

CLASSICAL THEORETICAL PHYSICS



Greiner

CLASSICAL
ELECTRODYNAMICS

经典电动力学

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Walter Greiner

CLASSICAL ELECTRODYNAMICS

Foreword by D. Allan Bromley

With 284 Figures

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Library of Congress Cataloging-in-Publication Data
Greiner, Walter, 1935–

[Klassische Elektrodynamik. English]

Classical electrodynamics/Walter Greiner.

p. cm.—(Classical theoretical physics)

Includes bibliographical references and index.

ISBN 0-387-94799-X (softcover: alk. paper)

I. Electrodynamics. I. Title. II. Series.

QC631.G79513 1996

537.6—dc20

96-15530

Printed on acid-free paper.

First German edition, *Klassische Elektrodynamik*, © 1991 Verlag Harri Deutsch.

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ISBN 0-387-94799-X Springer-Verlag New York Berlin Heidelberg

Foreword

More than a generation of German-speaking students around the world have worked their way to an understanding and appreciation of the power and beauty of modern theoretical physics—with mathematics, the most fundamental of sciences—using Walter Greiner's textbooks as their guide.

The idea of developing a coherent, complete presentation of an entire field of science in a series of closely related textbooks is not a new one. Many older physicians remember with real pleasure their sense of adventure and discovery as they worked their ways through the classic series by Sommerfeld, by Planck, and by Landau and Lifshitz. From the students' viewpoint, there are a great many obvious advantages to be gained through the use of consistent notation, logical ordering of topics, and coherence of presentation; beyond this, the complete coverage of the science provides a unique opportunity for the author to convey his personal enthusiasm and love for his subject.

These volumes on classical physics, finally available in English, complement Greiner's texts on quantum physics, most of which have been available to English-speaking audiences for some time. The complete set of books will thus provide a coherent view of physics that includes, in classical physics, thermodynamics and statistical mechanics, classical dynamics, electromagnetism, and general relativity; and in quantum physics, quantum mechanics, symmetries, relativistic quantum mechanics, quantum electro- and chromodynamics, and the gauge theory of weak interactions.

What makes Greiner's volumes of particular value to the student and professor alike is their completeness. Greiner avoids the all too common "it follows that . . .," which conceals several pages of mathematical manipulation and confounds the student. He does not hesitate to include experimental data to illuminate or illustrate a theoretical point, and these data, like the theoretical content, have been kept up to date and topical through frequent revision and expansion of the lecture notes upon which these volumes are based.

Moreover, Greiner greatly increases the value of his presentation by including something like one hundred completely worked examples in each volume. Nothing is of greater importance to the student than seeing, in detail, how the theoretical concepts and tools

under study are applied to actual problems of interest to working physicists. And, finally, Greiner adds brief biographical sketches to each chapter covering the people responsible for the development of the theoretical ideas and/or the experimental data presented. It was Auguste Comte (1789–1857) in his *Positive Philosophy* who noted, “To understand a science it is necessary to know its history.” This is all too often forgotten in modern physics teaching, and the bridges that Greiner builds to the pioneering figures of our science upon whose work we build are welcome ones.

Greiner’s lectures, which underlie these volumes, are internationally noted for their clarity, for their completeness, and for the effort that he has devoted to making physics an integral whole. His enthusiasm for his sciences is contagious and shines through almost every page.

These volumes represent only a part of a unique and Herculean effort to make all of theoretical physics accessible to the interested student. Beyond that, they are of enormous value to the professional physicist and to all others working with quantum phenomena. Again and again, the reader will find that, after dipping into a particular volume to review a specific topic, he or she will end up browsing, caught up by often fascinating new insights and developments with which he or she had not previously been familiar.

Having used a number of Greiner’s volumes in their original German in my teaching and research at Yale, I welcome these new and revised English translations and would recommend them enthusiastically to anyone searching for a coherent overview of physics.

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Preface

Classical Electrodynamics contains the lectures that form part of the course of study in theoretical physics at the Johann Wolfgang Goethe University in Frankfurt am Main. There they are given for students in physics and mathematics in their third semester and are preceded by Theoretical Mechanics I (first semester) and Theoretical Mechanics II (second semester). Quantum Mechanics I—An Introduction then completes the first part of the lectures. Graduate course work continues with Quantum Mechanics II—Symmetries and Relativistic Quantum Mechanics (fifth semester), Thermodynamics and Statistics, Quantum Electrodynamics, Field Quantization, Gauge Theory of Weak Interaction, Quantum Chromodynamics, General Relativity and Cosmology, Nuclear and Solid State Theory, and other, more specialized courses in Many Particle Theory, etc.

As in all other fields mentioned, we present classical electrodynamics according to the inductive method that comes closest to the methodology of the research physicist. Starting from some key experimental observations, the framework of the theory is developed step by step, and after the basic equations, that is, the Maxwell equations, have been obtained, new phenomena are investigated from thereon.

This leads to electrostatics and magnetostatics and their application to macroscopic problems and further to the theory of electromagnetic waves in vacuum, which are among the most fascinating consequences of Maxwell's equations. We follow Maxwell theory with respect to basic, field theoretical questions (energy, momentum of the field) and its application to establish optics (laws of reflection and refraction, frequency dependency and conductivity, polarization and index of refraction), as well as in the sector of practical application (propagation of waves, wave guides, resonance cavity, etc.). Also, the covariant formulation of electrodynamics in the framework of the theory of special relativity is presented, ending with the relativistically covariant Lagrange formalism. Many worked-out examples and exercises illustrate the general theory and its applications.

Finally, biographical and historical footnotes as well as an extra section on the history of electrodynamics anchor the scientific development within the general context of scientific progress and evolution. In this context, I thank the publishers Harri Deutsch and

F.A. Brockhaus (*Brockhaus Enzyklopädie*, F.A. Brockhaus, Wiesbaden—marked by BR) for giving permission to extract the biographical data of physicists and mathematicians from their publications.

The lectures are now up for their fifth German edition. Over the years, many students and collaborators have helped to work out exercises and illustrative examples. For the first English edition, I enjoyed the help of Ulrich Eichmann, Nils Hammon, Oliver Martin, and Panajotis Papazoglou. The coordinatory help of Sven Soff is particularly appreciated.

Finally, I am pleased to acknowledge the agreeable collaboration with Dr. Thomas von Foerster and his team at Springer-Verlag New York, Inc. The English manuscript was copyedited by Margaret Marynowski, and the production of the book was supervised by Francine McNeill.

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PART



ELECTROSTATICS

1 Introduction and Fundamental Concepts

In the investigation of the properties of charged bodies at rest the following results have been obtained experimentally: charged bodies (charges) exert a force on each other. There are two kinds of charges, positive and negative ones. Unlike charges attract each other, like charges repel each other. The force between two charges q_1 and q_2 is proportional to their product:

$$F_{12} \sim q_1 q_2$$

The force decreases with the square of the mutual distance, that is,

$$F_{12} \sim \frac{1}{|\mathbf{r}_1 - \mathbf{r}_2|^2}$$

The electrostatic forces are central forces. Thus, for the force exerted by the charge 2 on the charge 1 we can write

$$\mathbf{F}_{12} = k q_1 q_2 \frac{\mathbf{r}_1 - \mathbf{r}_2}{|\mathbf{r}_1 - \mathbf{r}_2|^3} \quad (1.1)$$

k is a constant of proportionality still to be fixed. See Figure 1.1. This equation for the

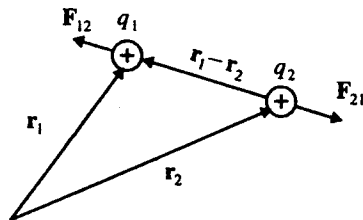


Figure 1.1. On Coulomb's law: Like charges repel each other.

force acting between two charges is called *Coulomb's law*. Furthermore, the principle of *superposition* is valid: The electric forces exerted by several charges q_2, q_3, \dots on a test charge q_1 superpose each other undisturbed without the force between q_1 and a certain charge (e.g., q_2) being changed due to the presence of other charges. In particular, this implies that the forces between charges can be merely *two-body forces*; many-body forces do not occur. For *many-body forces* the force between two bodies 1 and 2 depends also on the positions of the other bodies r_3, r_4, \dots . For example, a three-body force would be

$$\mathbf{F}_{12} = kq_1q_2 \frac{\mathbf{r}_1 - \mathbf{r}_2}{\left| (\mathbf{r}_1 - \mathbf{r}_2) \left(1 + \frac{q_3^2}{q_1q_2} \frac{|\mathbf{r}_1 - \mathbf{r}_2|}{|\mathbf{r}_s - \mathbf{r}_3|^3} \right) \right|^3} \quad (1.2)$$

Here, \mathbf{r}_s is the center of gravity between q_1 and q_2 . See Figure 1.2. This three-body force would tend to a two-body force (1.1) as $\mathbf{r}_3 \rightarrow \infty$, as should be. Microscopically, one can imagine that the force (i.e., a force field) originates from the virtual exchange of particles. These are thrown back and forth between the centers, like tennis balls, and in this way they bind the centers to each other. For two-body forces, this exchange proceeds between two centers only; for three- (many-)body forces, a detour via the third center (or several centers) occurs. (See Figure 1.3.) In the Coulomb interaction photons are exchanged; in the weak interaction, Z- and W-bosons; in the gravitational interaction, gravitons; and in the strong (nuclear) interaction, π -mesons (or, on a deeper level, gluons). The photons and gravitons have a rest mass equal to zero. Therefore, these forces are of infinite range. On the other hand, the short range of the strong interaction ($\sim 2\text{fm} = 2 \cdot 10^{-13}\text{cm}$) is based on the finite rest mass of the π -meson. Nowadays, we know that pions and nucleons are built up out of quarks. The quarks interact by the exchange of heavy photons (interacting intensively among each other and coupling to the so-called glue-balls). These heavy photons are called

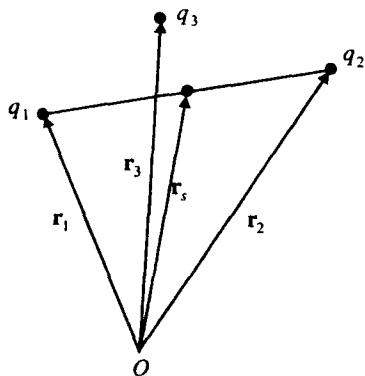


Figure 1.2. For the explanation of a three-body force: The charges q_i are placed at the position vectors \mathbf{r}_i . The vector to the center of gravity of the charges q_1 and q_2 is \mathbf{r}_s .

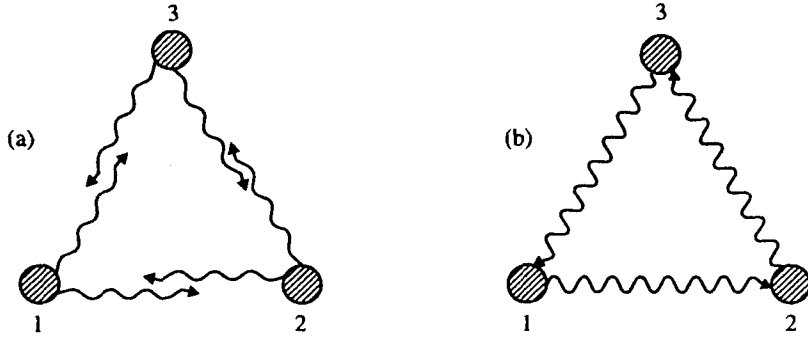


Figure 1.3. (a) Exchange of particles in a two-body interaction. (b) Exchange of particles in a three-body force.

gluons. For the ordinary Coulomb force in the presence of further charges q_i the force exerted on charge q_1 reads

$$\mathbf{F} = kq_1 \sum_{i=2}^N q_i \frac{\mathbf{r}_1 - \mathbf{r}_i}{|\mathbf{r}_1 - \mathbf{r}_i|^3} \quad (1.3)$$

In this form, Coulomb's law is valid exactly only for point charges and for uniformly charged spherical bodies. For charges of arbitrary shape, deviations appear which will be discussed later on. Nevertheless, one should wonder about the $1/r^2$ -dependence of the Coulomb force. This particular force law is related to the fact that the rest mass of the photons exchanged by the charges is zero. According to Heisenberg's uncertainty relation they can then be produced virtually with a long range R . The uncertainty relation states

$$\begin{aligned} \Delta E \Delta t &\sim \hbar, & \Delta E = \text{uncertainty of energy} &\approx \mu c^2 \\ \Delta t &\sim \frac{\hbar}{\Delta E}, & \Delta t = \text{uncertainty of time} &\approx \frac{\hbar}{\mu c^2} \end{aligned} \quad (1.4)$$

The latter expression gives the lifetime of a virtual particle having the rest mass $\Delta E \approx \mu c^2$, and thus the range is

$$R = c\Delta t \sim \frac{\hbar c}{\Delta E}$$

This explains the long range of the Coulomb force. If the photon had the rest mass μ , then the Coulomb potential (compare the following pages) would have to be of the Yukawa type, namely,

$$V(r) \sim \frac{e^{-r/\lambda}}{r} \quad (1.5)$$

Here, $\lambda = \hbar/\mu c$ would be the so-called Compton wavelength of a photon having the rest mass μ . For $\mu = 0$ one obtains the Coulomb potential of a point charge. Nowadays, the best

precision measurements of the photon mass yield a value of $\mu c^2 \leq 5 \cdot 10^{-16} \text{eV}$.¹ The charge represents a new physical property of bodies. Now, we may introduce a new dimension for the charge, or express it in terms of the dimensions mass, length, and time used in mechanics. The product kq_1q_2 is fixed in equation (1.1). Subject to this condition the dimension of the single factors, charge and constant of proportionality, can still be chosen freely. *Depending on the choice of k we get different systems of units.* In textbooks, mainly two different systems of units are still used nowadays, the Gaussian and the rationalized system of units. In the *Gaussian system of units* the constant of proportionality k takes the numerical value 1 and remains nondimensional. Then, the charge is no longer an independent unit. In the CGS-system, one obtains from equation (1.1) the unit $1 \text{cm}^{3/2} \text{g}^{1/2} \text{s}^{-1}$ for the charge, which is also denoted the electrostatic unit (esu) or statCoulomb. This explicit tracing back of electromagnetic quantities to mechanical units can be found virtually only in older textbooks; in more recent textbooks on atomic and nuclear physics or quantum mechanics using the Gaussian system of units the charge is handled like an independent unit; thereby the physical interrelations often become clearer. Setting $|\mathbf{r}_1 - \mathbf{r}_2| = r$, equation (1.1) takes the simple form:

$$F = \frac{q_1 q_2}{r^2} \quad (1.6)$$

The opposite line is taken in the so-called rationalized system of units. Here, the unit is fixed by the charge. Its value is determined by measuring the force exerted by two current-carrying conductors on each other. According to the definition, when a current of one Ampere is flowing through two parallel, rectilinear, infinitely long conductors placed at a distance of one meter from each other, a force of $2 \cdot 10^{-7}$ newton per meter of their length acts between them. The product of current and time gives the quantity of charge:

$$1 \text{ Coulomb (C)} = 1 \text{ Ampere second (As)}$$

By this (arbitrary) definition, the constant of proportionality k takes a dimension as well as a fixed numerical value; one sets

$$k = \frac{1}{4\pi\epsilon_0} \quad (1.7)$$

The constant ϵ_0 is called the *permittivity of vacuum*; it has the value

$$\epsilon_0 = 8.854 \cdot 10^{-12} \left[\frac{\text{As}}{\text{Vm}} \right] \approx \frac{1}{4\pi \cdot 9 \cdot 10^9} \left[\frac{\text{As}}{\text{Vm}} \right] \quad (1.8)$$

¹We refer to the seminar held by W. Martienssen and his graduate students P. Kurowski and J. Wagner; prepr. Physikalisches Institut, Universität Frankfurt/M. (1974). A lab test of Coulomb's law is described by E.R. Williams, J.E. Faller, and H.A. Hill in "New Experimental Test of Coulomb's Law: A Laboratory Upper Limit on the Photon Rest Mass," *Phys. Rev. Letters* 26 (1971) 721.

(For the moment, we consider the unit volt (V) as an abbreviation of $1V = 1A^{-1}m^2kg s^{-3} = 1NmC^{-1}$.) In the framework of this system of units, Coulomb's law reads

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1q_2}{r^2} \quad (1.9)$$

A comparison of (1.9) and (1.6) then yields the relation between the charges in the Gaussian system of units (q) and in the rationalized system of units (q^*) (also called the mksA—(meter kilogram second Ampere)—system). It reads

$$q = \frac{1}{\sqrt{4\pi\epsilon_0}} q^* \quad (1.10)$$

The unit of q^* is 1 Coulomb = 1 Ampere second. In the Gaussian system, this corresponds to

$$\begin{aligned} 1 \text{ Coulomb} \cdot \frac{1}{\sqrt{4\pi\epsilon_0}} &\approx 1As \frac{1}{\sqrt{4\pi \frac{1}{4\pi 9 \cdot 10^9} \frac{As}{Vm}}} \\ &= 1As \sqrt{9 \cdot 10^9 \frac{Vm}{As}} = \sqrt{9 \cdot 10^9 Vm As} \\ &= \sqrt{9 \cdot 10^9 \frac{m^3 kg}{s^2}} = \sqrt{9 \cdot 10^{18} \frac{cm^3 g}{s^2}} \\ &= 3 \cdot 10^9 \sqrt{\text{erg cm}} = 3 \cdot 10^9 \text{ cgs charge units} \\ &\equiv 3 \cdot 10^9 \text{ statCoulomb} \end{aligned} \quad (1.11)$$

In macroscopic physics and experimental physics, the rationalized system of units is used predominantly. In atomic physics, nuclear physics, and many textbooks in theoretical physics, the Gaussian system of units is used mostly. *Here, we will use exclusively the Gaussian system of units.*

The electric field intensity

To explain the notion of the electric field intensity, we start from the force \mathbf{F} exerted by a charge q_1 on a test charge q that is as small as possible. The field intensity caused by q_1 at the position \mathbf{r} of the charge q is defined by the quotient:

$$\mathbf{E}(\mathbf{r}) = \frac{\mathbf{F}}{q} \quad (1.12)$$

Since, in general, the electric field is altered by the test charge q , we take the limit of an infinitely small charge:

$$\mathbf{E} = \lim_{\Delta q \rightarrow 0} \frac{\Delta \mathbf{F}}{\Delta q} = \frac{d\mathbf{F}}{dq} \quad (1.13)$$

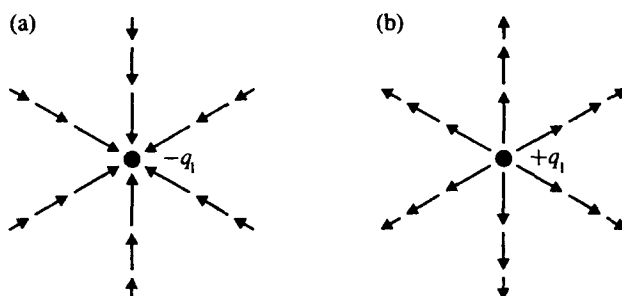


Figure 1.4. (a) E-field of a negative point charge. (b) E-field of a positive point charge.

With Coulomb's law (1.1) the electric field of a point charge q_1 is

$$\mathbf{E}(\mathbf{r}) = \frac{q_1(\mathbf{r} - \mathbf{r}_1)}{|\mathbf{r} - \mathbf{r}_1|^3} \quad (1.14)$$

The electric field vector $\mathbf{E}(\mathbf{r})$ of a positive point charge is directed radially outward; that of a negative point charge is directed radially inward, as depicted in Figure 1.4. According to the *principle of superposition* (1.3), for a system of point charges we have

$$\mathbf{E}(\mathbf{r}) = \sum_i \frac{q_i(\mathbf{r} - \mathbf{r}_i)}{|\mathbf{r} - \mathbf{r}_i|^3} = \sum_i \mathbf{E}_i \quad (1.15)$$

This is shown in Figure 1.5. In the presence of a continuous charge distribution, we have to go from a summation over the point charges to an integration over the spatial distribution (see Figure 1.6). Instead of the point charge q_i we have to insert the charge element $\rho(\mathbf{r}') dV'$. Here, ρ and dV are the charge density and the volume element, respectively:

$$\mathbf{E}(\mathbf{r}) = \int \rho(\mathbf{r}') \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3} dV' \quad (1.16)$$

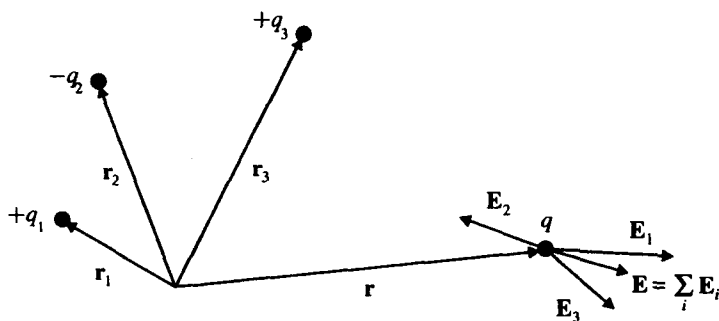


Figure 1.5. The electric field intensity at the position \mathbf{r} for a sum of point charges q_i .