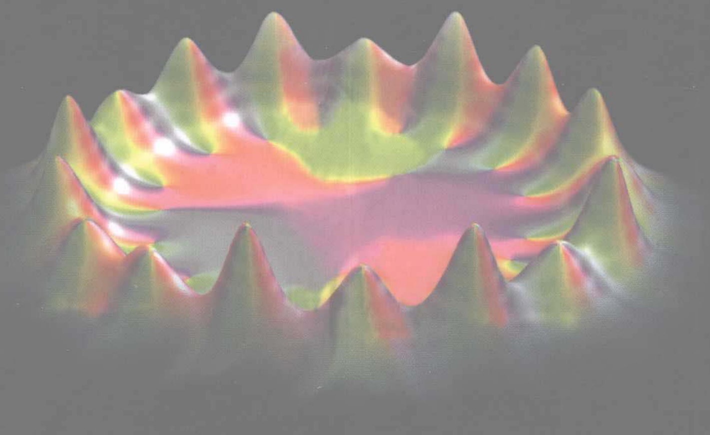
回顾与展望丛书（影印版）

50 YEARS OF YANG-MILLS THEORY

杨－米尔斯理论50年

Edited by Gerardus't Hooft



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内 容 简 介

在杨－米尔斯理论发表50周年之际，这本无价的文集回顾了由这一美妙思想引发出来的基本粒子物理的发展和成就。

在过去的50年，作为理论物理不可否认的最重要的基础，杨－米尔斯理论得到了广阔的发展。从各种视角对这一理论进行的研究，使理论以许多新的、没有预想到的面貌被揭示出来。在最近几十年，从高能物理延伸出去，该理论已经活跃地应用在物理学的其他分支中，诸如统计物理、凝聚态物理和非线性系统等，使这一理论成为所有从事物理学工作的人无法回避的课题。

在这本文集上发表文章或作更详细的技术上说明的是一个国际的专家团队，他们中的每一位专家都曾在这一非凡理论的发展中留下过自己的足迹。这些文章又从各位专家独到的视角凸现了这些新发现。

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Preface

At the time of the publication of this volume, fifty years have passed since the appearance of an article in *The Physical Review* by Chen Ning Yang and Robert L. Mills, entitled "Conservation of Isotopic Spin and Isotopic Gauge Invariance". This book on the one hand serves as a tribute to that monumental piece of work, and on the other intends to show how its subject has evolved since that time, highlighting the landmarks that followed after the original paper emerged, and allowing its authors to indulge in new ideas and concepts. Gauge Theory has indeed grown into a pivotal concept in the Theory of Elementary Particles, and it is expected to play an equally essential role in even more basic theoretical constructions that are speculated upon today, with the aim of providing an all-embracing picture of the universal Laws of Physics.

Some of the chapters in this book are contributions that have appeared elsewhere; most of the contributions are original pieces of work. All are accompanied by brief comments by the Editor. Needless to state that this volume is far from complete. There are numerous well-known landmarks that we could not cover. Furthermore, like most developments in Science, progress not only comes from the relatively small set of papers by famous authors that enjoy enormous scores on citation indices, but it predominantly comes from the large crowds of scientists who confirm and reproduce the original research while adding inconspicuous but essential bits of understanding, not only by writing papers, but also by lecturing to students, by performing experiments and doing calculations. Without them, this book could not have been written.

Contents

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Introduction

Yang and Mills' paper, originally published in *The Physical Review* [1] has been reproduced several times, such as in C. N. Yang's collection of Selected Papers [2]. Let us here briefly summarize the idea.

It had been established that the strong interactions appear to be fairly accurately invariant under isospin transformations. These transformations, which mathematically form the Lie group $SO(3)$, act as rotations in an internal space, such that the three pion states, (π^+, π^0, π^-), form a *vector*, and the nucleons, (p, n) form a *spinor*. If all states involved in a strong interaction process are rotated by the same angle around the same axis in isospin space, the laws of physics are observed to be practically the same as before.

This concept of isospin appears to make sense only if the states *after* an interaction are rotated the same way as the states *before* the interaction. We call such s symmetry a *global* symmetry: the $SO(3)$ rotation has to be performed everywhere in space-time in the same way. Yang and Mills pointed out that this seems odd. "It seems that this is not consistent with the localized field concept that underlies the usual physical theories." At this point, not everyone agrees. It is easy enough to write down equations for perfectly localized fields that show only global, continuous symmetries such as isospin. As often happens with obviously false statements, they were eagerly embraced by some enthusiastic followers; yet this cannot be the real reason why the theory became as important as it is today. I think that what Yang and Mills really wanted to say is this: global symmetries are fine, but could it not be so that there exist more delicate varieties of symmetries? Could one have an isospin-*like* symmetry that allows the $SO(3)$ rotation to be different at different points in space-time? There were at least two examples known of forces in Nature that indeed are associated with *local* symmetries: electromagnetism (where we have a local $U(1)$ invariance), and gravity (where the group of Lorentz transformations is replaced by general, *local* coordinate transformations).

And so it happened that, by asking a rather ill-posed question, Yang and Mills made a momentous discovery: electro-magnetism and gravity are not the only force laws one can write down that have a local symmetry at their basis. One can start with any compact Lie group (of which $SO(3)$ may be regarded as the prototype), and build a generalized theory of electromagnetism, now called Yang–Mills theory.

In electromagnetism, the group of transformations considered consists of multiplying the wave functions $\psi(x)$ for charged particles by an arbitrary phase factor $e^{i\Lambda}$, and if Λ is allowed to be x-dependent, then the gradient of $\Lambda(x)$ is to be added to the vector potential field $A_\mu(x)$. In the Yang–Mills theory, we consider multiplets of fields $\psi^i(x)$, where i stands for some 'internal' index counting different species of particles. The group of gauge transformations consists of that of the unitary transformations

$$\psi^i(x) = S^i_j \psi'^j(x)\,, \tag{1}$$

or a subgroup of these transformations. In the simplest non-trivial case, we have the group $SU(2)$ of transformations on doublet states ($i = 1, 2$).

In analogy with the electro-magnetic case, we modify the field equations for these multiplets of fields, by replacing all gradients $\partial_\mu \psi^i$, wherever they occur in the field equations or in the Lagrangian, by the so-called covariant derivative:

$$D_\mu \psi^i(x) \stackrel{\text{def}}{=} \partial_\mu \psi^i(x) - ieB_{\mu}{}^i_j \psi^j(x)\,, \tag{2}$$

where $B_{\mu}{}^i_j$ is a new set of vector fields, transforming as the adjoint representation under the global gauge transformations. Invariance under *local* gauge transformations requires that

$$(\partial_\mu - ieB_\mu)\psi = S(\partial_\mu - ieB'_\mu)\psi'\,, \tag{3}$$

where we suppressed the indices.

Combining Eqs. (1) and (3), the isotopic gauge transformation on B_μ is obtained:

$$B_\mu = SB'_\mu S^{-1} + \frac{i}{e} S \partial_\mu S^{-1}\,. \tag{4}$$

Noticing that

$$[D_\mu, D_\nu]\psi = -ie(\partial_\mu B_\nu - \partial_\nu B_\mu - ie[B_\mu, B_\nu])\psi\,, \tag{5}$$

one finds that the field combination

$$F_{\mu\nu} = \partial_\nu B_\mu - \partial_\mu B_\nu + ie[B_\mu, B_\nu] \tag{6}$$

transforms covariantly under local gauge transformations:

$$F_{\mu\nu} = SF_{\mu\nu}{}'S^{-1}\,, \tag{7}$$

which of course can also be verified directly by applying Eq. (4). This field looks very similar to the Maxwell field $F_{\mu\nu}$, apart from the commutator term. The commutator term is one of the prime novelties of the Yang–Mills theory.

The B field transforms as the adjoint representation of the global part of the gauge group, so, in the simplest non-Abelian case of $SO(3)$, it forms a triplet. Such an isospin-vector, Lorentz-vector field was new to Yang and Mills, so they turned their attention to the physical significance of this field. One *could* regard it as a mere mathematical artifact; today we would call such a field a 'background field'. They emphasize that this would be physically unacceptable. If these fields exist at all, they must be endowed with dynamical properties and obey field equations. Fortunately, many clues could be found in Maxwell's equations, which we are only too familiar with. Take as our Lagrangian:

$$\mathcal{L} = -\tfrac{1}{4}\,\mathrm{Tr}\,(F_{\mu\nu}F_{\mu\nu})\,. \tag{8}$$

This is by far the simplest expression one can write down to generalize the Maxwell equations in a gauge-invariant way, and one has to accept the presence of the commutator term in Eq. (6). Indeed, when the issue of renormalizability is raised, it is the *only* acceptable kinetic term for the lagrangian. Adding the Dirac Lagrangian for a fermion doublet, with the gradients duly modified into covariant derivatives, one has

$$\mathcal{L} = -\tfrac{1}{4}\,\mathrm{Tr}\,(F_{\mu\nu}F_{\mu\nu}) - \bar{\psi}\gamma_\mu(\partial_\mu - ieB_\mu)\psi - m\bar{\psi}\psi\,. \tag{9}$$

The field equations are

$$\partial_\nu F_{\mu\nu} - ie[B_\nu, F_{\mu\nu}] = -J_\mu\,; \quad J_{\mu i}^{\,j} = ie\bar{\psi}_i\gamma_\mu\psi^j\,, \tag{10}$$

$$\gamma_\mu(\partial_\mu - ieB_\mu)\psi + m\psi = 0\,. \tag{11}$$

It was realized, from the start, that this system of equations should be subject to quantization, and the quanta of the B field should be added to the existing spectrum of elementary particles. The B quanta would be expected to be exchanged between any pair of particles carrying isospin, generating not only a force much like the electro-magnetic force, but also a force that rotates these particles in isospin space, which means that elementary reactions involving the transmutation of particles into their isospin partners will result. A novelty in the Yang–Mills theory was that the B quanta are predicted to interact directly with one another. These interactions originate from the commutator term in the $F_{\mu\nu}$ field in Eq. (10), but one can also understand physically why such interactions have to occur: in contrast with ordinary photons, the Yang–Mills quanta also carry isospin, so they will undergo isospin transitions themselves, and furthermore, some of them are charged, so the neutral components of the Yang–Mills fields cause Coulomb-like interactions between these charged objects.

Two fundamental problems were duly recognized by Yang and Mills in their paper. First, we have the divergences. Primitive Feynman diagrams tend to lead to divergent integrals, so some kind of renormalization procedure is required. This problem was a familiar one, at that time, and it was known that it could be addressed, at least in the electro-magnetic case. At that time, however, it was generally believed that more advanced theories would be developed in the future, where, somehow, the difficulty of the infinities would be avoided altogether. Many theoreticians expected that these smarter theories would completely replace our 'primitive' quantum field theories.

A second problem was a novel one, and it was very disconcerting. The Yang–Mills equations resemble the Maxwell equations a bit too much: just like photons, the Yang–Mills B quanta would be massless particles. It seemed that there simply exists no mass term compatible with local gauge symmetry. The lightest particles with isospin are the pions, and they are copiously produced in high-energy collisions between nucleons. The only limiting factor appears to be the energy required for their production, and an essential part of this energy is in the mass of these pions. If Yang–Mills particles would be massless while carrying isospin, they would have to be produced even more abundantly than the pions. In reality, no such particles are ever seen to be produced. We only have the pions, and it was established, beyond any doubt, that they have zero spin, unlike the Yang–Mills bosons, which should have spin one. Thus, abundant experimental evidence appeared to indicate that Yang–Mills particles do not exist.

The only remedy to this problem appeared to be that, somehow, some dynamical mechanism generates mass terms for the Yang–Mills quanta. If one tries to write down such terms, they invariably violate gauge-invariance. This had already been noted by Pauli, as Yang recalls in his comments added in his collected papers. Could one combine this problem with the first, that of the divergent integrals? Questions of this sort were justly postponed for future generations to investigate. Indeed, we now have most of the answers to these questions, and, with the proper adjustments, Yang–Mills theory is now recognized as an essential ingredient in all our theories for sub-atomic particles. We know that these fields are there, that our theories not only look elegant with these fields incorporated, but that these fields *have* to be included in any system of particles as soon as the interactions tend to become strong.

Indeed, one of the reasons why, up till the early '70s, the notion of quantized fields was rejected by many experts in particle theory, was the so-called Landau ghost. It was the 'certainty' that, when extrapolated to higher energies, the interactions among the fields, due to non-linearities, would explode to infinity. Not only would this render any decent calculation hopelessly complicated, but it would even jeopardize the very foundation of such a theory, since one would have expected that, at least at the very tiniest distance scales, the interactions should be under control, to some extent. Well, today we still think that this objection holds, but only if one excludes Yang–Mills fields. The Yang–Mills field interactions tend to extenuate this divergence, through a mechanism called 'asymptotic freedom'. But, this would not be known for nearly twenty years to come. In 1954, most of those investigators who still did adhere to quantum field theory were either stubborn, or ignorant, or both. Serendipity? Perhaps.

C. N. Yang's Earliest Calculations

Yang now was so kind as to offer copies of his 1947 notes. They were clearly unfinished, and reproduced in Chapter 1. These were the notes of a graduate student still struggling with the concept of gauge invariance, a long way off from the masterpiece of 1954.

Robert L. Mills passed away on October 27, 1999. Mills had developed an excellent reputation as a mathematical physicist while studying at Columbia and at Cambridge University. He was still a PhD student while writing his paper with C. N. Yang. Chapter 1 also contains a letter written by Frank Yang, for *Physics Today*.

[1] C. N. Yang and R. L. Mills, *Phys. Rev.* **95**, 631 (1954).

[2] C. N. Yang, *Collected Papers 1945–1980*, with Commentary, W. H. Freeman and Co., San Francisco, 1983.

Chapter 1

Gauge Invariance and Interactions

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Gerardus 't Hooft wants me to write something about the early origin of non-Abelian gauge theory. I searched through my notes and found a few pages which I now contribute to this volume that he is editing.

These pages were written in March 1947 when I was a graduate student at the University of Chicago. Like graduate students of my generation, I was familiar with Pauli's description of gauge theory in his 1933 *Handbuch der Physik* article [1] and his 1941 review in the *Reviews of Modern Physics* [2], but not very much the 1929 article of Weyl [3].

I was clearly focusing on a very important problem. Unfortunately the mathematical calculations that I had carried out repeatedly in subsequent years I could not find today. They always had ended in more and more complicated formulae and total frustration. It was only in 1953–1954 when Bob Mills and I revisited the problem and tried adding quadratic terms to the field strength $F_{\mu\nu}$ that an elegant theory emerged. For Mills and me it was many years later that we realized the quadratic terms were in fact *natural* from the mathematical viewpoint.

[1] W. Pauli, *Handbuch der Physik* **24** (1933).
[2] W. Pauli, *Rev. Mod. Phys.* **13**, 203 (1941).
[3] H. Weyl, *Zeitschr. f. Phys.* **56**, 330 (1929).

March 47 (VI)

Gauge Invariance And Interact.

I. The gauge invariance in electromagnetic theory serves for two purp

(i) Give rise to the definition of a charge + current density.

(ii) Fix the interaction between an arbitrary field and the photon field, ~~...~~ by the requirement of the gauge invariance.

~~II We can easily~~ By the second purpose, e.g. a real field cannot interact with the photon. (But notice that a complex field that does not undergo a ~~transfor~~ change under a gauge transformat. will not interact with the photon either. e.g. the neutron field belong to this class)

II. We can easily formulate a " theory for meson gauge transformation by requiring that

$$U_k \to U_k e^{i\alpha\varepsilon_k e} \qquad U_k^* \to U_k^* e^{-i\alpha\varepsilon_k e}$$

and get a ~~meson~~ charge & current" density when α is a constant, (ε_k = meson charge number of the particle U_k)

In order to fix the interact. between an arbitrary field & the meson field a gauge transformat. of the second kind for the meson field is necessary. This can be done in the case of neutral mesons by writing

Incorrect ? see III

$$\mathcal{L} = -\frac{1}{2} F_{\nu\mu}^* F_{\nu\mu} - K^2 U_\nu^* U_\nu + \partial A_\nu^* M_\nu \quad \cancel{\ldots} \quad -\frac{2}{K} B M_{\nu\nu}$$

where A_ν is Stueckelberg's potential (cf. Pauli R.M.P.) & regard A_ν & B as independent variables. Gauge transf.: $A_\nu \to A_\nu + \frac{1}{K} f_{,\nu}$, $B \to B - Kf$.

For charged mesons, the theory perhaps works too, with α = an operator not commutable with T_3 (isotopic spin).

III. Notice that for photon field, $\varepsilon_k = 0$ or 1 because all particles are either uncharged or have charge e in photon field. But for meson field $\frac{g}{e} = \varepsilon_k \cong 3$. if U_k = heavy particle field

$\frac{g}{e} = \varepsilon_k \cong 10^{-8}$ if U_k = light particle field.

IV. The gauge transformation described in II will not lead to any fixation of the interaction because the field part of the L and the particle part are respectively invariant under the transformation, while in the e.m. case they ~~are~~ both vary.

This constitutes perhaps a fundamental difference between a field with $\kappa = 0$ & one with $\kappa \neq 0$.

$\boxed{\text{Masses of Particles}}$

V. The present theory cannot give a satisfactory account of the masses of the different particles because of infinite difficulties and incompleteness of the theory. e.g. We cannot expect to be able to derive the mass ratio of the proton & the electron, or of the proton & the neutron on the present theory.

In a completely satisfactory theory there should be ~~three~~ only three universal constants: $\hbar$, c, G (gravitational const.) ~~and all other~~ which in natural units are all 1. The mass or compton wave length of the particles can be derived.

We see that a theory of such kind is necessarily a merge-together of the Q. theory & general relativity (in which the masses of the particles are defined)

Point worth noticing: if electron radius = range of meson force, $M_{meson} = 137\, m$!