TEXTBOOKS IN MATHEMATICS

# Analysis for Engineers

Methods and Applications
SECOND EDITION

Bilal M. Ayyub Richard H. McCuen



# Numerical Analysis for Engineers Methods and Applications SECOND EDITION



Richard H. McCuen

University of Maryland College Park, USA



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# Numerical Analysis for Engineers

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To my wife, Deena.
—Bilal M. Ayyub

# **Preface**

In preparing this book, we strove to achieve a set of educational objectives that include (1) introducing numerical methods to engineering students and practicing engineers, (2) emphasizing the practical aspects of the use of these methods, and (3) establishing the limitations, advantages, and disadvantages of the methods. Although the book was developed with emphasis on engineering and technological problems, the numerical methods can also be used to solve problems in other fields of science.

The book demonstrates the power of numerical methods in the context of solving complex engineering and scientific problems. Studies show that engineering practice will continue to tackle more complex design problems, thus necessitating increased reliance on numerical methods because of the lack of analytical solution methodologies. Therefore, this book is intended to better prepare future engineers, as well as assist practicing engineers, in understanding the fundamentals of numerical methods, especially their applications, limitations, and potentials.

### STRUCTURE, FORMAT, AND MAIN FEATURES

We have developed this book with a dual use in mind: as a self-learning guidebook and as a required textbook for a course. In either case, the text has been designed to achieve important educational objectives.

The 11 chapters of the book cover the following subjects: (1) introduction to the text, (2) matrix analysis, (3) introduction to numerical methods, (4) the identification of the roots of equations, (5) the solution of linear simultaneous equations, (6) the interpolation of values of a dependent variable for a given set of discrete measurements, (7) approximation of the differential or integral of an unknown function for a given set of discrete measurements from the function, (8) solutions to differential equations, (9) describing data graphically and computing important characteristics of sample measurements and making basic statistical computations, (10) fitting a curve or a model to data, and (11) numerical optimization. The book was designed for an introductory course in numerical analysis with emphasis on applications. In developing the book, a set of educational outcomes as detailed in Chapter 1 motivated the structure and content of its text. Ultimately, serious readers will find the material very useful in engineering problem solving and decision making.

Each chapter of the book contains many computational examples, as well as a section on applications that contain additional engineering examples. Also, each chapter includes a set of exercise problems that cover the materials of the chapter. The problems were designed carefully to meet the needs of instructors in assigning homework and the readers in practicing the fundamental concepts.

The book can be covered in one semester or two semesters in conjunction with a programming language. The chapter sequence can be followed as a recommended sequence. However, if needed, instructors can choose a subset of the chapters of the book for courses that do not

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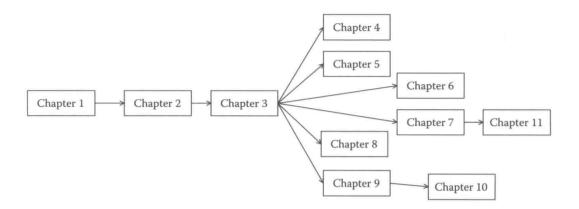


FIGURE P.1 Logical sequence of chapters in the book in terms of their interdependencies.

permit complete coverage of the materials or require coverage that cannot follow the presented order. After completing Chapters 1, 2, and 3, the readers will have sufficient background to follow and understand the materials in Chapters 4 through 8 in any sequence. However, before reading Chapter 10, Chapter 9 should be studied. Figure P.1 illustrates the logical sequence of these chapters.

# Acknowledgments

This book was developed over several years and draws on our experiences in teaching courses on numerical methods and statistics. The first edition of this book was used repeatedly in several courses at the University of Maryland for about two decades and has been used at many institutions worldwide. This usage has proved to be a very valuable tool in establishing the second edition's contents and final format and structure.

We would like to acknowledge all the students who participated in the testing of the first edition of the book for their comments, input, and suggestions. The students who took courses on computational methods in civil engineering in the semesters of 1992 and 1994 contributed to this endeavor. Their feedback was very helpful and greatly contributed to the final product. Also, we would like to acknowledge the instructors who used the book during this testing period, Drs. P. Johnson and R. Muhanna. The assistance of Ru-Jen Chao, Dr. Maguid Hassan, and Dr. Ibrahim Assakkaf in critically reviewing the problems at the ends of the chapters and developing some of the example problems and problem solutions is gratefully acknowledged. The reviewers' comments that were provided to us by the publisher were used to improve the book to meet the needs of instructors and enhance the education process. We acknowledge the publisher and the reviewers for these comments.

We would like to invite users of this book, including both readers and instructors, to send us any comments on the book by e-mail at ba@umd.edu. These comments will be used in developing future editions of the book.

Bilal M. Ayyub Richard H. McCuen

# **Authors**

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Richard H. McCuen, PhD, is the Ben Dyer Professor of civil and environmental engineering at the University of Maryland, College Park. Dr. McCuen earned degrees from Carnegie Mellon University and the Georgia Institute of Technology. He received the Icko Iben Award from the American Water Resource Association and was co-recipient of the 1988 outstanding research award from the American Society of Civil Engineers Water Resources Planning and Management Division. He received the 2015 VenTe Chow Award from the American Society of Civil Engineers. Topics in statistical hydrology and storm water management are his primary research interest. Dr. McCuen is the author of 26 books and over 250 professional papers, including Modeling Hydrologic Change (CRC Press, 2002); Hydrologic Analysis and Design, third edition (Prentice Hall, 2005); The Elements of Academic Research (ASCE Press, 1996); Estimating Debris Volumes for Flood Control (Lighthouse Publications, 1996; with T. V. Hromadka); and Dynamic Communication for Engineers (ASCE Press, 1993; with P. Johnson and C. Davis).

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