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Geometric and Cohomological Group Theory

Edited by
Peter H. Kropholler, Ian J. Leary,
Conchita Martínez-Pérez
and Brita E. A. Nucinkis

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Geometric and Cohomological Group Theory

Edited by Peter H. Kropholler, Ian J. Leary, Conchita Martínez-Pérez and Brita E. A. Nucinkis

This volume provides state-of-the-art accounts of exciting recent developments in the rapidly-expanding fields of geometric and cohomological group theory. The research articles and surveys collected here demonstrate connections to such diverse areas as geometric and low-dimensional topology, analysis, homological algebra and logic. Topics include various constructions of Thompson-like groups, Wise's theory of special cube complexes, groups with exotic homological properties, the Farrell–Jones assembly conjectures and new applications of Garside structures. Its mixture of surveys and research makes this book an excellent entry point for young researchers as well as a useful reference work for experts in the field. This is the proceedings of the 100th meeting of the London Mathematical Society series of Durham Symposia.

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Kropholler, Leary, Martínez-Pérez and Nucinkis

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Preface

This proceedings volume results from the fourth London Mathematical Society Durham Symposium of an influential series that belongs to the mathematical territory that we now see as part of Geometric Group Theory. Notably it was also the 100th in the entire series of LMS Durham Symposia. The first of these four meetings was held in 1976 organised by Scott and Wall, the second in 1994 organised by Kropholler and Stöhr (with Niblo as an additional editor for the proceedings volume), and the third in 2003 organised by Bridson, Kropholler and Leary. Proceedings volumes for these three meetings appeared in the *London Mathematical Society Lecture Notes* series as volumes 36, 252 and 358, and we are pleased to be able to continue this tradition.

This fourth meeting drew together some 80 mathematicians from around the world. It shared with the earlier meetings the high standards and significance of its main lecture series. These lecture series were delivered by Kai-Uwe Bux, Desi Kochloukova, Jon McCammond, Justin Moore, Piotr Przytycki, and Holger Reich. There was also room in the schedule for individual invited lectures from Azer Akhmedov, Collin Bleak, Brian Bowditch, Martin Bridson, Michael Davis, Ioannis Emmanouil, Dan Farley, Ross Geoghegan, Martin Kassabov, Conchita Martínez-Pérez, Volodymyr Nekrashevych, Nansen Petrosyan, Colva Roney-Dougal, Mark Sapir, Karen Vogtmann, Christian Wegner, John Wilson, Henry Wilton, and Stefan Witzel.

The titles of the four proceedings volumes that have flowed from these symposia have evolved in a way that mirrors the evolution of the subject: over a period of almost four decades, the role of geometry in group theory has grown hugely. This change was evident at the symposium and can also be seen in the present volume.

We thank all the authors who have contributed to this volume. We thank the London Mathematical Society and the Engineering and Physical Sciences Research Council for their support both in terms of advice and financially. The organisational burden that has in the past fallen on the scientific committee associated with an LMS Durham Symposium has now largely been replaced by the very supportive and tireless work of the administrative staff in the Mathematics Department of Durham University and we gratefully acknowledge this contribution, which has gone a long way to making these symposia run smoothly and ensuring that the focus is on the important science at the heart of our work.

The warm and friendly atmosphere of the meeting itself led to many useful interactions and a flow of ideas. We hope that the reader will find some of this excitement is reflected in the present volume.

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Contents

Obstructions for subgroups of Thompson's group V

José Burillo* Sean Cleary* Claas E. Röver*

Abstract

Thompson's group V has a rich variety of subgroups, containing all finite groups, all finitely generated free groups and all finitely generated abelian groups, the finitary permutation group of a countable set, as well as many wreath products and other families of groups. Here, we describe some obstructions for a given group to be a subgroup of V.

1 Introduction

Thompson constructed a finitely presented group now known as V as an early example of a finitely presented infinite simple group. The group V contains a remarkable variety of subgroups, such as the finitary infinite permutation group S_∞, and hence all (countable locally) finite groups, finitely generated free groups, finitely generated abelian groups, Houghton's groups, copies of Thompson's groups F, T and V, and many of their generalizations, such as the groups $G_{n,r}$ constructed by Higman [9]. Moreover, the class of subgroups of V is closed under direct products and restricted wreath products with finite or infinite cyclic top group.

In this short survey, we summarize the development of properties of V focusing on those which prohibit various groups from occurring as subgroups of V.

Thompson's group V has many descriptions. Here, we simply recall that V is the group of right-continuous bijections from the unit interval $[0, 1]$ to itself, which map dyadic rational numbers to dyadic rational numbers, which are differentiable except at finitely many dyadic rational numbers, and with slopes, when defined, integer powers of 2. The elements of this group can

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be described by reduced tree pair diagrams of the type (S, T, π) where π is a bijection between the leaves of the two finite rooted binary trees S and T.

Higman [9] gave a different description of V, which he denoted as $G_{2,1}$ in a family of groups generalizing V.

2 Obstructions

Higman [9] described several important properties of V which can serve as obstructions to subgroups occurring in V.

Theorem 2.1 ([9]) *An element of infinite order in V has only finitely many roots.*

This prevents all Baumslag-Solitar groups $B_{m,n} = \langle a, b \mid a^n b = ba^m \rangle$ from occurring as subgroups of V, if m properly divides n; see [13].

Theorem 2.2 ([9]) *Torsion free abelian subgroups of V are free abelian, and their centralizers have finite index in their normalizers in V.*

This prevents $GL_n(\mathbb{Z})$ from occurring as a subgroup of V for $n \geq 2$.

A group is *torsion locally finite* if every torsion subgroup is locally finite. That is, if every finitely generated torsion subgroup is finite. Röver [12] showed

Theorem 2.3 ([12]) *Thompson's group V is torsion locally finite.*

This rules out many branch groups from occurring as subgroups of V, including the Grigorchuk groups of intermediate growth [7] and the Gupta-Sidki groups [8]. It also rules out Burnside groups.

Holt and Röver [10] showed that V has indexed co-word problem.

Theorem 2.4 ([10]) *The set of words (over an arbitrary but fixed finite generating set) which do not represent the identity in V is an indexed language, and hence can be recognized by a nested-stack automaton.*

This property is not easy to verify, however. But it is inherited by finitely generated subgroups (see [10]), and hence groups which do not have an indexed co-word problem cannot occur as a subgroup of V.

Lehnert and Schweitzer [11] improved this result.

Theorem 2.5 ([11]) *The set of words (over an arbitrary but fixed finite generating set) which do not represent the identity in V is a context-free language, and hence can be recognized by a pushdown automaton.*

Again, this property is inherited by finitely generated subgroups, but the condition is still not easy to verify.

More recently, Bleak and Salaza-Díaz [4] and subsequently Corwin [6], using similar techniques showed

Theorem 2.6 **([4, 6])** *Neither the free product $\mathbb{Z} * \mathbb{Z}^2$ nor the standard restricted wreath product $\mathbb{Z} \wr \mathbb{Z}^2$ with $\mathbb{Z}^2$ as top group are subgroups of V.*

One theorem of Higman [9] together with a metric estimate of Birget [1] gives another obstruction.

Theorem 2.7 **([9])** *For any element v of infinite order in V, there is a power v^n such that for the reduced tree pair diagram (S, T, π) for v^n, there is a leaf i in the source tree S which is paired with a leaf j in the target tree T so that j is a child of of i.*

Theorem 2.8 **([1])** *For any finite generating set of V, There are constants C and C' such that word length $|v|$ of an element of V with respect to that generating set satisfies $Cn \leq |v| \leq C'n \log n$ where n is the size of the reduced tree pair diagram representing v.*

Since the powers of v^n will have length thus growing linearly, these two theorems give as a consequence the following.

Theorem 2.9 *Cyclic subgroups of V are undistorted.*

We note that this argument applies as well to generalizations of V where there is a linear lower bound on word length in terms of the number of carets, such as braided versions of V [5].

This last theorem has an obvious corollary.

Corollary 2.10 *If a group embeds in V, its cyclic subgroups must be undistorted.*

The reason for this is that in a chain of subgroups $G \supset H \supset K$ the distortion of K in H cannot be larger than the distortion of K in G.

We note that Bleak, Bowman, Gordon, Graham, Hughes, Matucci and J. Sapir [3] used Brin's methods of revealing pairs for elements of V to show that cyclic subgroups of V are undistorted.

This result excludes all Baumslag-Solitar groups with $|n| \neq |m|$, as these have distorted cyclic subgroups. It also rules out nilpotent groups which are not virtually abelian. An alternative argument excluding the Baumslag-Solitar groups is due to Bleak, Matucci and Neunhöffer [2].

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