follows:[‡]

Definition 1. A **complex number** is an expression of the form $a + bi$, where a and b are real numbers. Two complex numbers $a + bi$ and $c + di$ are said to be equal ($a + bi = c + di$) if and only if $a = c$ and $b = d$.

The operations of addition and subtraction of complex numbers are given by

$$(a + bi) \pm (c + di) := (a \pm c) + (b \pm d)i,$$

where the symbol := means "is defined to be."

In accordance with the distributive law and the proviso that $i^2 = -1$, we postulate the following:

The multiplication of two complex numbers is defined by

$$(a + bi)(c + di) := (ac - bd) + (bc + ad)i.$$

To compute the quotient of two complex numbers, we again "rationalize the denominator":

$$\frac{a+bi}{c+di} = \frac{a+bi}{c+di}\,\frac{c-di}{c-di} = \frac{ac+bd}{c^2+d^2} + \frac{bc-ad}{c^2+d^2}\,i.$$

Thus we formally postulate the following:

The division of complex numbers is given by

$$\frac{a+bi}{c+di} := \frac{ac+bd}{c^2+d^2} + \frac{bc-ad}{c^2+d^2}\,i \qquad (\text{if } c^2 + d^2 \neq 0).$$

These are rules for computing in the complex number system. The usual algebraic properties (commutativity, associativity, etc.) are easy to verify and appear as exercises.

[†]The algebraic aspects of extending a number field are discussed in Ref. 5 at the end of this chapter.

[‡]Karl Friedrich Gauss (1777–1855) was the first mathematician to use complex numbers freely and give them full acceptance as genuine mathematical objects.

Example 1

Find the quotient

$$\frac{(6+2i)-(1+3i)}{(-1+i)-2}.$$

Solution.

$$\begin{aligned}\frac{(6+2i)-(1+3i)}{(-1+i)-2} &= \frac{5-i}{-3+i} = \frac{(5-i)}{(-3+i)}\frac{(-3-i)}{(-3-i)} \\ &= \frac{-15-1-5i+3i}{9+1} \\ &= -\frac{8}{5}-\frac{1}{5}i. \quad \blacksquare\end{aligned} \tag{7}$$

(A slug marks the end of solutions or proofs throughout the text.)

Historically, i was considered as an "imaginary" number because of the blatant impossibility of solving Eq. (6) with any of the numbers at hand. With the perspective we have developed, we can see that this label could also be applied to the numbers $\sqrt{2}$ or $\sqrt[4]{17}$; like them, i is simply one more symbol appended to a given number system to create a richer system. Nonetheless, tradition dictates the following designations:†

> **Definition 2.** The **real part** of the complex number $a + bi$ is the (real) number a; its **imaginary part** is the (real) number b. If a is zero, the number is said to be a **pure imaginary number**.

For convenience we customarily use a single letter, usually z, to denote a complex number. Its real and imaginary parts are then written Re z and Im z, respectively. With this notation we have $z = \text{Re}\, z + i\ \text{Im}\, z$.

Observe that the equation $z_1 = z_2$ holds if and only if $\text{Re}\, z_1 = \text{Re}\, z_2$ and $\text{Im}\, z_1 = \text{Im}\, z_2$. Thus any equation involving complex numbers can be interpreted as a pair of real equations.

The set of all complex numbers is sometimes denoted as **C**. Unlike the real number system, there is no natural ordering for the elements of **C**; it is meaningless, for example, to ask whether $2 + 3i$ is greater than or less than $3 + 2i$. (See Prob. 30.)

EXERCISES 1.1

1. Verify that $-i$ is also a root of Eq. (6).
2. Verify the commutative, associative, and distributive laws for complex numbers.

---

†René Descartes introduced the terminology "real" and "imaginary" in 1637. W. R. Hamilton referred to a number's "imaginary part" in 1843.

3. Notice that 0 and 1 retain their "identity" properties as complex numbers; that is, $0 + z = z$ and $1 \cdot z = z$ when z is complex.

   (a) Verify that complex subtraction is the inverse of complex addition (that is, $z_3 = z_2 - z_1$ if and only if $z_3 + z_1 = z_2$).

   (b) Verify that complex division, as given in the text, is the inverse of complex multiplication (that is, if $z_2 \neq 0$, then $z_3 = z_1/z_2$ if and only if $z_3 z_2 = z_1$).

4. Prove that if $z_1 z_2 = 0$, then $z_1 = 0$ or $z_2 = 0$.

In Problems 5–13, write the number in the form $a + bi$.

5. (a) $-3\left(\frac{i}{2}\right)$ (b) $(8 + i) - (5 + i)$ (c) $\frac{2}{i}$

6. (a) $(-1 + i)^2$ (b) $\frac{2 - i}{\frac{1}{3}}$ (c) $i(\pi - 4i)$

7. (a) $\frac{8i - 1}{i}$ (b) $\frac{-1 + 5i}{2 + 3i}$ (c) $\frac{3}{i} + \frac{i}{3}$

8. $\frac{(8 + 2i) - (1 - i)}{(2 + i)^2}$

9. $\frac{2 + 3i}{1 + 2i} - \frac{8 + i}{6 - i}$

10. $\left[\frac{2 + i}{6i - (1 - 2i)}\right]^2$

11. $i^3(i + 1)^2$

12. $(2 + i)(-1 - i)(3 - 2i)$

13. $((3 - i)^2 - 3)i$

14. Show that $\text{Re}(iz) = -\text{Im}\, z$ for every complex number z.

15. Let k be an integer. Show that

$$i^{4k} = 1, \qquad i^{4k+1} = i, \qquad i^{4k+2} = -1, \qquad i^{4k+3} = -i.$$

16. Use the result of Problem 15 to find

   (a) i^7 (b) i^{62} (c) i^{-202} (d) i^{-4321}

17. Use the result of Problem 15 to evaluate

$$3i^{11} + 6i^3 + \frac{8}{i^{20}} + i^{-1}.$$

18. Show that the complex number $z = -1 + i$ satisfies the equation

$$z^2 + 2z + 2 = 0.$$

19. Write the complex equation $z^3 + 5z^2 = z + 3i$ as two real equations.

20. Solve each of the following equations for z.

(a) $iz = 4 - zi$ **(b)** $\dfrac{z}{1-z} = 1 - 5i$

(c) $(2-i)z + 8z^2 = 0$ **(d)** $z^2 + 16 = 0$

21. The complex numbers z_1, z_2 satisfy the system of equations

$$(1-i)z_1 + 3z_2 = 2 - 3i,$$
$$iz_1 + (1+2i)z_2 = 1.$$

Find z_1, z_2.

22. Find all solutions to the equation $z^4 - 16 = 0$.

23. Let z be a complex number such that $\operatorname{Re} z > 0$. Prove that $\operatorname{Re}(1/z) > 0$.

24. Let z be a complex number such that $\operatorname{Im} z > 0$. Prove that $\operatorname{Im}(1/z) < 0$.

25. Let z_1, z_2 be two complex numbers such that $z_1 + z_2$ and $z_1 z_2$ are each negative real numbers. Prove that z_1 and z_2 must be real numbers.

26. Verify that

$$\operatorname{Re}\Big(\sum_{j=1}^{n} z_j\Big) = \sum_{j=1}^{n} \operatorname{Re} z_j$$

and that

$$\operatorname{Im}\Big(\sum_{j=1}^{n} z_j\Big) = \sum_{j=1}^{n} \operatorname{Im} z_j.$$

[The real (imaginary) part of the sum is the sum of the real (imaginary) parts.] Formulate, and then *disprove*, the corresponding conjectures for multiplication.

27. Prove the *binomial formula* for complex numbers:

$$(z_1 + z_2)^n = z_1^n + \binom{n}{1} z_1^{n-1} z_2 + \cdots + \binom{n}{k} z_1^{n-k} z_2^k + \cdots + z_2^n,$$

where n is a positive integer, and the *binomial coefficients* are given by

$$\binom{n}{k} := \frac{n!}{k!(n-k)!}.$$

28. Use the binomial formula (Prob. 27) to compute $(2-i)^5$.

29. Prove that there is no rational number x that satisfies $x^2 = 2$. [HINT: Show that if p/q were a solution, where p and q are integers, then 2 would have to divide both p and q. This contradicts the fact that such a ratio can always be written without common divisors.]