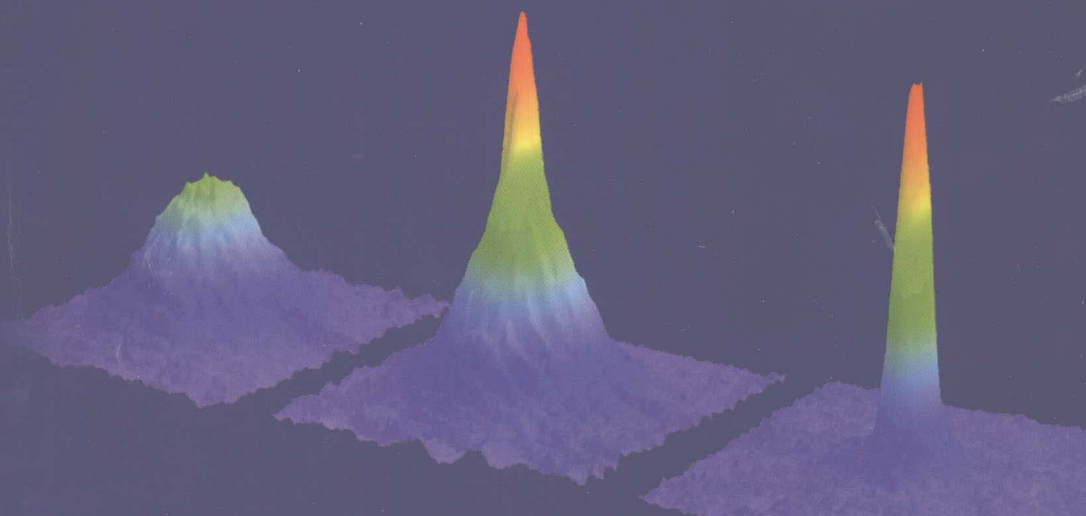
Bose–Einstein Condensation in Dilute Gases

稀化气体中的玻色——爱因斯坦凝聚



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Bose–Einstein Condensation in Dilute Gases

In 1925 Einstein predicted that at low temperatures particles in a gas could all reside in the same quantum state. This peculiar gaseous state, a Bose–Einstein condensate, was produced in the laboratory for the first time in 1995 using the powerful laser-cooling methods developed in recent years. These condensates exhibit quantum phenomena on a large scale, and investigating them has become one of the most active areas of research in contemporary physics.

The study of Bose–Einstein condensates in dilute gases encompasses a number of different subfields of physics, including atomic, condensed matter, and nuclear physics. The authors of this textbook explain this exciting new subject in terms of basic physical principles, without assuming detailed knowledge of any of these subfields. This pedagogical approach therefore makes the book useful for anyone with a general background in physics, from undergraduates to researchers in the field.

Chapters cover the statistical physics of trapped gases, atomic properties, the cooling and trapping of atoms, interatomic interactions, structure of trapped condensates, collective modes, rotating condensates, superfluidity, interference phenomena and trapped Fermi gases. Problem sets are also included in each chapter.

CHRISTOPHER PETHICK graduated with a D.Phil. in 1965 from the University of Oxford, and he had a research fellowship there until 1970. During the years 1966–69 he was a postdoctoral fellow at the University of Illinois at Urbana–Champaign, where he joined the faculty in 1970, becoming Professor of Physics in 1973. Following periods spent at the Landau Institute for Theoretical Physics, Moscow and at Nordita (Nordic Institute for Theoretical Physics), Copenhagen, as a visiting scientist, he accepted a permanent position at Nordita in 1975, and divided his time for many years between Nordita and the University of Illinois. Apart from the subject of the present book, Professor Pethick's main research interests are condensed matter physics (quantum liquids, especially ^{3}He, ^{4}He and superconductors) and astrophysics (particularly the properties of dense matter and the interiors of neutron stars). He is also the co-author of *Landau Fermi-Liquid Theory: Concepts and Applications* (1991).

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The two authors have worked together on problems in low-temperature physics, in particular on the superfluid phases of liquid ^{3}He, superconductors and dilute quantum gases. This book derives from graduate-level lectures given by the authors at the University of Copenhagen.

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Preface

The experimental discovery of Bose–Einstein condensation in trapped atomic clouds opened up the exploration of quantum phenomena in a qualitatively new regime. Our aim in the present work is to provide an introduction to this rapidly developing field.

The study of Bose–Einstein condensation in dilute gases draws on many different subfields of physics. Atomic physics provides the basic methods for creating and manipulating these systems, and the physical data required to characterize them. Because interactions between atoms play a key role in the behaviour of ultracold atomic clouds, concepts and methods from condensed matter physics are used extensively. Investigations of spatial and temporal correlations of particles provide links to quantum optics, where related studies have been made for photons. Trapped atomic clouds have some similarities to atomic nuclei, and insights from nuclear physics have been helpful in understanding their properties.

In presenting this diverse range of topics we have attempted to explain physical phenomena in terms of basic principles. In order to make the presentation self-contained, while keeping the length of the book within reasonable bounds, we have been forced to select some subjects and omit others. For similar reasons and because there now exist review articles with extensive bibliographies, the lists of references following each chapter are far from exhaustive. A valuable source for publications in the field is the archive at Georgia Southern University: http://amo.phy.gasou.edu/bec.html

This book originated in a set of lecture notes written for a graduate-level one-semester course on Bose–Einstein condensation at the University of Copenhagen. We have received much inspiration from contacts with our colleagues in both experiment and theory. In particular we thank Gordon Baym and George Kavoulakis for many stimulating and helpful discussions over the past few years. Wolfgang Ketterle kindly provided us with the cover illustration and Fig. 13.1. The illustrations in the text have been prepared by Janus Schmidt, whom we thank for a pleasant collaboration. It is a pleasure to acknowledge the continuing support of Simon Capelin and Susan Francis at the Cambridge University Press, and the careful copy-editing of the manuscript by Brian Watts.

Copenhagen Christopher Pethick Henrik Smith

Contents

1

Introduction

Bose–Einstein condensates in dilute atomic gases, which were first realized experimentally in 1995 for rubidium [1], sodium [2], and lithium [3], provide unique opportunities for exploring quantum phenomena on a macroscopic scale.[1] These systems differ from ordinary gases, liquids, and solids in a number of respects, as we shall now illustrate by giving typical values of some physical quantities.

The particle density at the centre of a Bose–Einstein condensed atomic cloud is typically 10^{13}–10^{15} cm^{-3}. By contrast, the density of molecules in air at room temperature and atmospheric pressure is about 10^{19} cm^{-3}. In liquids and solids the density of atoms is of order 10^{22} cm^{-3}, while the density of nucleons in atomic nuclei is about 10^{38} cm^{-3}.

To observe quantum phenomena in such low-density systems, the temperature must be of order 10^{-5} K or less. This may be contrasted with the temperatures at which quantum phenomena occur in solids and liquids. In solids, quantum effects become strong for electrons in metals below the Fermi temperature, which is typically 10^4–10^5 K, and for phonons below the Debye temperature, which is typically of order 10^2 K. For the helium liquids, the temperatures required for observing quantum phenomena are of order 1 K. Due to the much higher particle density in atomic nuclei, the corresponding degeneracy temperature is about 10^{11} K.

The path that led in 1995 to the first realization of Bose–Einstein condensation in dilute gases exploited the powerful methods developed over the past quarter of a century for cooling alkali metal atoms by using lasers. Since laser cooling alone cannot produce sufficiently high densities and low temperatures for condensation, it is followed by an evaporative cooling stage, in

[1] Numbers in square brackets are references, to be found at the end of each chapter.

which the more energetic atoms are removed from the trap, thereby cooling the remaining atoms.

Cold gas clouds have many advantages for investigations of quantum phenomena. A major one is that in the Bose–Einstein condensate, essentially all atoms occupy the same quantum state, and the condensate may be described very well in terms of a mean-field theory similar to the Hartree–Fock theory for atoms. This is in marked contrast to liquid ^{4}He, for which a mean-field approach is inapplicable due to the strong correlations induced by the interaction between the atoms. Although the gases are dilute, interactions play an important role because temperatures are so low, and they give rise to collective phenomena related to those observed in solids, quantum liquids, and nuclei. Experimentally the systems are attractive ones to work with, since they may be manipulated by the use of lasers and magnetic fields. In addition, interactions between atoms may be varied either by using different atomic species, or, for species that have a Feshbach resonance, by changing the strength of an applied magnetic or electric field. A further advantage is that, because of the low density, 'microscopic' length scales are so large that the structure of the condensate wave function may be investigated directly by optical means. Finally, real collision processes play little role, and therefore these systems are ideal for studies of interference phenomena and atom optics.

The theoretical prediction of Bose–Einstein condensation dates back more than 75 years. Following the work of Bose on the statistics of photons [4], Einstein considered a gas of non-interacting, massive bosons, and concluded that, below a certain temperature, a finite fraction of the total number of particles would occupy the lowest-energy single-particle state [5]. In 1938 Fritz London suggested the connection between the superfluidity of liquid ^{4}He and Bose–Einstein condensation [6]. Superfluid liquid ^{4}He is the prototype Bose–Einstein condensate, and it has played a unique role in the development of physical concepts. However, the interaction between helium atoms is strong, and this reduces the number of atoms in the zero-momentum state even at absolute zero. Consequently it is difficult to measure directly the occupancy of the zero-momentum state. It has been investigated experimentally by neutron scattering measurements of the structure factor at large momentum transfers [7], and the measurements are consistent with a relative occupation of the zero-momentum state of about 0.1 at saturated vapour pressure and about 0.05 near the melting curve [8].

The fact that interactions in liquid helium reduce dramatically the occupancy of the lowest single-particle state led to the search for weakly-interacting Bose gases with a higher condensate fraction. The difficulty with most substances is that at low temperatures they do not remain gaseous, but form solids, or, in the case of the helium isotopes, liquids, and the effects of interaction thus become large. In other examples atoms first combine to form molecules, which subsequently solidify. As long ago as in 1959 Hecht [9] argued that spin-polarized hydrogen would be a good candidate for a weakly-interacting Bose gas. The attractive interaction between two hydrogen atoms with their electronic spins aligned was then estimated to be so weak that there would be no bound state. Thus a gas of hydrogen atoms in a magnetic field would be stable against formation of molecules and, moreover, would not form a liquid, but remain a gas to arbitrarily low temperatures.

Hecht's paper was before its time and received little attention, but his conclusions were confirmed by Stwalley and Nosanow [10] in 1976, when improved information about interactions between spin-aligned hydrogen atoms was available. These authors also argued that because of interatomic interactions the system would be a superfluid as well as being Bose–Einstein condensed. This latter paper stimulated the quest to realize Bose–Einstein condensation in atomic hydrogen. Initial experimental attempts used a high magnetic field gradient to force hydrogen atoms against a cryogenically cooled surface. In the lowest-energy spin state of the hydrogen atom, the electron spin is aligned opposite the direction of the magnetic field (H$\downarrow$), since then the magnetic moment is in the same direction as the field. Spin-polarized hydrogen was first stabilized by Silvera and Walraven [11]. Interactions of hydrogen with the surface limited the densities achieved in the early experiments, and this prompted the Massachusetts Institute of Technology (MIT) group led by Greytak and Kleppner to develop methods for trapping atoms purely magnetically. In a current-free region, it is impossible to create a local maximum in the magnitude of the magnetic field. To trap atoms by the Zeeman effect it is therefore necessary to work with a state of hydrogen in which the electronic spin is polarized parallel to the magnetic field (H$\uparrow$). Among the techniques developed by this group is that of evaporative cooling of magnetically trapped gases, which has been used as the final stage in all experiments to date to produce a gaseous Bose–Einstein condensate. Since laser cooling is not feasible for hydrogen, the gas is precooled cryogenically. After more than two decades of heroic experimental work, Bose–Einstein condensation of atomic hydrogen was achieved in 1998 [12].

As a consequence of the dramatic advances made in laser cooling of alkali atoms, such atoms became attractive candidates for Bose–Einstein condensation, and they were used in the first successful experiments to produce a gaseous Bose–Einstein condensate. Other atomic species, among them noble gas atoms in excited states, are also under active investigation, and in 2001 two groups produced condensates of metastable ^{4}He atoms in the lowest spin-triplet state [13, 14].

The properties of interacting Bose fluids are treated in many texts. The reader will find an illuminating discussion in the volume by Nozières and Pines [15]. A collection of articles on Bose–Einstein condensation in various systems, prior to its discovery in atomic vapours, is given in [16], while more recent theoretical developments have been reviewed in [17]. The 1998 Varenna lectures describe progress in both experiment and theory on Bose–Einstein condensation in atomic gases, and contain in addition historical accounts of the development of the field [18]. For a tutorial review of some concepts basic to an understanding of Bose–Einstein condensation in dilute gases see Ref. [19].

1.1 Bose–Einstein condensation in atomic clouds

Bosons are particles with integer spin. The wave function for a system of identical bosons is symmetric under interchange of any two particles. Unlike fermions, which have half-odd-integer spin and antisymmetric wave functions, bosons may occupy the same single-particle state. An order-of-magnitude estimate of the transition temperature to the Bose–Einstein condensed state may be made from dimensional arguments. For a uniform gas of free particles, the relevant quantities are the particle mass m, the number density n, and the Planck constant $h = 2\pi\hbar$. The only energy that can be formed from $\hbar$, n, and m is $\hbar^2 n^{2/3}/m$. By dividing this energy by the Boltzmann constant k we obtain an estimate of the condensation temperature T_c,

$$T_c = C\frac{\hbar^2 n^{2/3}}{mk}. \tag{1.1}$$

Here C is a numerical factor which we shall show in the next chapter to be equal to approximately 3.3. When (1.1) is evaluated for the mass and density appropriate to liquid ^{4}He at saturated vapour pressure one obtains a transition temperature of approximately 3.13 K, which is close to the temperature below which superfluid phenomena are observed, the so-called lambda point[2] (T_λ= 2.17 K at saturated vapour pressure).

An equivalent way of relating the transition temperature to the particle density is to compare the thermal de Broglie wavelength λ_T with the

[2] The name lambda point derives from the measured shape of the specific heat as a function of temperature, which near the transition resembles the Greek letter λ.