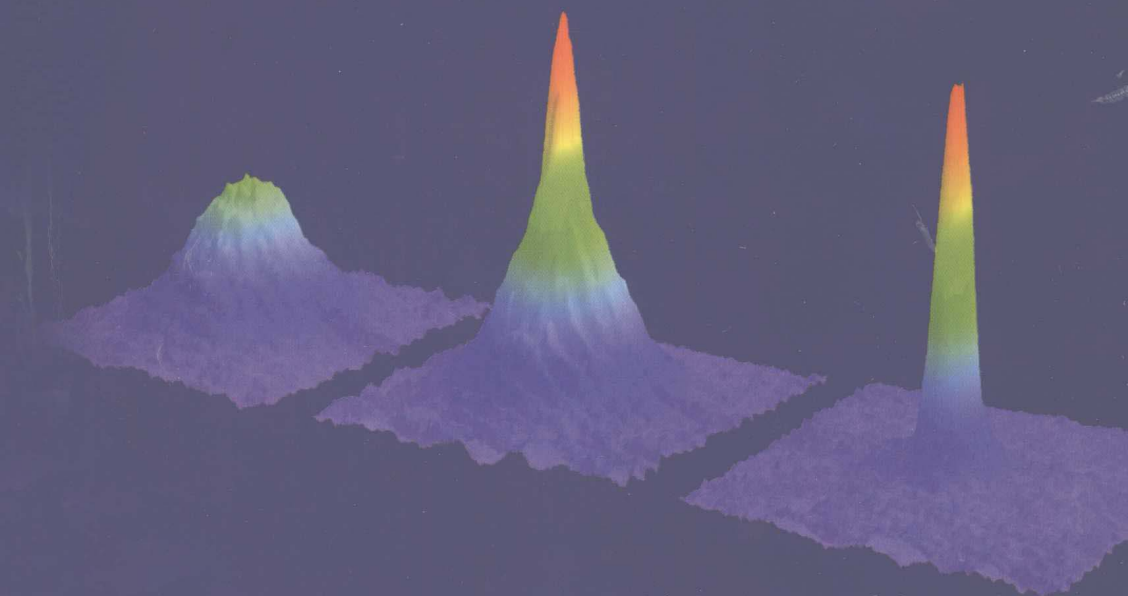


Bose-Einstein Condensation in Dilute Gases

稀化气体中的玻色 ——爱因斯坦凝聚



C. J. Pethick & H. Smith

Cambridge

世界图书出版公司
www.wpcbj.com.cn

Bose–Einstein Condensation in Dilute Gases

C. J. Pethick

Nordita

H. Smith

University of Copenhagen



世界图书出版公司

书 名: Bose-Einstein Condensation in Dilute Gases
作 者: C. J. Pethick, H. Smith
中 译 名: 稀化气体中的玻色-爱因斯坦凝聚
出 版 者: 世界图书出版公司北京公司
印 刷 者: 北京世图印刷厂
发 行: 世界图书出版公司北京公司 (北京朝内大街 137 号 100010)
联系电话: 010-64015659, 64038347
电子信箱: kjsk@vip.sina.com
开 本: 24 开 印 张: 17.5
出版年代: 2005 年 6 月第 1 版 2006 年 2 月第 2 次印刷
书 号: 7-5062-7272-5 / O · 539
版权登记: 图字: 01-2005-2490
定 价: 68.00 元

世界图书出版公司北京公司已获得 Cambridge University Press 授权在中国大陆
独家重印发行。

Bose–Einstein Condensation in Dilute Gases

In 1925 Einstein predicted that at low temperatures particles in a gas could all reside in the same quantum state. This peculiar gaseous state, a Bose–Einstein condensate, was produced in the laboratory for the first time in 1995 using the powerful laser-cooling methods developed in recent years. These condensates exhibit quantum phenomena on a large scale, and investigating them has become one of the most active areas of research in contemporary physics.

The study of Bose–Einstein condensates in dilute gases encompasses a number of different subfields of physics, including atomic, condensed matter, and nuclear physics. The authors of this textbook explain this exciting new subject in terms of basic physical principles, without assuming detailed knowledge of any of these subfields. This pedagogical approach therefore makes the book useful for anyone with a general background in physics, from undergraduates to researchers in the field.

Chapters cover the statistical physics of trapped gases, atomic properties, the cooling and trapping of atoms, interatomic interactions, structure of trapped condensates, collective modes, rotating condensates, superfluidity, interference phenomena and trapped Fermi gases. Problem sets are also included in each chapter.

CHRISTOPHER PETHICK graduated with a D.Phil. in 1965 from the University of Oxford, and he had a research fellowship there until 1970. During the years 1966–69 he was a postdoctoral fellow at the University of Illinois at Urbana–Champaign, where he joined the faculty in 1970, becoming Professor of Physics in 1973. Following periods spent at the Landau Institute for Theoretical Physics, Moscow and at Nordita (Nordic Institute for Theoretical Physics), Copenhagen, as a visiting scientist, he accepted a permanent position at Nordita in 1975, and divided his time for many years between Nordita and the University of Illinois. Apart from the subject of the present book, Professor Pethick's main research interests are condensed matter physics (quantum liquids, especially ^3He , ^4He and superconductors) and astrophysics (particularly the properties of dense matter and the interiors of neutron stars). He is also the co-author of *Landau Fermi-Liquid Theory: Concepts and Applications* (1991).

HENRIK SMITH obtained his mag. scient. degree in 1966 from the University of Copenhagen and spent the next few years as a postdoctoral fellow at Cornell University and as a visiting scientist at the Institute for Theoretical Physics, Helsinki. In 1972 he joined the faculty of the University

of Copenhagen where he became dr. phil. in 1977 and Professor of Physics in 1978. He has also worked as a guest scientist at the Bell Laboratories, New Jersey. Professor Smith's research field is condensed matter physics and low-temperature physics including quantum liquids and the properties of superfluid ^3He , transport properties of normal and superconducting metals, and two-dimensional electron systems. His other books include *Transport Phenomena* (1989) and *Introduction to Quantum Mechanics* (1991).

The two authors have worked together on problems in low-temperature physics, in particular on the superfluid phases of liquid ^3He , superconductors and dilute quantum gases. This book derives from graduate-level lectures given by the authors at the University of Copenhagen.

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS

The Edinburgh Building, Cambridge CB2 2RU, UK
40 West 20th Street, New York, NY 10011-4211, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
Ruiz de Alarcón 13, 28014, Madrid, Spain
Dock House, The Waterfront, Cape Town 8001, South Africa
<http://www.cambridge.org>

© C. J. Pethick, H. Smith 2002

This book is in copyright. Subject to statutory exception
and to the provisions of relevant collective licensing agreements,
no reproduction of any part may take place without
the written permission of Cambridge University Press.

First published 2002
Reprinted 2004

Typeface Computer Modern 11/14pt. *System* L^AT_EX 2_ε [DBD]

A catalogue record of this book is available from the British Library

Library of Congress Cataloguing in Publication Data

Pethick, Christopher.

Bose–Einstein condensation in dilute gases / C. J. Pethick, H. Smith.
p. cm.

Includes bibliographical references and index.

ISBN 0 521 66194 3 – ISBN 0 521 66580 9 (pb.)

1. Bose–Einstein condensation. I. Smith, H. 1939– II. Title.

QC175.47.B65 P48 2001

530.4'2–dc21 2001025622

ISBN 0 521 66194 3 hardback

ISBN 0 521 66580 9 paperback

This reprint edition is published with the permission of the Syndicate of the Press of the
University of Cambridge, Cambridge, England.

THIS EDITION IS LICENSED FOR DISTRIBUTION AND SALE IN THE PEOPLE'S
REPUBLIC OF CHINA ONLY, EXCLUDING TAIWAN, HONG KONG AND MACAO
AND MAY NOT BE DISTRIBUTED AND SOLD ELSEWHERE.

Preface

The experimental discovery of Bose–Einstein condensation in trapped atomic clouds opened up the exploration of quantum phenomena in a qualitatively new regime. Our aim in the present work is to provide an introduction to this rapidly developing field.

The study of Bose–Einstein condensation in dilute gases draws on many different subfields of physics. Atomic physics provides the basic methods for creating and manipulating these systems, and the physical data required to characterize them. Because interactions between atoms play a key role in the behaviour of ultracold atomic clouds, concepts and methods from condensed matter physics are used extensively. Investigations of spatial and temporal correlations of particles provide links to quantum optics, where related studies have been made for photons. Trapped atomic clouds have some similarities to atomic nuclei, and insights from nuclear physics have been helpful in understanding their properties.

In presenting this diverse range of topics we have attempted to explain physical phenomena in terms of basic principles. In order to make the presentation self-contained, while keeping the length of the book within reasonable bounds, we have been forced to select some subjects and omit others. For similar reasons and because there now exist review articles with extensive bibliographies, the lists of references following each chapter are far from exhaustive. A valuable source for publications in the field is the archive at Georgia Southern University: <http://amo.phy.gasou.edu/bec.html>

This book originated in a set of lecture notes written for a graduate-level one-semester course on Bose–Einstein condensation at the University of Copenhagen. We have received much inspiration from contacts with our colleagues in both experiment and theory. In particular we thank Gordon Baym and George Kavoulakis for many stimulating and helpful discussions over the past few years. Wolfgang Ketterle kindly provided us with the

cover illustration and Fig. 13.1. The illustrations in the text have been prepared by Janus Schmidt, whom we thank for a pleasant collaboration. It is a pleasure to acknowledge the continuing support of Simon Capelin and Susan Francis at the Cambridge University Press, and the careful copy-editing of the manuscript by Brian Watts.

Copenhagen

Christopher Pethick

Henrik Smith

Contents

<i>Preface</i>	<i>page xi</i>
1 Introduction	1
1.1 Bose–Einstein condensation in atomic clouds	4
1.2 Superfluid ^4He	6
1.3 Other condensates	8
1.4 Overview	10
Problems	13
References	14
2 The non-interacting Bose gas	16
2.1 The Bose distribution	16
2.1.1 Density of states	18
2.2 Transition temperature and condensate fraction	21
2.2.1 Condensate fraction	23
2.3 Density profile and velocity distribution	24
2.3.1 The semi-classical distribution	27
2.4 Thermodynamic quantities	29
2.4.1 Condensed phase	30
2.4.2 Normal phase	32
2.4.3 Specific heat close to T_c	32
2.5 Effect of finite particle number	35
2.6 Lower-dimensional systems	36
Problems	37
References	38
3 Atomic properties	40
3.1 Atomic structure	40
3.2 The Zeeman effect	44

3.3	Response to an electric field	49
3.4	Energy scales	55
	Problems	57
	References	57
4	Trapping and cooling of atoms	58
4.1	Magnetic traps	59
4.1.1	The quadrupole trap	60
4.1.2	The TOP trap	62
4.1.3	Magnetic bottles and the Ioffe–Pritchard trap	64
4.2	Influence of laser light on an atom	67
4.2.1	Forces on an atom in a laser field	71
4.2.2	Optical traps	73
4.3	Laser cooling: the Doppler process	74
4.4	The magneto-optical trap	78
4.5	Sisyphus cooling	81
4.6	Evaporative cooling	90
4.7	Spin-polarized hydrogen	96
	Problems	99
	References	100
5	Interactions between atoms	102
5.1	Interatomic potentials and the van der Waals interaction	103
5.2	Basic scattering theory	107
5.2.1	Effective interactions and the scattering length	111
5.3	Scattering length for a model potential	114
5.4	Scattering between different internal states	120
5.4.1	Inelastic processes	125
5.4.2	Elastic scattering and Feshbach resonances	131
5.5	Determination of scattering lengths	139
5.5.1	Scattering lengths for alkali atoms and hydrogen	142
	Problems	144
	References	144
6	Theory of the condensed state	146
6.1	The Gross–Pitaevskii equation	146
6.2	The ground state for trapped bosons	149
6.2.1	A variational calculation	151
6.2.2	The Thomas–Fermi approximation	154
6.3	Surface structure of clouds	158
6.4	Healing of the condensate wave function	161

Problems	163
References	163
7 Dynamics of the condensate	165
7.1 General formulation	165
7.1.1 The hydrodynamic equations	167
7.2 Elementary excitations	171
7.3 Collective modes in traps	178
7.3.1 Traps with spherical symmetry	179
7.3.2 Anisotropic traps	182
7.3.3 Collective coordinates and the variational method	186
7.4 Surface modes	193
7.5 Free expansion of the condensate	195
7.6 Solitons	196
Problems	201
References	202
8 Microscopic theory of the Bose gas	204
8.1 Excitations in a uniform gas	205
8.1.1 The Bogoliubov transformation	207
8.1.2 Elementary excitations	209
8.2 Excitations in a trapped gas	214
8.2.1 Weak coupling	216
8.3 Non-zero temperature	218
8.3.1 The Hartree–Fock approximation	219
8.3.2 The Popov approximation	225
8.3.3 Excitations in non-uniform gases	226
8.3.4 The semi-classical approximation	228
8.4 Collisional shifts of spectral lines	230
Problems	236
References	237
9 Rotating condensates	238
9.1 Potential flow and quantized circulation	238
9.2 Structure of a single vortex	240
9.2.1 A vortex in a uniform medium	240
9.2.2 A vortex in a trapped cloud	245
9.2.3 Off-axis vortices	247
9.3 Equilibrium of rotating condensates	249
9.3.1 Traps with an axis of symmetry	249
9.3.2 Rotating traps	251

9.4	Vortex motion	254
9.4.1	Force on a vortex line	255
9.5	The weakly-interacting Bose gas under rotation	257
	Problems	261
	References	262
10	Superfluidity	264
10.1	The Landau criterion	265
10.2	The two-component picture	267
10.2.1	Momentum carried by excitations	267
10.2.2	Normal fluid density	268
10.3	Dynamical processes	270
10.4	First and second sound	273
10.5	Interactions between excitations	280
10.5.1	Landau damping	281
	Problems	287
	References	288
11	Trapped clouds at non-zero temperature	289
11.1	Equilibrium properties	290
11.1.1	Energy scales	290
11.1.2	Transition temperature	292
11.1.3	Thermodynamic properties	294
11.2	Collective modes	298
11.2.1	Hydrodynamic modes above T_c	301
11.3	Collisional relaxation above T_c	306
11.3.1	Relaxation of temperature anisotropies	310
11.3.2	Damping of oscillations	315
	Problems	318
	References	319
12	Mixtures and spinor condensates	320
12.1	Mixtures	321
12.1.1	Equilibrium properties	322
12.1.2	Collective modes	326
12.2	Spinor condensates	328
12.2.1	Mean-field description	330
12.2.2	Beyond the mean-field approximation	333
	Problems	335
	References	336

13	Interference and correlations	338
13.1	Interference of two condensates	338
13.1.1	Phase-locked sources	339
13.1.2	Clouds with definite particle number	343
13.2	Density correlations in Bose gases	348
13.3	Coherent matter wave optics	350
13.4	The atom laser	354
13.5	The criterion for Bose–Einstein condensation	355
13.5.1	Fragmented condensates	357
	Problems	359
	References	359
14	Fermions	361
14.1	Equilibrium properties	362
14.2	Effects of interactions	366
14.3	Superfluidity	370
14.3.1	Transition temperature	371
14.3.2	Induced interactions	376
14.3.3	The condensed phase	378
14.4	Boson–fermion mixtures	385
14.4.1	Induced interactions in mixtures	386
14.5	Collective modes of Fermi superfluids	388
	Problems	391
	References	392
	<i>Appendix. Fundamental constants and conversion factors</i>	394
	<i>Index</i>	397

1

Introduction

Bose–Einstein condensates in dilute atomic gases, which were first realized experimentally in 1995 for rubidium [1], sodium [2], and lithium [3], provide unique opportunities for exploring quantum phenomena on a macroscopic scale.¹ These systems differ from ordinary gases, liquids, and solids in a number of respects, as we shall now illustrate by giving typical values of some physical quantities.

The particle density at the centre of a Bose–Einstein condensed atomic cloud is typically 10^{13} – 10^{15} cm^{-3} . By contrast, the density of molecules in air at room temperature and atmospheric pressure is about 10^{19} cm^{-3} . In liquids and solids the density of atoms is of order 10^{22} cm^{-3} , while the density of nucleons in atomic nuclei is about 10^{38} cm^{-3} .

To observe quantum phenomena in such low-density systems, the temperature must be of order 10^{-5} K or less. This may be contrasted with the temperatures at which quantum phenomena occur in solids and liquids. In solids, quantum effects become strong for electrons in metals below the Fermi temperature, which is typically 10^4 – 10^5 K, and for phonons below the Debye temperature, which is typically of order 10^2 K. For the helium liquids, the temperatures required for observing quantum phenomena are of order 1 K. Due to the much higher particle density in atomic nuclei, the corresponding degeneracy temperature is about 10^{11} K.

The path that led in 1995 to the first realization of Bose–Einstein condensation in dilute gases exploited the powerful methods developed over the past quarter of a century for cooling alkali metal atoms by using lasers. Since laser cooling alone cannot produce sufficiently high densities and low temperatures for condensation, it is followed by an evaporative cooling stage, in

¹ Numbers in square brackets are references, to be found at the end of each chapter.

which the more energetic atoms are removed from the trap, thereby cooling the remaining atoms.

Cold gas clouds have many advantages for investigations of quantum phenomena. A major one is that in the Bose–Einstein condensate, essentially all atoms occupy the same quantum state, and the condensate may be described very well in terms of a mean-field theory similar to the Hartree–Fock theory for atoms. This is in marked contrast to liquid ^4He , for which a mean-field approach is inapplicable due to the strong correlations induced by the interaction between the atoms. Although the gases are dilute, interactions play an important role because temperatures are so low, and they give rise to collective phenomena related to those observed in solids, quantum liquids, and nuclei. Experimentally the systems are attractive ones to work with, since they may be manipulated by the use of lasers and magnetic fields. In addition, interactions between atoms may be varied either by using different atomic species, or, for species that have a Feshbach resonance, by changing the strength of an applied magnetic or electric field. A further advantage is that, because of the low density, ‘microscopic’ length scales are so large that the structure of the condensate wave function may be investigated directly by optical means. Finally, real collision processes play little role, and therefore these systems are ideal for studies of interference phenomena and atom optics.

The theoretical prediction of Bose–Einstein condensation dates back more than 75 years. Following the work of Bose on the statistics of photons [4], Einstein considered a gas of non-interacting, massive bosons, and concluded that, below a certain temperature, a finite fraction of the total number of particles would occupy the lowest-energy single-particle state [5]. In 1938 Fritz London suggested the connection between the superfluidity of liquid ^4He and Bose–Einstein condensation [6]. Superfluid liquid ^4He is the prototype Bose–Einstein condensate, and it has played a unique role in the development of physical concepts. However, the interaction between helium atoms is strong, and this reduces the number of atoms in the zero-momentum state even at absolute zero. Consequently it is difficult to measure directly the occupancy of the zero-momentum state. It has been investigated experimentally by neutron scattering measurements of the structure factor at large momentum transfers [7], and the measurements are consistent with a relative occupation of the zero-momentum state of about 0.1 at saturated vapour pressure and about 0.05 near the melting curve [8].

The fact that interactions in liquid helium reduce dramatically the occupancy of the lowest single-particle state led to the search for weakly-interacting Bose gases with a higher condensate fraction. The difficulty with

most substances is that at low temperatures they do not remain gaseous, but form solids, or, in the case of the helium isotopes, liquids, and the effects of interaction thus become large. In other examples atoms first combine to form molecules, which subsequently solidify. As long ago as in 1959 Hecht [9] argued that spin-polarized hydrogen would be a good candidate for a weakly-interacting Bose gas. The attractive interaction between two hydrogen atoms with their electronic spins aligned was then estimated to be so weak that there would be no bound state. Thus a gas of hydrogen atoms in a magnetic field would be stable against formation of molecules and, moreover, would not form a liquid, but remain a gas to arbitrarily low temperatures.

Hecht's paper was before its time and received little attention, but his conclusions were confirmed by Stwalley and Nosanow [10] in 1976, when improved information about interactions between spin-aligned hydrogen atoms was available. These authors also argued that because of interatomic interactions the system would be a superfluid as well as being Bose-Einstein condensed. This latter paper stimulated the quest to realize Bose-Einstein condensation in atomic hydrogen. Initial experimental attempts used a high magnetic field gradient to force hydrogen atoms against a cryogenically cooled surface. In the lowest-energy spin state of the hydrogen atom, the electron spin is aligned opposite the direction of the magnetic field ($H\downarrow$), since then the magnetic moment is in the same direction as the field. Spin-polarized hydrogen was first stabilized by Silvera and Walraven [11]. Interactions of hydrogen with the surface limited the densities achieved in the early experiments, and this prompted the Massachusetts Institute of Technology (MIT) group led by Greytak and Kleppner to develop methods for trapping atoms purely magnetically. In a current-free region, it is impossible to create a local maximum in the magnitude of the magnetic field. To trap atoms by the Zeeman effect it is therefore necessary to work with a state of hydrogen in which the electronic spin is polarized parallel to the magnetic field ($H\uparrow$). Among the techniques developed by this group is that of evaporative cooling of magnetically trapped gases, which has been used as the final stage in all experiments to date to produce a gaseous Bose-Einstein condensate. Since laser cooling is not feasible for hydrogen, the gas is precooled cryogenically. After more than two decades of heroic experimental work, Bose-Einstein condensation of atomic hydrogen was achieved in 1998 [12].

As a consequence of the dramatic advances made in laser cooling of alkali atoms, such atoms became attractive candidates for Bose-Einstein condensation, and they were used in the first successful experiments to produce a gaseous Bose-Einstein condensate. Other atomic species, among them

noble gas atoms in excited states, are also under active investigation, and in 2001 two groups produced condensates of metastable ^4He atoms in the lowest spin-triplet state [13, 14].

The properties of interacting Bose fluids are treated in many texts. The reader will find an illuminating discussion in the volume by Nozières and Pines [15]. A collection of articles on Bose–Einstein condensation in various systems, prior to its discovery in atomic vapours, is given in [16], while more recent theoretical developments have been reviewed in [17]. The 1998 Varenna lectures describe progress in both experiment and theory on Bose–Einstein condensation in atomic gases, and contain in addition historical accounts of the development of the field [18]. For a tutorial review of some concepts basic to an understanding of Bose–Einstein condensation in dilute gases see Ref. [19].

1.1 Bose–Einstein condensation in atomic clouds

Bosons are particles with integer spin. The wave function for a system of identical bosons is symmetric under interchange of any two particles. Unlike fermions, which have half-odd-integer spin and antisymmetric wave functions, bosons may occupy the same single-particle state. An order-of-magnitude estimate of the transition temperature to the Bose–Einstein condensed state may be made from dimensional arguments. For a uniform gas of free particles, the relevant quantities are the particle mass m , the number density n , and the Planck constant $\hbar = 2\pi\hbar$. The only energy that can be formed from \hbar , n , and m is $\hbar^2 n^{2/3}/m$. By dividing this energy by the Boltzmann constant k we obtain an estimate of the condensation temperature T_c ,

$$T_c = C \frac{\hbar^2 n^{2/3}}{mk}. \quad (1.1)$$

Here C is a numerical factor which we shall show in the next chapter to be equal to approximately 3.3. When (1.1) is evaluated for the mass and density appropriate to liquid ^4He at saturated vapour pressure one obtains a transition temperature of approximately 3.13 K, which is close to the temperature below which superfluid phenomena are observed, the so-called lambda point² ($T_\lambda = 2.17$ K at saturated vapour pressure).

An equivalent way of relating the transition temperature to the particle density is to compare the thermal de Broglie wavelength λ_T with the

² The name lambda point derives from the measured shape of the specific heat as a function of temperature, which near the transition resembles the Greek letter λ .